A CRITICAL EDITION, ENGLISH TRANSLATION AND COMMENTARY

of the

Upodghāta, Ṣaḍvidhaprakaraṇa and Kuṭṭakādhikāra

of

THE *SŪRYAPRAKĀŚA*

of

SŪRYADĀSA

(A Commentary on Bhāskarācārya's *Bījagaṇita*)

by

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A Critical Edition, English Translation and Commentary of the Upodghāta, Ṣaḍvidhaprakaraṇa and Kuṭṭakādhikāra of the Sūryaprakāśa of Sūryadāsa (A Commentary on Bhāskarācārya's Bījagaṇita)

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ABSTRACT

The present study is a critical edition and translation into English of the first three chapters, the upodghāta, the ṣaḍvidhaprakaraṇa and the kuṭṭakādhikāra, of the Sanskrit commentary, the *SŪRYAPRAKĀŚA* (सूर्यप्रकाश), of SŪRYADĀSA (सूर्यदास) on BHĀSKARA's (भास्कर) Sanskrit classic on algebra, the *BĪJAGAṆITA* (बीजगणित) written in the A.D. 1140's.

This is the first edition and the first translation of this work. The edition of this portion of the text, which constitutes about a third of the commentary, is a large step toward an edition of the entire commentary. A mathematical and historical commentary is also included in this thesis.

Bhāskara (b. A.D. 1114), a native of Vijjaḍaviḍa in the Sahyādris, was one of the most renowned Indian astronomers and mathematicians. His works were held in high esteem and studied in India for many centuries. The first known commentary on his *Bījagaṇita* (which was also a standard textbook on algebra) is the *Sūryaprakāśa*, written in Śaka 1460 (A.D. 1538) by Sūryadāsa (or Sūrya Paṇḍita), a native of Pārthapura near the confluence of the Godāvarī and Vidarbhā rivers. In the *Sūryaprakāśa*, Sūryadāsa explains every verse and solves almost every example of Bhāskara's text in order to teach the student how to apply the underlying rules (or sūtras).

The mathematical content of the portion which has been edited is as follows: arithmetical operations involving positive and negative numbers, zero, colours (unknowns), and karaṇī (surds); and the kuṭṭaka (pulverizer), which involves the solution of indeterminate equations of the first degree.

To establish a critical edition of the first three chapters of the text of Sūryadāsa's commentary, twelve manuscripts (out of some twenty-four listed in catalogues, some of which may no longer be extant) have been collated, and an Apparatus Criticus prepared.

On the basis of a comparison of the readings of these twelve manuscripts, via the Apparatus Criticus, a stemma has been drawn in which the relationships of the manuscripts to each other and to the archetypes are established. On the basis of these relationships it has been possible to constitute a text, namely the *Text Alpha*, that is close to the original *Sūryaprakāśa*. However, it is not likely to be exactly Sūryadāsa's original text since the reconstruction of the original is in principle impossible.

DEDICATION

In loving memory of my father.

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ABBREVIATIONS OF JOURNALS, SERIALS AND SANSKRIT TEXTS

The following abbreviations will be used throughout the thesis.[1]

AB	*Āryabhaṭīya*
ABB	*Āryabhaṭīya-Bhāṣya*
AR	*Asiatic (k) Researches* (continues as *JAS Bengal*)
Archeion	*Archeion*
ASS	*Ānandāśrama Sanskrit Series*
BCMS	*Bulletin of the Calcutta Mathematical Society*
BenSS	*Benares Sanskrit Series*
BG	*Bījagaṇita*
BGV	*Bījagaṇitāvataṃsa*
BMAUA	*Bulletin of the Mathematical Association of the University of Allahabad*
BP	*Bījapallava*
BSS	*Brāhmasphuṭasiddhānta*
Centaurus	*Centaurus*
CESS	*Census of the Exact Sciences in Sanskrit*
CORS	*Chaukhambha Oriental Research Studies*
CPG	*Caukhambhā Prācyavidyā Granthamālā*
DSB	*Dictionary of Scientific Biography*
EI	*Epigraphia Indica*
G	*Gaṇita*
GB	*Gaṇita Bhāratī*

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1. Complete citations, including the editions of the ancient Sanskrit texts will be found among the references.

GD	*Golādhyāya*
GG	*Grahagaṇitādhyāya*
GK	*Gaṇitakaumudī*
GMK	*Gaṇitāmṛtakūpikā*
GOS	*Gaekwad's Oriental Series*
GPV	*Gaṇitapañcaviṃśī*
GSS	*Gaṇitasārasaṅgraha*
HM	*Historia Mathematica*
HSS	*Haridas Sanskrit Series*
IJHS	*Indian Journal of the History of Science*
JAS Bengal	*Journal of the Asiatic Society of Bengal*
JDL/UC	*Journal of the Department of Letters, University of Calcutta*
JIMS	*Journal of the Indian Mathematical Society*
JIMS/NQ	*Journal of the Indian Mathematical Society: Notes and Questions*
JJG	*Jīvarāja Jaina Granthamālā*
JOI Baroda	*Journal of the Oriental Institute, Baroda*
JSHS	*Japanese Studies in the History of Science*
JUB	*Journal of the University of Bombay*
KHKH	*Khaṇḍakhādyaka*
KK	*Karaṇakutūhala*
KSS	*Kāśī Sanskrit Series*
L	*Līlāvatī*
LB	*Laghubhāskarīya*
M	*Mathematica*
Madras GOS	*Madras Government Oriental Series*
MB	*Mahābhāskarīya*
ME	*Mathematics Education*

MK	*Mitākṣarā*
MS	*Mahāsiddhānta*
PG	*Pāṭīgaṇita*
PO	*Poona Orientalist*
PWSBT	*Princess of Wales Sarasvatī Bhavana Texts*
RMS	*Russian Mathematical Surveys*
SB	*Siddha Bhāratī*
SDVT	*Śiṣyadhīvṛddhidatantra*
SM	*Scripta Mathematica*
SP	*Sūryaprakāśa*
SSE	*Siddhāntaśekhara*
SSI	*Siddhāntaśiromaṇi*
SSU	*Siddhāntasundara*
TS	*Triśatikā*
TSMS	*Tanjore Saraswathi Mahal Series*
VB	*Vāsanābhāṣya*
VIS	*Vishveshvaranand Indological Series*

CHAPTER I

INTRODUCTION

1. *Sūryadāsa (1507 – 1588 A.D.*[2] *) and His Work*

Sūryadāsa is also known as Sūrya Paṇḍita, Sūryadeva, Sūrya Kavi, Sūrya Gaṇaka, Sūrya Sūri, Daivajña Paṇḍita Sūrya, Daivajña Sūrya Paṇḍita, Ācārya Sūrya or simply, Sūrya. These names are revealed in the verses at the end of his compositions. He was the son and pupil of the astronomer Jñānarāja (fl. 1503 A.D.), who was the son of the astronomer Nāganātha (fl. ca. 1480 A.D.).

A. *Family Background and Native Place*

Sūryadāsa comes from a celebrated family of the Bhāradvājagotra which was settled at Pārthapura on the Godāvarī and flourished from ca. 1300 A.D. (Pingree, 1981b). Pārthapura has been identified with Pathri in the Parbhani District about eighty miles southeast of Devagiri and was under the Muslim rule for the better part of the medieval period. This family produced several prominent scholars and possessed a family library. Some of these scholars wrote works which were sequels to Bhāskara's works (p. 120).

For example, Jñānarāja wrote an astronomical course entitled *Siddhāntasundara* or *Sundarasiddhānta* in 1503 A.D. (Pingree, 1976). This work was commented on by his son Cintāmaṇi (fl. ca. 1530 A.D.). Also, Jñānarāja wrote an algebraical supplement, the *Bījādhyāya*, to the *Siddhāntasundara* (*CESS A 3*, pp. 75a, 76b). The *Bījādhyāya* has been cited a few times by his son Sūryadāsa, as will be seen in our commentary on the *Text Alpha* which we have edited.

Shankar Balkrishna Dikshit, a modern historian of Indian mathematics, has prepared a lineage of this family with information provided by Kāśīnātha Śāstrī (b. 1846 A.D.), a surviving member of the family, in Śaka 1817 or A.D. 1895 (Pingree, 1976,

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2. See "Jñānarāja" in Pingree, David. (1976). *CESS A 3*. Philadelphia: American Philosophical Society. P. 75a.

CESS A 3, pp. 75a-b). The interested readers may refer to Professor Pingree's *CESS A 3*, 1976, pp. 75a-b, and the *Jyotiḥśāstra*, 1981, p. 124, Table 9.

B. *Sūryadāsa's Works*

Sūryadāsa was a versatile genius who wrote on a wide variety of topics (Sarma, 1946). He was not only a great astronomer, but also a great poet. Furthermore, he composed philosophic works and commented on the Vedas. His commentary, the *Sūryaprakāśa*, is the first known commentary on the *Bījagaṇita* of Bhāskara. It was written in Śaka 1460 (A.D. 1538) when he was only 31 years old. This indicates that Sūryadāsa was born in 1507 – 08 A.D. (*PO 11*, p. 54).

Kāśīnātha Śāstrī, the descendant of Sūryadāsa who provided information about his lineage to Shankar Balkrishna Dikshit, has also provided a list of works composed by Sūryadāsa. The following fourteen works of Sūryadāsa have been mentioned by Dikshit (Sarma, 1950, *SB VIS 2*, pp. 222-223):

1. *Bhāskarīya-bījabhāṣya (Sūrya-prakāśa)*
2. *Līlāvatī-ṭīkā (Gaṇitāmṛtakūpikā)*
3. *Śrīpatipaddhati-gaṇita*
4. *Bīja-gaṇita*
5. *Tājika-grantha (Tājikālaṅkāra)*
6-7. *Kāvya-dvaya* (names not given)
8. *Bodha-sudhākara*
9. *Padyāmṛta-taraṅgiṇī*
10. *Rāmakṛṣṇaviloma-kāvya*
11. *Śaṅkarābharaṇa*
12. *Nṛsiṃha-campū*
13. *Vighna-mocana*
14. *Bhagavatī-gītā*

The above list is, however, not exhaustive, because Colebrooke, Aufrecht and Sarma have recorded some other works as well, some of which have not been seen by any other scholars (Sarma, 1950). Sarma has found and edited a manuscript of another work composed by Sūryadāsa, which was also recorded by Colebrooke (and then by Aufrecht), and is entitled *Siddhānta-saṃhitā-sāra-samuccaya*. This work contains, among other topics, a discussion on gravity and astronomical interpretations of some of the Vedic mantras (*SB VIS 2*, pp. 223, 225).

Another work, which was unknown to other scholars and was found and edited by Sarma (1946), is the *Bhāskara-bhūṣaṇa (Bhāskarābharaṇa)* of Sūrya Paṇḍita written in 1572 A.D. It contains 101 verses. They are astronomical and devotional and are addressed to the Sun as the deity. The last verse of this work contains an allusion to another eight poems written by Sūryadāsa as follows—three on the deity Viṣṇu, one each on the deities Śiva, Sūrya and Gaṇapati and two on the deity Gaurī (*PO 11*, pp. 54, 66):

यः पञ्चायतनप्रसादविधये काव्यान्यथाष्टौ व्यधात्
तत्र त्रीणि हरेर्हरस्य तरणेरेकैकमम्बाजनेः।
द्वे गौर्याः परिवर्णने स विधिवत्सूर्याभिधानः कविः
काव्यं भास्करभूषणाख्यमकरोज्ज्ञानाधिराजात्मजः ॥१०१॥

The translation is: The poet named Sūrya, the son of Jñānādhirāja, who, in conformity to prescribed rules, wrote eight poems as an action of favour to the five seats (i.e. the deities), (wrote) three (poems) in the description of Viṣṇu, one each of Śiva, of the Sun, of Gaṇeśa, (and) two of Gaurī. (In addition,) he (Sūrya) composed one poem called the *Bhāskarabhūṣaṇa*.

Sarma (1950) maintains that a reference to the above eight poems written by Sūryadāsa exists also in his *Bīja-gaṇita*, (which is the fourth work in the list given by Dikshit,) as follows (*SB VIS 2*, p. 222):

टीके वासनयान्विते गणितयोर्लीलावतीबीजयो-
स्तद्वच्छ्रीपतिपद्धतेश्च गणितं बीजं तथैकं व्यधात्।
एतत्ताजिकमच्युतार्थमपरं काव्याष्टकं प्रौढधीः
सूर्यो बोधसुधाकराख्यमकरोदध्यात्मशास्त्रेऽपरम् ॥

The translation is: Sūrya, of lofty intellect, wrote two commentaries together with demonstration(s) pertaining to the two (types of) mathematics in the *Līlāvatī* and the *Bījagaṇita*; and likewise, (he composed) the *gaṇita* of the *Śrīpatipaddhati,* and a *Bīja*. (Furthermore), he composed the (work called the) *Tājika(-grantha)* for the sake of God Viṣṇu and another (work called) the *Kāvyāṣṭaka* (i.e. an octad of poems). He composed another (work) called the *Bodhasudhākara* on the science of self (ātman).

Sūryadāsa's exceptional skill in poetic compositions is revealed by his poem *Rāmakṛṣṇaviloma-kāvya* (the tenth work in Dikshit's list). In this poem, the second line of each verse is obtained by reversing the order of the syllables in the first line. Furthermore, the first lines relate incidents in the life of Lord Rāma, the second lines those of Lord Kṛṣṇa. For example (see Miśra, 1970, *HSS 288*, p. 2):

तं भूसुतामुक्तिमुदारहासं वन्दे यतो भव्यभवं दयाश्रीः।
श्रीयादवं भव्यभतोयदेवं संहारदामुक्तिमुतासुभूतम् ॥१॥

The first line, which is in the direction of Lord Rāma, means: I salute him (Lord Rāma), who is the source of the liberation of the daughter of the earth (i.e. Sītā, whom Lord Rāma freed from the clutches of Rāvaṇa), who has a broad smile, (and) who is the source of (all) existence, compassion and wealth.

The second line, which is in the direction of Lord Kṛṣṇa, means: I salute Śrī Yādava (the superhuman power, Kṛṣṇa), who is the god of the glorious (planet) Venus and of water, who is the source of emancipation of the one who was the giver of destruction (i.e. Pūtana), (and) who is all life.

In addition to the above works, Sūryadāsa wrote a few commentaries which are considered to be fine pieces of work. Among his Vedic commentaries are those on the *Ṛk*, *Yajus* and *Sāma Vedas* (Sarma, 1950, *SB VIS 2*, p. 224).

Among Sūryadāsa's mathematical commentaries are those on Bhāskara's works: the *Sūryaprakāśa* (written in 1538 A.D. on the *Bījagaṇita*) and the *Gaṇitāmṛtakūpikā* (or *Amṛtakūpikā* written in 1541 A.D. on the *Līlāvatī*). (Nos. 1 and 2 in Dikshit's list.)

C. *Meaning of the Word Commentary*

One of the dictionary meanings of the word 'commentary' is (Woolf, 1979, p. 223b): "a systematic series of explanations or interpretations (as of a writing)." This basically describes the *Sūryaprakāśa.*

Radhakrishna Sastri (1958), an editor of the commentary *Bījapallava* on Bhāskara's *Bījagaṇita* (which was composed by Kṛṣṇa in ca. 1600 A.D.), related that the author of the original text gives only the general enunciations in the mūlagrantha (original textbook). A commentary, which consists of the explanatory statements and demonstrations (not rigorous proofs as found, for instance, in Euclidean geometry) of the general enunciations, is generally written by a disciple or an earnest scholar of the subject. Usually the "demonstrations" are merely verifications (by examples) in order to understand the text correctly. The process of analysis is usually absent from these writings. Medieval Indian mathematics did not contain a system of rigorous proof. The disciples received their education in the gurukulas where the gurus transmitted their experiences to their disciples. Thus it was only necessary that the texts contain the rules and not the processes by which they were obtained or proved. (Preface, pp. ii-iii)

D. *The Distinctive Features of Sūryadāsa's Commentaries on Bhāskara's Works*

Sūryadāsa's commentaries are clear interpretations of Bhāskara's text. They contain concise explanations and demonstrations of the 'sūtras' (rules), in addition to the solutions of most of the examples. Furthermore, Sūrya's commentaries contain systematic expositions. These and some other facts will be seen in reading of the *Sūryaprakāśa*, a brief summary of which follows.

E. *The Sūryaprakāśa*

This commentary begins with a 'maṅgalācaraṇa' (auspicious introduction) which contains religious tributes to the deities Gaṇeśa and Sarasvatī, and to the elders. These tributes are followed by six verses, which are in turn followed by a homage to Brahman which is the unmanifest all-pervading spirit of the universe.

The six verses are peculiar in the sense that they contain double (i.e. poetical and mathematical) meanings. This use of paronomasia is a distinctive feature of Sanskrit writings in general, and Sūryadāsa's writings in particular. In these six verses, Sūryadāsa pays homage to: (i) Gaṇapati (which implies Śiva and Gaṇeśa); (ii) Kṛṣṇa (which implies the Supreme Spirit and algebra); (iii) Jñānarāja (which implies Sūryadāsa's father and guru who taught pāṭī, kuṭṭaka and bīja to Sūrya); (iv) the rising of 'sūrya' (which implies the rising of the Sun or Bhāskara who is the author of the *Bījagaṇita*) which is a destroyer of confusion and thus a symbol of inspiration and knowledge; (v) the bīja (i.e. 'seed,' which implies computation of the unmanifest i.e. algebra and the Supreme Spirit because the symbolic single syllables employed in algebra are as hard to grasp as the first syllable of a mantra); and (vi) the commentary *Sūryaprakāśa* (which, according to Sūryadāsa, is a boat of liberation for those whose souls seek emancipation by crossing the worldly ocean and thus by merging in the unmanifest Supreme Spirit, and on the other hand it implies a means of learning tedious methods of algebra for those who are bewildered and desire to cross the ocean of algebra).

After these six verses, Sūryadāsa pays homage to the unmanifest Supreme Spirit, which is known as Brahman. Sūryadāsa maintains that Brahman assumed a body in the form of Brahmā as a favour to the entire universal creation. Then Brahmā created Jyotiḥśāstra which is the foremost of all the sciences (āgamas) and aṅgas (of the Veda). Also, Brahmā created the sun, the radiance of the rays of which destroys the darkness caused by night. On the other hand, Brahmā created scholars such as Bhāskara and

Sūryadāsa, so that the world could be uplifted by their teachings when it had nearly been destroyed by the power of the Kaliyuga.

Thus, Sūryadāsa's maṅgalācaraṇa reveals his deep sense of devotion for the Supreme Spirit, and his father and guru Jñānarāja. In addition, the extreme importance attached by Sūryadāsa to algebra and to kuṭṭaka in particular is also apparent, as is his use of pun.

Next Sūryadāsa introduces the first verse of Bhāskara's *Bījagaṇita*, which happens to be Bhāskara's verse of maṅgalācaraṇa. Sūryadāsa explains this verse at length. Sūryadāsa's commentary on this verse shows his great scholarship. Sūryadāsa refers to Sāṅkhya philosophy, āgamas and śāstras (i.e. traditional doctrines and teachings), smṛti (tradition), Nyāyaśāstra (logic), Pāṇini's *Vyākaraṇa* (grammar), the unmanifest and manifest, computation (i.e. mathematics) of the unmanifest and manifest, the deity Gaṇapati, Bhāskara's *Siddhāntaśiromaṇi* and Jñānarāja's *Siddhāntasundara*.

Now, as regards Sūryadāsa's style of presentation, Sūryadāsa follows a logical and consistent program of exposition and explanation throughout his *Sūryaprakāśa* (and his *Gaṇitāmṛtakūpikā*). Having provided the necessary background or introduction before each verse of Bhāskara's mūla (text), Sūryadāsa quotes the lemma pertaining to each verse of the mūla. Then Sūryadāsa supplies the necessary explanations. His presentations of the sūtras (and sometimes of the examples) are generally divided into three parts which are marked by the following grammatical indicators:

(i). Part 1 is the syntactic connection, i.e. a rearrangement of the words of a verse in prose form. This part usually ends in 'iti saṃbandhaḥ' (इति संबंधः) though sometimes in 'ityanvayaḥ' (इत्यन्वयः).

(ii). Part 2 explains the meaning of the verse. It ends in 'ityarthaḥ' (इत्यर्थः).

(iii). Part 3 deals with the demonstration of the sūtras contained in a verse. This part ends in 'ityupapannam' (इत्युपपन्नम्).

Sometimes there is another part between parts 2 and 3 which has the indicator 'ityāśaṅkyāha' (इत्याशंक्याह, i.e., suspecting that ... he states ...). In this part, Sūryadāsa states and explains the alternative statements made by Bhāskara in order to support the ones he has already discussed (for example, see Sūryadāsa's commentary on Bhāskara's verse 1 of the *Bījagaṇita*).

In his demonstration, Sūryadāsa quite often refers to some sūtras of the mūla other than the one on which he is commenting. In some situations, Sūryadāsa makes reference to or quotes from some works of Bhāskara other than the *Bījagaṇita*. At times, Sūryadāsa supplies his own example. Allusion is also made by Sūryadāsa to the works of authors other than Bhāskara, as will be seen later, in the sources used by Sūryadāsa (section 1.G. below). Sūryadāsa's demonstrations are clear and skillful.

Generally Sūryadāsa explains a sūtra completely. But in case of an example, sometimes he writes: "It is clear" (see *Text Alpha*, verse 8a-d); "It has a clear meaning" (verse 9c-d); "It has a clear meaning. It is also exemplified in the demonstration" (verse 6c-7b); "It all has a clear meaning" (verse 15a-b); "The whole has a clear meaning, and is understood from the book" (verse 5a-b). Sūryadāsa does so presumably when he thinks that an example is too trivial to spend time on or when the solution of an example has already been discussed by Bhāskara in the mūla.

Moreover, Sūryadāsa discusses the examples of the mūla in order to elucidate and apply the principles underlying the various sūtras. The sūtras which are being used in the solution of a problem are also articulated by Sūrya.

In the beginning of each sub-section, Sūryadāsa provides a brief introduction about the topic to be covered in it. Similarly, at the end, the name of the topic, which was covered, is provided.

Sūryadāsa marks the end of his commentary on various chapters of Bhāskara's text by the verses of 'upasaṃhāra' (colophon) and 'upaupasaṃhāra' (post-colophon), for example, see Sūrya's verses after his commentary on Bhāskara's verses 45b-46a and

67a-d, respectively. These verses contain the name of the author of the commentary, Sūrya, the name of Sūrya's father, Sūrya's qualifications, the name and subject of the commentary, the name of the work on which the commentary is written and the name of the author of that work, and the titles of the topics discussed in a particular chapter.

Sarma (1950) has mentioned that there exists a verse at the end of Sūryadāsa's two works entitled the *Sūryaprakāśa* and the *Gaṇitāmṛtakūpikā* respectively, which gives the first eight works mentioned by Dikshit (*SB VIS 2*, p. 222). Obviously Sarma is referring to the following verse:

> व्याख्ये वासनयान्विते गणितयोर्लीलावतीबीजयो-
> स्तद्वच्छ्रीपतिपद्धतेश्च गणितं बीजं तथैकं व्यधात्।
> एकं ताजिकमच्युतार्थमपरं काव्यद्वयं प्रौढधीः
> सूर्यो बोधसुधाकराख्यमकरोदध्यात्मशास्त्रेऽष्टमम् ॥७॥

This verse may be translated as: Sūrya, of lofty intellect, wrote two commentaries together with demonstration(s) pertaining to the two (types of) mathematics in the *Līlāvatī* and the *Bījagaṇita*; and likewise, (he composed) the *gaṇita* of the *Śrīpatipaddhati*, and a *Bīja*. (Furthermore), he composed one (work called) the *Tājika(-grantha)* for the sake of God Viṣṇu and another (work called) the *Kāvyadvaya* (i.e. a pair of poems). He composed the eighth (work) called the *Bodhasudhākara* on the science of self (ātman).

But this verse is missing from some of the manuscripts of the *Sūryaprakāśa* which we have collated. This fact will support a claim concerning Sūryadāsa's revising an earlier version of his text of the *Sūryaprakāśa* and thus creating more than one recension. (See Chapter I, section 3.C.(c) below.)

F. *The Relationship of Sūryadāsa's Text to Bhāskara's Text and to the Text of the Later Commentator Kṛṣṇa*

For the mūla, i.e. Bhāskara's *Bījagaṇita*, we have used the edition entitled *Bījagaṇita: A treatise on algebra by Bhāskarācārya,* which is edited by Jībānanda Vidyāsāgara, 1878, Calcutta. This edition contains Bhāskara's own commentary, in addition to Bhāskara's verses. It is this edition on which we have based the numbering of the verses pertaining to the mūla in our *Text Alpha* which we have edited, because Sūryadāsa does not assign any numbering to these verses. Vidyāsāgara's edition is not a critical edition in the modern sense. This editor does not record any variant readings. Indeed, it is clear that there exists no critical edition of Bhāskara's *Bījagaṇita.*

The other available text containing the verses of Bhāskara's *Bījagaṇita* is the text with the commentary of Kṛṣṇa (ca. 1600 A.D.) (Pingree, 1971, 1981a). Kṛṣṇa was patronized by the Mughal Emperor Jahāngīr, and belonged to the Devarātragotra of Dadhigrāma on the Payoṣṇī river (i.e. the modern Tāptī river, or more correctly, its tributary the Purnā river which flows from the Vindhya mountains). Kṛṣṇa's commentary is entitled *Bījāṅkura* or *Navāṅkura* or *Bījapallava* or *Kalpalatāvatāra* or *Bījalīlāvatī*. There are two editions (and hence two versions) of this text of Kṛṣṇa: (i) edition by Āpaṭe, Dattātreya, entitled *Bhāskarīyabījagaṇitam with the vyākhyā Navāṅkura of Kṛṣṇa*, published as *ASS 99*, Poona, 1930; (ii) edition by Radhakrishna Sastri, T. V., entitled *Bījapallavam with introduction by T. V. Radhakrishna Sastri*, published as *Madras GOS 67, TSMS 78*, Madras-Tanjore, 1958. (*CESS A 2*, pp. 53a-b, 54b; *CESS A 4*, p. 311b)

For Kṛṣṇa's commentary, we have chosen the edition by Radhakrishna Sastri (unless otherwise stated) because it seems to be based on the earliest known manuscript of Kṛṣṇa. It is based on the manuscript *Tanjore D 11523,* which was copied at Kāśī by Tryambaka on April 11, 1601 in Kṛṣṇa's lifetime (Pingree, 1971, *CESS A 2*, pp. 53b, 54b; Pingree, 1981a, *CESS A 4*, p. 308b). On the other hand, the edition by Āpaṭe is based on the manuscripts (Pingree, 1971): (i) *India Office 2830* which was copied at Prayāga in A.D. 1704; (ii) *BORI (Bhandarkar Oriental Research Institute) 287 of Vishrambag I* which was copied on February 7, 1826; (iii) *Ānandāśrama 2005;* (iv) *Ānandāśrama 4357* and (v) a *Benares manuscript*. The last three manuscripts were copied in the eighteenth or nineteenth century (*CESS A 2*, pp. 53b-54b). Therefore the text of the edition by Radhakrishna Sastri (1958) is more authentic than that of the edition by Āpaṭe (1930). Also the edition by Radhakrishna Sastri has a better layout and has fewer mistakes than the edition by Āpaṭe. These two editions contain different numberings for the same verses pertaining to Bhāskara's *Bījagaṇita*.

Now with respect to the recensions of Bhāskara's *Bījagaṇita*, it appears that the recension used by Sūryadāsa is different from that used by Vidyāsāgara or Kṛṣṇa, because there are some differences in the readings at some places. For example, in verse 48b, Vidyāsāgara's reading is संज्ञितौ (*BG*, p. 26) but Sūrya (see *Text Alpha*) and Kṛṣṇa (*BP*, 26d, p. 87) have the reading संज्ञकौ। Similarly, in verse 51a where Vidyāsāgara reads यदागतौ (*BG*, p. 27), Sūrya (*Text Alpha*) and Kṛṣṇa (*BP*, 29c, p. 89) read यथागतौ। Likewise, in verse 27b, Vidyāsāgara's reading is गुणोऽथवा (*BG*, p. 13, and the same is in Āpaṭe, 1930, *ASS 99*, p. 39), Kṛṣṇa's reading is गुण्योऽथवा (*BP*, 13d, p. 56), but Sūrya (*Text Alpha*) reads गुण्ये अथवा which is correct in the light of the solution. (Note that the multiplicand and multiplier given by these mathematicians are formed with the same karaṇīs.)

That these mathematicians used different recensions of Bhāskara's *Bījagaṇita* is also suggested by the difference in the order of some of the verses. For instance, in the *Sūryaprakāśa*, the explanation of verse 53a (i.e. Vidyāsāgara's ५२ऽऽ) is followed by that of 54b, but Vidyāsāgara's arrangement is different (see *BG*, p. 27) as is evident from the numbering of these verses. In this regard, Sūrya's arrangement is more logical because it preserves continuity of the context. Here again Kṛṣṇa (see *BP*, 32a-b, p. 108) follows Sūrya. On the other hand, in terms of Vidyāsāgara's numbering, Kṛṣṇa states verse 53b-

54a before 52c (see *BP*, 31a-b, p. 106; 31c-d, p. 108), whereas Sūryadāsa and Vidyāsāgara keep the opposite order. Kṛṣṇa's order seems less logical due to the fact that in any problem involving kuṭṭaka, according to Bhāskara's method, the quotient and multiplier for the positive case are to be found first (which involve the requirement of equal results in division of the pair of numbers by their respective takṣaṇas). The subtraction (of the quotient and the multiplier) from respective takṣaṇas for the purposes (of obtaining solutions in case) of a negative additive or a negative dividend is done afterwards.

As will be seen in our commentary, *Text Alpha* has a lacuna at some places. Some of these lacunae indicate that Sūryadāsa had a manuscript of Bhāskara's *Bījagaṇita* which missed some verses partly or completely but contained Bhāskara's commentary on them. Presumably, it is for this reason that Sūryadāsa's text does not have a lemma for the verse consisting of lines 53a and 54b, nor a lemma for verse 67a-d; but Sūryadāsa does provide a correct solution to the problem given by verse 67a-d.

Since there is occasional difficulty in locating Bhāskara's mūla in the text of the *Sūryaprakāśa*, it appears that Sūryadāsa was using an imperfect manuscript of Bhāskara's *Bījagaṇita*. Furthermore, in view of the differences in the readings as well as in the order of some of the verses of the mūla, one can safely conclude that Sūryadāsa, Kṛṣṇa and Vidyāsāgara were not using the same recension of Bhāskara's *Bījagaṇita* text.

As far as Sūryadāsa's presentation in relation to that of Bhāskara is concerned, Sūryadāsa is following Bhāskara's text closely. Not only did he use the verses of Bhāskara, but also he took prose examples and prose commentaries of Bhāskara. Also he borrowed expressions such as 'nyāsaḥ' and 'yoge jātam'. (Bhāskara is not the inventor of these expressions because they occur already in earlier works, such as the *Bakhshālī Manuscript*, the works of Śrīdhara etc.) One may see the similarities in the commentaries of Bhāskara pertaining to verses *BG*, 3c-4b, p. 1; *BG*, 57c-58b, pp. 28-29; *BG*, 61c-62a, p. 34, and those of Sūryadāsa pertaining to the same verses.

It is also evident that Sūryadāsa avoids minute details if they exist in Bhāskara's text. For example, in the solution pertaining to verse 58c-59b, Sūryadāsa skips the pair of numbers 2430, 1530 which Bhāskara has recorded (*BG*, p. 29). Also, as described before (see our summary of the *Sūryaprakāśa*), Sūryadāsa does not spend time in solving problems which he thinks to be self-explanatory or straightforward or which are trivial and have already been solved by Bhāskara. In the latter case, sometimes Sūryadāsa refers his students to Bhāskara's *Bījagaṇita* and thus expects them to achieve understanding themselves, as in the case of Bhāskara's *BG*, 5a-b, p. 2 which Bhāskara has already solved. For this verse, Sūryadāsa's commentary is: "The whole has a clear meaning, and is understood from the book." Kṛṣṇa explains this problem along Bhāskara's lines and also in terms of Eastern and Western regions (see *BP*, without number, pp. 14-15). Likewise, Sūryadāsa does not solve problems pertaining to verses 6a-7b and 8a-d, while Bhāskara (*BG*, pp. 2-4) and Kṛṣṇa (*BP*, 2, pp. 15-16; 3, pp. 18-19; 4, p. 20) do. Sūryadāsa discusses only one case pertaining to the example following the demonstration to verse 19a-d, and leaves the remaining cases to the reader. Kṛṣṇa discusses all of them very briefly (*BP*, pp. 42-43), as does Bhāskara (*BG*, pp. 8-9). As another illustration, after stating *BG*, 21b, p. 9, Bhāskara discusses in one sentence (*BG*, p. 10, 1-2) the square-root pertaining to the problem in *BG*, 20a-b, p. 9 (though square-root is not required in this problem). Sūryadāsa gives only a hint concerning verse 20a-b, but he makes no reference to the square-root. Kṛṣṇa discusses this square-root in detail (*BP*, pp. 44-45). Further, with reference to verse 58c-59b, Sūryadāsa omits the last case when the dividend is 10, divisor is 7 and additive is 1. Bhāskara discusses this case as well (*BG*, p. 31) and so does Kṛṣṇa (*BP*, p. 114). As another illustration, having solved the problem given in *BG*, 59c-60b, p. 32, Bhāskara adds further discussion (*BG*, p. 33). Kṛṣṇa (*BP*, 24, p. 115) elaborates on Bhāskara's discussion. But after giving the solution of the problem given in 59c-60b, Sūryadāsa says: "The remainder, which is clear, is understood from the treatise also."

On the other hand, normally, Sūryadāsa expands on and supplements Bhāskara's explanations. For example, for the first problem given by *BG*, 25c-d, p. 12, Bhāskara gives only answers, but Sūryadāsa supplies detailed solutions. Further, for *BG*, 59c-60b, p. 32 and for the example subsequent to *BG*, 63c-64b, p. 37 (in the setting out of which the dividend is 17, additive is 1 and divisor is 15), Bhāskara does not supply the chains of quotients, while Sūryadāsa does. Moreover, in order to explain *BG*, 64c-65d, p. 38, Sūryadāsa gives an example, likely composed by himself. In this example, Sūryadāsa discusses: how to determine the remainder of seconds when the revolutions, civil days and elapsed days of a planet are given (to be 3, 11 and 3 respectively). Conversely, how to find the position of a planet from the remainder of seconds. Kṛṣṇa too discusses a similar example (*BP*, pp. 123-127). But Bhāskara (*BG*, p. 39, 1) only mentions that examples are in the Tripraśnādhyāya. (The notation "p. 39, 1" in the last sentence means "page 39, line 1". This form will be used generally for Sanskrit texts.)

From this comparison between the commentaries of Sūryadāsa and Kṛṣṇa, it is evident that Sūryadāsa's explanations are concise while Kṛṣṇa's are generally detailed. Sūryadāsa skips some solutions if they are self-evident or exist in Bhāskara's mūla, but Kṛṣṇa tends to solve almost every problem of the mūla. Though the grammatical indicators used by Sūryadāsa in his expositions (as of verse 5c-d) do not exist in Kṛṣṇa's commentary, yet, because of many similarities (see section 1.I.(ii) of this chapter), it seems that Kṛṣṇa does look at the *Sūryaprakāśa*. On the other hand, Kṛṣṇa seems to have made some changes. For instance (in terms of Vidyāsāgara's numbering), Kṛṣṇa's 5c is followed by 6a-b, which is followed by 5d (see *BP*, pp. 15, 18); perhaps because the problem given in 6a-b involves an application of the sūtra given in 5c only.

G. *The Sources Used by Sūryadāsa*

Sūryadāsa mentions the following works in his *Sūryaprakāśa*:

(i). The *Bījagaṇita* of Bhāskara—Sūryadāsa has used the verses and prose commentaries from this work.

(ii). The Āgamas and Śāstras—Sūryadāsa mentions these sources in his commentary on Bhāskara's verse 1. (See *Text Alpha*, Preface.)

(iii). The Smṛtis—From this source, Sūryadāsa has cited the verse about devotion to one's teacher as to one's god. (See *Text Alpha*, Preface.)

(iv). The *Vyākaraṇa* of Pāṇini—Rules 4, 1, 83; 4, 2, 59; 3, 1, 136 and 3, 3, 106 have been cited from this grammar. (See *Text Alpha*, Preface.)

(v). The *Golādhyāya* of Bhāskara's *Siddhāntaśiromaṇi*—Sūryadāsa has quoted the first verse from this work (see e.g. Āpaṭe, 1943, *GD I*, *ASS 122*, p. 21). In this verse, Bhāskara describes the creation of the universe, which is similar to that described by the Sāṅkhya philosophers (see *Text Alpha*, Preface).

(vi). The *Siddhāntasundara* of Jñānarāja—Sūryadāsa quotes verse 9a pertaining to Prakṛti and Puruṣa (see *Text Alpha*, Preface), from the section Bhuvanakośa of the *Golādhyāya* part of this astronomical course.

(vii). The *Bījādhyāya* of Jñānarāja—Sūryadāsa quotes the rule pertaining to the approximate square-root of a non-square number from this algebraical supplement (see e.g. the manuscript *Berlin 833*, f. 3v., 10-13), in his commentary on verse 44b-45a of the mūla.

(viii). The *Grahagaṇitādhyāya* of Bhāskara's *Siddhāntaśiromaṇi*—Allusions to verse 64 of *GG I* (see Āpaṭe, 1939, *ASS 110*, p. 125) and to verse 13 of *GG II* (see Āpaṭe, 1941, *ASS 110*, p. 86) seem to have been made by Sūryadāsa in his demonstrations pertaining to verse 3a-b of Bhāskara's *Bījagaṇita* and the unnumbered verse following Bhāskara's verse 3a-b, respectively. Also verse 4a from this work (see Āpaṭe, 1939, *GG I*, *ASS 110*, p. 30) has been quoted for demonstrating the sūtras in Bhāskara's verses 64c-65d.

(ix). The *Līlāvatī* of Bhāskara—The verses cited by Sūryadāsa from this source include (see e.g. Āpaṭe, 1937, *L I, ASS 107)*:

verse 18a, p. 18 - in Sūryadāsa's commentary on verse 5d (of the mūla)

verse 19a, p. 19 - in Sūryadāsa's commentary on verse 7c-d and verse 31c-32d

verse 14b, p. 14 - in Sūryadāsa's commentary on verse 17c-d

verse 22a, p. 21 - in Sūryadāsa's commentary on verse 27c-28b

verse 56, p. 54 - in Sūryadāsa's commentary on verse 33a-34d

(x). The *Bījagaṇitāvataṃsa* of Nārāyaṇa Paṇḍita (fl. 1356 A.D.)—Verse 14, p. 6 and verse 25a, p. 13 (see Shukla, 1970) have been quoted from this work while Sūryadāsa comments on verses 10a-b and 23c-24b respectively, of the mūla.

(xi). The *Thirteenth Book of the Mahābhārata*—Sūryadāsa cites verse 135, 11 from this source (see e.g. Dandekar, 1966, *Vol. 17, Part II*, p. 705) in his commentary on the verse 11a-d of the mūla.

(xii). The *Amarakośa* of Amarasiṃha—Sūryadāsa has cited a saying from this work in his commentary on verse 12a-d of the mūla.

(xiii). The *Gaṇita* of Sūryadāsa—After his commentary on verse 50c-51b of the mūla, Sūryadāsa has given nine concise verses of his own creation pertaining to the subject of kuṭṭaka. Sūryadāsa cites the ninth verse again, in his example which follows verses 64c-65d of the mūla as well as in his commentary, the *Gaṇitāmṛtakūpikā* (see the manuscript *Wai, Prājña Pāṭhaśālā Maṇḍala (PPM) 9762*, f. 118v., 5-6). In the latter, Sūryadāsa mentions that he has quoted this (ninth) verse from his *Gaṇita* as follows: तदुक्तमस्माभिः स्वगणिते ।

लब्धयो विषमा यत्र क्षेपशुद्धिर्भवेद्यदि ।
यौ तत्र लब्धिगुणकौ तावेव हि परिस्फुटाविति ॥

Possibly Sūryadāsa is referring to his own *Bīja-gaṇita* listed above, though Colebrooke (1817) mentions a distinct work on calculation by Sūryadāsa, entitled *Gaṇita-mālatī* (see p. xxv). This latter work has also been noticed by Aufrecht but no other scholar appears to have seen it (Sarma, 1950, *SB VIS 2*, p. 223).

(xiv). The *Algebra* of Śrīdhara (ca. eighth century A.D.)—Sūryadāsa mentions this source in his commentary on the section called madhyamāharaṇa as follows (see e.g. the manuscript *India Office (IO), London, 2825 (789)*, f. 82v., 6-9; or the manuscript *Stadtsbibliothek, Berlin, 832*, f. 85v., 2-5): अथ अव्यक्तवर्गादि यदावशेषमिति सूत्रक्रमात्पक्षौ केनचित्संगुण्य किंचित्क्षिप्य मूलं ग्राह्यमिति प्राप्ते केन गुणनीयं किं वा क्षेप्यमिति मुग्धछात्राणां संदेहो वृत्तस्तद्व्याकरणार्थं श्रीधरोक्तं सूत्रं लिख्यते।

चतुराहतवर्गसमै रूपैः पक्षद्वयं गुणयेत्।
अव्यक्तवर्गरूपैर्युक्तौ पक्षौ ततो मूलमिति ॥

H. *The Innovations Made by Sūryadāsa (Through Kuṭṭaka)*

Surely there exist similarities between the commentaries of Bhāskara and Sūryadāsa, however, Sūryadāsa did introduce novelties, some of which are the following:

(i). Sūryadāsa's organization of his succinct exposition into various parts which bear special designations: "(syntactic) connection" (i.e. rearrangement of the words of a verse in the prose form), "meaning" (of a verse along with the meanings of various technical terms contained in it), "demonstration" (of the principles underlying a verse), and sometimes "suspecting that" (there may be the possibility of some alternative interpretation pertaining to the verse under discussion), "he states" (the alternative explanation).

(ii). Sūryadāsa's use of the approximate square-roots of karaṇīs 8 and 2 (in the sexagesimal system), to demonstrate the validity of the rule concerning the sum and difference of two karaṇīs in Bhāskara's verse 23c-24b (see *Text Alpha*, Sūryadāsa's demonstration under Bhāskara's verse 24c-25b).

(iii). Sūryadāsa's use of a unique term 'rūḍha,' when he comments with reference to verse 46b-47b (see *Text Alpha)*: "अत्र कुट्टक इति रूढः शब्दः" which means: "Here "pulverizer" is a conventional word" (that is, it refers, by convention, to a mathematical process).

(iv). The nine verses of Sūryadāsa's own creation, after his commentary on verse 50c-51b (see *Text Alpha)*, as mentioned before.

(v). Sūryadāsa's introduction of an example and its solution in connection with his commentary on verses 64c-65d (see *Text Alpha*). A reference is made to this example in the *Gaṇitāmṛtakūpikā* by Sūryadāsa as follows (see e.g. the manuscript *Wai, PPM 9762*, f. 121v., 9): तद्बीजभाष्ये सोदाहरणत्वेन व्याख्यातमतोऽत्र संक्षिप्योक्तम्।

I. *The Later Uses of the Sūryaprakāśa*

(i). Use by Sūryadāsa himself—For example, Sūryadāsa quotes a part of his commentary on verse 10a-b of Bhāskara's mūla in his *Gaṇitāmṛtakūpikā* (*Wai, PPM 9762*, f. 21v., 5) as follows: तदुक्तं बीजभाष्ये। शून्यस्य स्वातंत्र्येण संख्याविषयत्वाभावादिति भाव इति।

(ii). Use by the Commentator Kṛṣṇa—Kṛṣṇa's *Bījapallava* does not contain (direct) citations from Sūryadāsa's *Sūryaprakāśa*. Nonetheless, there exist several striking similarities in the style of exposition in these two commentaries. Like Sūryadāsa, Kṛṣṇa too provides the necessary background or introduction before each verse of Bhāskara's mūla. After that Sūryadāsa gives only the lemma (see section E. above) but Kṛṣṇa gives the complete verse. Sūryadāsa's presentations of the sūtras are generally divided into three parts while those of Kṛṣṇa contain only two parts. Part 1 (in which Sūryadāsa describes the syntactic connection) does not exist in the *Bījapallava*. Parts 2 and 3 are present in the *Bījapallava* though Kṛṣṇa does not end these parts using the grammatical indicators of Sūryadāsa (except in a very few places; see *BP*, p. 24, 17 and p. 109, 13). On the other hand, Kṛṣṇa usually begins part 2 with अस्यार्थः whereas Sūryadāsa says either nothing or sometimes अयमर्थः। Both commentators begin part 3 with the indicator अत्रोपपत्तिः। Occasionally Kṛṣṇa includes another part between parts 2 and 3 as does Sūryadāsa. Following Sūryadāsa, Kṛṣṇa begins this part with the clause ननु, but does not make use of Sūryadāsa's indicator इत्याशंक्याह। Furthermore, in order to introduce a citation, Sūryadāsa states अत उक्तं or उक्तं च or simply अथ (see Sūryadāsa's commentary on verse 27c-d in *Text Alpha*), while Kṛṣṇa employs the expression एतदुक्तं भवति (see *BP*, p. 15, 17). At the end of the citation, Sūryadāsa variously uses इत्युक्तत्वात् , इत्यादिना, इत्यनेन, इति प्रकारेण, and इत्यादिसूत्रक्रमात्। Here Kṛṣṇa uses either just इति or one of the expressions of Sūryadāsa.

Further evidence that Kṛṣṇa has used the *Sūryaprakāśa* is supplied by the fact that Kṛṣṇa has given an example which is similar to the one given by Sūryadāsa in order to explain verses 64c-65d of Bhāskara's mūla. The only difference is that Kṛṣṇa chooses the constants 9, 19, 13 (see *BP*, p. 123, 22) instead of Sūryadāsa's 3, 11, 3.

Like Sūryadāsa, Kṛṣṇa mentions Śrīdhara in his *Navāṅkura* as follows (see Āpaṭe, 1930, *ASS 99*, p. 139, 19-22): तत्र केन पक्षौ गुणनीयौ किं वा तयोः क्षेप्यमिति बालावबोधार्थं श्रीधराचार्यकृतमुपायं दर्शयति—

> चतुराहतवर्गसमै रूपैः पक्षद्वयं गुणयेत्।
> पूर्वाव्यक्तस्य कृतेः समरूपाणि क्षिपेत्तयोरेव इति ॥

It is interesting to note the differences between the second line here and that of the same quote given by Sūryadāsa (see the end of section G. above).

Thus from the above discussion, it is clearly possible that Kṛṣṇa has been influenced by the *Sūryaprakāśa*.

(iii). Transmission to Scribes—Some twenty-four manuscripts of the *Sūryaprakāśa* (some of which may no longer be extant) are known to have been written by scribes.

(iv). Future Work for Historians of Indian Mathematics and Astronomy—For example, Colebrooke (1817) mentions both *Sūryaprakāśa* and *Gaṇitāmṛtakūpikā* in his Dissertation (p. xxvi).

(v). Extracts from the *Sūryaprakāśa* in the Works of Other Commentators—Parts of the *Sūryaprakāśa* on Bhāskara's verse 1 are found in the commentary *Vimlā* which is written in Sanskrit and Hindi. This commentary is on Bhāskara's *Bījagaṇita* and was composed by Acyutānanda Jhā in 1949 A.D. The following contains some of these extracts in the *Vimlā* (see Jhā, 1949, *KSS 148*, pp. 3-4):

> अहं बुद्धेरीशं गणाधिपतिं वंदे। नमस्करोमीत्यर्थः। ... तं किंभूतम् ? कृत्स्नस्य व्यक्तस्योत्पादकं कर्तारमिति, समस्तस्य व्यक्तस्य स्थूलस्य कार्यस्य भूभूधरादेरुत्पत्तिकारकमित्यर्थः।

> अथ "संख्यावान्पंडितः कवि"रित्यमरोक्तेः सांख्याः कवयो यदव्यक्तं तत्तेन सत्पुरुषेणाधिष्ठितं प्रवदन्ति, अव्यक्तममूर्तं व्योमादिकं येन व्याप्तमिति, अस्यायमर्थः। जायमानं कार्यं कर्तारमाक्षिपतीति न्यायः, यथा क्षित्यादिकं सकर्तृकं कार्यत्वाद्घटवदिति तत्र कर्ता च परेश एव। ... कथंभूतं तं गणितम्। ... पुनः किंभूतमेकबीजमिति। एकं बीजमक्षरं यस्य सः। तथा तम् एकाक्षरगणपतिमंत्राभिप्रायेणैतदुक्तमिति ध्येयम्।

> उत्पादकं। उत्पादयतीत्युत्पादकः पिता। तमुत्पादकं पितरं वंदे। कथंभूतं तं बुद्धेरीशम्। बुद्धेरिति पंचम्यर्थबलाद्धशत्वोपस्थितौ ज्ञानवशादपीशमिति। तथा च ज्ञानहेतुतया गुरुत्वे व्यवस्थिते नतिरपि तस्य युक्तेति। ... पितुः गुरुत्वमभिव्यक्तीकृतमेव ... सांख्याः संख्यानं

गणनम्। तच्छीलाः सांख्या ज्यौतिषिकाः। यदव्यक्तगणितं बीजाख्यं तत्तेन सत्पुरुषेणाधिष्ठितं प्रवदंति। ... अव्यक्तं कथंभूतम्। कृत्स्नस्य व्यक्तस्य पाटीगणितस्यैकबीजमुपजीव्यमिति यावत्।

(vi). Use by Various Institutions—For example, the Oriental Institute, Baroda; the Prājña Pāṭhaśālā Maṇḍala, Wai; the Wellcome Institute for the History of Medicine, London, England. Some of our manuscripts of the *Sūryaprakāśa* were procured from these institutes (see section 3.A. of this chapter, Overview of the Manuscripts).

2. *Bhāskara (b. 1114 A.D.) and His Work*

Bhāskara, also known as Bhāskarācārya (ācārya meaning teacher, learned, venerable), was the most renowned Indian astronomer and mathematician of the medieval period. He wrote his excellent astronomical treatise, the *Siddhāntaśiromaṇi*, in the thirty-sixth year of his life (Śāstrī, 1893). He was the son of a very great Paṇḍita (i.e. scholar) as well as a poet, an excellent astronomer, and an expert scientist, named Maheśvara. Maheśvara had attained the title of 'ācārya' in the assembly of the Paṇḍitas. Bhāskara had learned all the sciences through his father Maheśvara (*JAS Bengal 62*, p. 224).

A. *Bhāskara's Date of Birth*

Bhāskara himself gives his date of birth in one of the concluding verses of the *Golādhyāya II* of his *Siddhāntaśiromaṇi* as follows (see e.g. Āpaṭe, 1952, *GD II* Praśnādhyāya, 58, *ASS 122*, p. 520):

रसगुणपूर्णमही १०३६ समशकनृपसमयेऽभवन्ममोत्पत्तिः।
रसगुण ३६ वर्षेण मया सिद्धान्तशिरोमणी रचितः ॥५८॥

This verse says that Bhāskara was born in the year 1036 of the Śaka Era (which began 78 years after the Christian Era), that is in 1114 A.D. He wrote his *Siddhāntaśiromaṇi* at the age of 35 in the year 1072 of the Śaka Era.

B. *Family Background and Native Place*

In verses 61 and 62 (pp. 522-523) of the previously mentioned *GD II* Praśnādhyāya, *ASS 122*, Bhāskara says:

आसीत् सह्यकुलाचलाश्रितपुरे त्रैविद्यविद्वज्जने
नानासज्जनधाम्नि विज्जडविडे शाण्डिल्यगोत्रो द्विजः।

श्रौतस्मार्तविचारसारचतुरो निःशेषविद्यानिधिः
साधूनामवधिर्महेश्वरकृती दैवज्ञचूडामणिः ॥६१॥

तज्जस्तच्चरणारविन्दयुगलप्राप्तप्रसादः सुधी-
र्मुग्धोद्बोधकरं विदग्धगणकप्रीतिप्रदं प्रस्फुटम्।
एतद्व्यक्तसदुक्तियुक्तिबहुलं हेलावगम्यं विदां
सिद्धान्तग्रथनं कुबुद्धिमथनं चक्रे कविर्भास्करः ॥६२॥

These verses inform us, among other things, that Bhāskara's father Maheśvara was a Brāhmaṇa of the Śāṇḍilyagotra, and was well-versed in the Vedas and the Śrutis. Further, Maheśvara was a native of the city of Vijjaḍaviḍa in the Sahyādris (a part of the Western Ghāts, north of the river Godāvarī and south of the river Tāptī). Having touched his father's feet, that is, having obtained his father's blessings, Bhāskara composed his *Siddhāntaśiromaṇi.*

Bhāskara's lineage consists of generations of prominent scholars some of whom had close connections with local political powers. This is revealed by an inscription which was installed by Bhāskara's grandson Caṅgadeva in a village known as Pāṭṇā near Chalisgaon in East Khandesh in the state of Mahārāṣṭra (Pingree, 1976, *CESS A 3*, p. 39b). This inscription is still extant in the temple of Bhavānī in Pāṭṇā (Miśra, 1979). The inscription was discovered and edited (in 1865 A.D.) by the scholar Dr. Bhāu Dājī, who was a native of Kailash (*CPG 13*, p. 2).

The inscription was reedited by F. Kielhorn in the *Epigraphia Indica 1*, 1892, pp. 338-346 (Pingree, 1970a). In addition to Bhāskara's genealogy, the inscription records that King Soīdeva endowed, on 9 August 1207, a maṭha (i.e. educational institution) for the study of Bhāskara's works to its founder Caṅgadeva. In the inscription, Bhāskara's genealogy begins with a kavicakravartī (i.e. emperor of poets) called Trivikrama who belonged to the Śāṇḍilyagotra. His son Bhāskara Bhaṭṭa was given the title of 'Vidyāpati' by the Paramāra king named Bhojarāja who ruled over Dhārā from ca. 995 A.D. to ca. 1056 A.D. The representatives of the next generations in succession were Govinda, Prabhākara, Manoratha, and Maheśvara (Bhāskara II's father). Bhāskara's son Lakṣmīdhara was the chief of the Paṇḍitas in the court of the Yādava king, Jaitrapāla (1191 – 1209 A.D.). Lakṣmīdhara's son Caṅgadeva (fl. ca. 1200/1220 A.D.) was the chief astrologer to the Yādava king, Siṅghaṇa, who ruled over Devagiri from 1209/1210 A.D. (*DSB* 2, pp. 115a-b)

For a quick glance at Bhāskara's genealogy, the reader may refer to Professor Pingree's *Jyotiḥśāstra*, 1981, p. 124, Table 8.

Note: Above and in what follows, unless otherwise specified, the name Bhāskara means Bhāskara II (b. 1114 A.D.). The "II" normally appears in the historical literature because of an earlier mathematician, Bhāskara I, not related to Bhāskara, who lived about 600 A.D.

C. *Bhāskara's Education*

Bhāskara studied mathematics, astronomy, astrology, philosophy, literature, poetry and religion. The following verse, which is added anonymously to the end of the *Līlāvatī*, describes the several facets of Bhāskara's education and his keen intellect (see Sharma, 1987, p. 211):

अष्टौ व्याकरणानि षट् च भिषजां व्याचष्ट ताः संहिताः
षट् तर्कान्गणितानि पंच चतुरो वेदानधीते स्म यः।
रत्नानां त्रितयं द्वयं च बुबुधे मीमांसयोरन्तरं
सद्ब्रह्मैकमगाध बोधमहिमा सोऽस्याः कविर्भास्करः ॥

This verse states, in essence, that the poet Bhāskara could explain the eight (kinds of) Vyākaraṇas (i.e. grammars) and the six Saṃhitās (i.e. compendiums) of medical science.

He had studied the six (kinds of) Logic, the five (branches of) Gaṇita and the four Vedas. Furthermore, he understood the difference between the two (branches of) Mīmāṃsās (i.e. a system of Indian philosophy which deals with the correct interpretation of the Vedic texts and the Vedic rituals). He recognized the three-fold and two-fold of "jewels". He understood the supreme reality which is Brahman (i.e. the essence from which all created things are produced and into which they are absorbed). He was the one who had profound knowledge and glory.

As mentioned in the beginning, Bhāskara studied these sciences with his father (who was also his guru) Maheśvara or Maheśvara Upādhyāya (teacher). Maheśvara, who flourished about 1114 A.D., was the author of (i) the *Karaṇaśekhara*, (ii) the *Pratiṣṭhāvidhidīpaka*, (iii) the *Vṛttaśataka*, and (iv) the *Laghujātakaṭīkā*; of which only the last two survive (Pingree, 1981a). The *Vṛttaśataka* is a phalagrantha (book on astrological results), while the *Laghujātakaṭīkā* is a commentary on the *Laghujātaka*, of the astronomer Varāhamihira who flourished about 550 A.D. (*CESS A 4*, pp. 397b-398b). The great wisdom of Maheśvara is reflected in the tribute paid to him by Bhāskara in the conclusion of his *Bījagaṇita* (see Vidyāsāgara, 1878, *BG*, 207, p. 162):

आसीन्महेश्वर इति प्रथितः पृथिव्या-
माचार्यवर्यपदवीं विदुषां प्रपन्नः।
लब्ध्वावबोधकलिकां तत एव चक्रे
तज्जेन बीजगणितं लघु भास्करेण ॥२०७॥

Here Bhāskara proclaims that his father earned the epithet "best of the ācāryas of the wise." Having obtained a minute quantity of knowledge from him, Bhāskara made *Bījagaṇita* easy. Another verse, which describes some of the attributes of Maheśvara, is the concluding verse of adhyāya 10 of Bhāskara's work entitled *Karaṇakutūhala* (see Purohita, 1989, *KK*, 4, p. 110):

आसीत्सज्जनधाम्नि गेहविवरे शाण्डिल्यगोत्रो द्विजः
श्रौतस्मार्तविचारसारचतुरः सौजन्यरत्नाकरः ।
ज्योतिर्वित्तिलको महेश्वर इति ख्यातः क्षितौ स्वैर्गुणै-
स्तत्सूनुः करणं कुतूहलमिदं चक्रे कविर्भास्करः ॥४॥

This verse proclaims that Maheśvara was the best of the astronomers, in addition to proclaiming his various other qualifications, religion and native place.

Furthermore, the verses 61-62 of Bhāskara's *Golādhyāya II* Praśnādhyāya, *ASS 122*, which were stated before, describe similar qualities of Maheśvara as well as Bhāskara's gratitude to his father.

D. *Bhāskara's Works*

Bhāskara composed six major works, which may be listed as follows (Pingree, 1970a):

(1). *Vivaraṇa* on the *Śiṣyadhīvṛddhidatantra* of Lalla (fl. eighth century A.D.).

(2). *Līlāvatī* or *Pāṭīgaṇita.*

(3). *Bījagaṇita.*

(4). *Siddhāntaśiromaṇi* (1150 A.D.).

(5). *Vāsanābhāṣya* on the *Siddhāntaśiromaṇi.*

(6). *Karaṇakutūhala* (1183 A.D.).

(*DSB* 2, p. 115b)

Now we discuss some details pertaining to the above works:

(1). The *Vivaraṇa* of Bhāskara on Lalla's *SDVT* has been recently studied and published.

(2). The *Līlāvatī*, which is often considered as the first part of the *Siddhāntaśiromaṇi*, usually consists of twenty-one prakaraṇas (divisions) (Pingree, 1981a). The contents of the *Līlāvatī* include arithmetic, geometry, and one chapter in algebra on the topic of kuṭṭaka. There exist hundreds of manuscript copies and many editions of the Sanskrit text of the *Līlāvatī*. Numerous commentaries have been written on this work in Sanskrit. In addition, there exist commentaries in other Indian languages. These include Hindi, Gujarātī, Marāṭhī and Telugu. The *Līlāvatī* has been translated into Hindi, Kaṇṇaḍa, Persian and English. (*CESS A 4*, pp. 299b-308a)

References to verses of *Bhāskara's Līlāvatī* in our thesis are from Dattātreya Āpaṭe's (1937) edition, *ASS 107*, 2 vols., Poona (unless otherwise stated). This edition contains *Bhāskara's* mūla as well as the ṭīkās *Buddhivilāsinī* (written in 1545 A.D. by Gaṇeśa) and *Līlāvatīvivaraṇa* (written in 1587 A.D. by Mahīdhara). Gaṇeśa (b. 1507 A.D.) was a native of Nandigrāma in Gujarat (Pingree, 1971). He was one of the most renowned astronomers of the sixteenth century (*CESS A 2*, p. 94a). On the other hand, Mahīdhara, who flourished from 1585 A.D. to 1599 A.D., composed a large number of works on a wide variety of subjects at Vārāṇasī (Pingree, 1981a, *CESS A 4*, p. 390a).

(3). The *Bījagaṇita* is often considered as the second part of the *Siddhāntaśiromaṇi*. It is divided into thirteen adhyāyas (chapters) which contain: the six-fold operation of positive and negative quantities, zero, unknowns and karaṇī (surds); the indeterminate equations of the first degree (kuṭṭaka) and second degree separately; linear and quadratic equations; linear and quadratic equations having more than one unknown; operations with products of several unknowns; a section about the author Bhāskara and his work.

The Sanskrit text of the *Bījagaṇita*, like that of the *Līlāvatī*, has been published many times (Pingree, 1970a, 1981a, 1981b). But the commentaries on the *Bījagaṇita* are far fewer than those on the *Līlāvatī* (perhaps because the *Bījagaṇita* is more difficult than the *Līlāvatī*). The commentaries on the *Bījagaṇita* include: *Sūryaprakāśa* (1538 A.D.) of

Sūryadāsa of Pārthapura; *Navāṅkura* (ca. 1600 A.D.) of Kṛṣṇa of Dadhigrāma (Vārāṇasī); *Bījavivaraṇa* (1639 A.D.) of Vīreśvara of Pārthapura; *Śiśubodhana* (1652 A.D.) of Bhāskara of Rājagiri; *Bījaprabodha* (1687 A.D.) of Rāmakṛṣṇa of Jalapura (Amarāvatī, in the Sahyādris); *Vāsanābhāṣya* of Haridāsa (before 1725 A.D.); *Bālabodhinī* (1792 A.D.) of Kṛpārāma of Ahmadābād; a commentary by Jīvanātha Jhā (fl. ca. 1846/1900 A.D.); *Bījālavāla* of Nijānanda; *Kalpalatā* of Paramaśukla (perhaps Kṛṣṇa's work?). (*DSB 2*, p. 117a; *CESS A 4*, p. 308b; pp. 63-64)

Besides the commentaries just mentioned, there exist commentaries with a few editions (not critical editions) of the *Bījagaṇita*. Some of these editions are as follows (Pingree, 1981a): edition of Jīvanātha Jhā with his own Sanskrit commentary, Benares, 1885; edition of Sudhākara Dvivedin with his own Sanskrit commentary, Benares, 1888; edition of Rādhāvallabha with his own Sanskrit commentary, Calcutta, 1917; edition of Muralīdhara Jhā with Sudhākara Dvivedin's as well as his own Sanskrit commentaries, *BenSS 40*, Benares, 1927; edition of Dattātreya Āpaṭe, with Kṛṣṇa's Sanskrit commentary (*Navāṅkura*), *ASS* 99, Poona, 1930; edition of Durgāprasāda Dvivedin with his own Sanskrit and Hindi commentaries, 3rd ed., Lakṣmaṇapura, 1941; edition of Acyutānanda Jhā with Jīvanātha Jhā's Sanskrit commentary (*Subodhinī*) and his own Sanskrit and Hindi commentaries (*Vimlā*), *KSS 148*, Benares, 1949; and edition of T. V. Rādhākṛṣṇa Śāstrin with Kṛṣṇa's Sanskrit commentary (*Bījapallava*), *Madras GOS 67*, *TSMS 78*, Madras-Tanjore, 1958 (*CESS A 4*, p. 311b). Also, there exists an edition of Gangādhara Miśra with his own Sanskrit commentary (*Bījavāsanā*), *HSS 124*, Benares, 1940.

The edition of Bhāskara's *Bījagaṇita*, by Jībānanda Vidyāsāgara, Calcutta, 1878, which we have referred to in our thesis, contains Bhāskara's own commentary. For the other editions containing verses of Bhāskara's *Bījagaṇita*, which we have considered for our purposes, see Chapter I, section 1.F. above.

The *Bījagaṇita* has been translated into Indian and foreign languages. In our thesis, we have referred to the English translation by Henry Thomas Colebrooke, entitled *Algebra*, *with Arithmetic and Mensuration, from the Sanscrit of Brahmegupta and Bhāscara,* London, 1817.

(4). The voluminous treatise *Siddhāntaśiromaṇi,* written in 1150 A.D., consists of two parts—the *Grahagaṇitādhyāya* (or *Gaṇitādhyāya*) and the *Golādhyāya* (Pingree, 1981a). The first part contains twelve adhikāras (chapters) on mathematical astronomy, and the second part contains thirteen adhikāras on the sphere. There exist several editions of these two parts, some combined, others separate. Furthermore, many commentaries have been written on the *Siddhāntaśiromaṇi* in Sanskrit, and some in other Indian languages. (*CESS A 4*, pp. 311b-319a)

References to verses of Bhāskara's *Siddhāntaśiromaṇi,* in our thesis are from the following sources: (i) The *Grahagaṇitādhyāya* with Bhāskara's own *Vāsanābhāṣya* and Gaṇeśa's (fl. ca. 1600/1650 A.D.) *Śiromaṇiprakāśa*, edited by Dattātreya Āpaṭe as *ASS 110,* 2 vols., Poona, 1939 – 1941; (ii) the *Golādhyāya* with Bhāskara's own *Vāsanābhāṣya* and Munīśvara's (b. 1603 A.D.) *Marīci,* edited by Dattātreya Āpaṭe as *ASS 122,* 2 vols., Poona, 1943 – 1952; and (iii) the *Siddhāntaśiromaṇi* with Bhāskara's own *Vāsanābhāṣya* and Nṛsiṃha's *Vāsanāvārttika* (1621 A.D.), edited by Muralīdhara Caturveda, Vārāṇasī, 1981. Caturveda's edition contains both *Grahagaṇitādhyāya* and *Golādhyāya.* We have also referred to the *Golādhyāya* with Bhāskara's own *Vāsanābhāṣya*, Munīśvara's (b. 1603 A.D.) *Marīci,* and Hindi commentary of the editor Kedāradatta Jośī, Dillī, 1988.

(5). The *Vāsanābhāṣya* or *Mitākṣarā* is Bhāskara's own ṭīkā on his *Siddhāntaśiromaṇi* (Pingree, 1970a, 1981b). A commentary on this commentary was also written in 1621 A.D. by Nṛsiṃha (b. 1586 A.D.) at Kāśī, (originally a native of Golagrāma, on the Godāvarī), and both commentaries have been edited. Nṛsiṃha's commentary is known as (see the previous paragraph) the *Vāsanāvārttika* (or *Vārttika*). (*DSB 2*, p. 119a; Table 11, p. 125)

(6). The *Karaṇakutūhala* or *Grahāgamakutūhala* or *Brahmatulya* or *Vidagdhabuddhivallabha* or *Kheṭakarma*, which was written in 1183 A.D., consists of ten chapters (or eleven adhyāyas) which contain rules for solving astronomical problems (Pingree, 1970a, 1981a). It has been edited twice. Several commentaries on this work have been written in Sanskrit. (*DSB 2*, p. 119a; *CESS A 4*, pp. 322a, 326a)

In our thesis, we have cited a verse (see section 2.C. above) from the *Karaṇakutūhala* with the *Gaṇakakumudakaumudī* ṭīkā of Sumatiharṣa Gaṇi (fl. 1622 A.D.), edited by Mādhava Śāstrī Purohita, Bombay, 1989. (First ed. 1901).

The ascription of a short astronomical text entitled the *Bījopanaya* along with the *Vāsanābhāṣya* to Bhāskara by some scholars, seems to have been shown to be a forgery of the nineteenth century; see e.g. Sastri, 1958/59. (Pingree, 1981a, *CESS A 4*, p. 326a)

For more detailed synopses of Bhāskara's works, the readers may look at the article by Tulsi Ram Gupta in the *BMAUA 1*, 1927 – 28, pp. 24-46.

For further information on the commentaries, editions, translations and articles pertaining to Bhāskara's major works, the reader is referred to Professor Pingree's article in the *DSB 2*, 1970, pp. 115a-120b. Another source is Professor Pingree's *CESS A 4*, 1981, pp. 299a-326a (supplemented by *CESS A 5*, 1994, pp. 254b-263a). This last source contains information about the author Bhāskara, about the manuscripts of Bhāskara's works and about commentaries, editions and translations of those works. It includes a bibliography as well. Furthermore, for a quick glance at the genealogical tables along with the titles of works composed by some of the astronomer-mathematicians considered in this section, the reader may refer to Professor Pingree's *Jyotiḥśāstra*, 1981, pp. 124-126.

E. *The Salient Features of Bhāskara's Writings*

Bhāskara was a compiler, a composer and a commentator. His works were so popular that they remained standard text books for several centuries due to the following reasons:

(1). Bhāskara, like his predecessors Śrīdhara (ca. eighth century A.D.) and Mahāvīra (ca. 850 A.D.), tried to make his works, specifically the *Līlāvatī*, interesting and easy by introducing examples and problems from the experiences of daily life and also of common interest. The exemplary problems in the *Līlāvatī* deal with mounds of grain, the sawing of wood, areas of fields, stacks of bricks, excavations, mixtures of different things, cisterns, interest, shadow of a gnomon etc. Brahmagupta (b. 598 A.D.) discusses almost all of these topics in the twelfth adhyāya (which is Gaṇitādhyāya) of his treatise entitled *Brāhmasphuṭasiddhānta*, but his treatment of these topics contains *only rules* and no examples or exemplary problems.

(2). Bhāskara's arrangement of the subject matter and the treatment thereof is very systematic. For example, in his *Līlāvatī* and *Bījagaṇita*, Bhāskara enunciates his sūtras (i.e. rules, principles) very clearly in a simple and charming Sanskrit rhythmical poetry. (Of course the use of Sanskrit rhythmical poetry was a common feature of mathematical and astronomical writings in those days.) In order to apply the underlying principles, procedures and operations, Bhāskara provides udāharaṇas (i.e. examples including problems). The statements of the sūtras and udāharaṇas are followed by a gloss, wherever necessary. This gloss contains brief solutions, hints or explanatory notes.

(3). Bhāskara's *Siddhāntaśiromaṇi* contains, in a very simple and lucid style, theories which do not exist in the treatises of the earlier astronomer-mathematicians, Āryabhaṭa and Śrīpati, for example (Sinha, 1951). Also, Bhāskara criticizes without any prejudice the theories propounded by his predecessors, and gives pertinent explanations to support his own views. (*BMAUA 15*, p. 11)

(4). Bhāskara's commentary *Vāsanābhāṣya* is an excellent piece of work (Sinha, 1951). It is simple enough to be understood by an average student. Also it includes a treatment of trigonometry (*BMAUA 15*, p. 11). It was perhaps this lucid gloss which made Bhāskara's *Siddhāntaśiromaṇi* so popular that it could almost replace the works of his predecessors.

(5). Another outstanding feature of Bhāskara's works is their depth, as was remarked by Professor Spottiswoode in 1859 in the *Journal of the Royal Astronomical Society* (cited in Sinha, 1951):

> It must be admitted that the penetration shown by Bhāskara in his Analysis is in the highest degree remarkable; that the formulae which he establishes and the method of establishing them, bear more than a resemblance—they bear an analogy—to the corresponding process in modern Astronomy, and the majority of scientific persons will learn with surprise the existence of such a method in the writings of so distant a period and so distant a region. (*BMAUA 15*, p. 12)

F. *Some Special Traits of Bhāskara*

In Bhāskara's view, the study of grammar is necessary before the study of any science, for he states in his *SSI Golādhyāya* Golapraśaṃsā, 8, p. 334 (see Caturveda, 1981) that:

यो वेद वेदवदनं सदनं हि सम्यग्ब्राह्मयाः स वेदमपि वेद किमन्यशास्त्रम्।
यस्मादतः प्रथममेतदधीत्य धीमान् शास्त्रान्तरस्य भवति श्रवणेऽधिकारी ॥८॥

The meaning of the verse is: Whoever knows grammar well, knows also the Veda which is the palace of Sarasvatī (the goddess of speech and learning). (Then) what is the (difficulty in learning some) other science (for such a person)? Therefore, a wise person is entitled to hear another science only after he has studied the science of grammar (a knowledge of which is necessary for the correct interpretation of the verses describing the science).

Bhāskara seems to have special regard for the deities Gaṇeśa and Sarasvatī as is evidenced by his invocations to these deities in the beginning of his works.

Another distinguishing feature of Bhāskara's personality is his esteem for the Sāṅkhya philosophy. This is reflected in his description of the universe in his *GD I*, Bhuvanakośapraśna, 1, *ASS 122*, p. 21 (see the Preface of our *Text Alpha* and our commentary thereon):

यस्मात्क्षुब्धप्रकृतिपुरुषाभ्यां महानस्य गर्भे-
ऽहंकारोऽभूत्खकशिखिजलोर्व्यस्ततः संहतेश्च ।
ब्रह्माण्डं यज्जठरगमहीपृष्ठनिष्ठाद्विरञ्चे-
र्विश्वं शश्वज्जयति परमं ब्रह्म तत्तत्त्वमाद्यम् ॥१॥

Another source is Bhāskara's *BG*, 1, p. 1:

उत्पादकं यत्प्रवदन्ति बुद्धेरधिष्ठितं सत्पुरुषेण साङ्ख्याः ।
व्यक्तस्य कृत्स्नस्य तदेकबीजमव्यक्तमीशं गणितं च वन्दे ॥१॥

The other distinguishing traits of Bhāskara's personality are his deep regard for his teacher who was his father Maheśvara. These traits are revealed by his *BG*, 1, p. 1 and by his *GD II*, Praśnādhyāya, 61-62, *ASS 122*, pp. 522-523 (quoted before in the section 2.B.). Verse 62 reflects not only Bhāskara's indebtedness to his father, but also Bhāskara's belief in the supremacy of a guru.

Bhāskara's *BG*, 1, p. 1 reveals an important aspect of Bhāskara's high literary ability; that is, the use of paronomasia. The pun-words in this verse are: 'utpādaka' (generator of intellect, which is Gaṇeśa himself *or* Maheśvara, here Maheśvara is Gaṇeśa's father or Bhāskara's father); 'buddhi' (the human intellect *or* the Mahattattva i.e. the second principle of the Sāṅkhya philosophy); 'satpuruṣa' (a wise man *or* the existing Puruṣa of the Sāṅkhya philosophy which is the unique source of everything); 'sāṅkhyāḥ' (astronomers *or* wise men *or* Sāṅkhya philosophers); 'vyakta' (vyaktagaṇita *or* the manifest material world); 'avyakta' (algebra *or* Prakṛti of the Sāṅkhya philosophy); 'ekabīja' (algebra *or* Gaṇapati); 'gaṇitam' (mathematics *or* Gaṇeśa).

In Bhāskara's opinion, avyaktagaṇita (algebra) is the unique source of vyaktagaṇita (Pāṭīgaṇita). This fact is clear from Bhāskara's *BG*, 1, p. 1. In his *BG*, 162, p. 122 Bhāskara defines algebra as "thought accompanied by various colours" as follows:

बीजं मतिर्विविधवर्णसहायनी हि मन्दावबोधविषये विबुधैर्निजाद्यैः।
विस्तारिता गणकतामरसांशुमद्भिर्या सैव बीजगणिताह्वयतामुपेता ॥१६२॥

The importance attached by Bhāskara to the study of algebra is also evident in his *BG*, 2, p. 1, where he proclaims that generally questions cannot be very well understood by the "dull-witted" without recourse to algebra:

पूर्वं प्रोक्तं व्यक्तमव्यक्तबीजं प्रायः प्रश्ना नो विनाऽव्यक्तयुक्त्या।
ज्ञातुं शक्या मन्दधीभिर्नितान्तं यस्मात्तस्माद्वच्मि बीजक्रियां च ॥२॥

Bhāskara suggests that the algebraic solution is manifold (see the section on the *Līlāvatī*, p. 28, in Colebrooke, 1817); and that in some cases, the algebraic demonstration must be shown to those who do not understand the geometrical one (see the section on the *Bījagaṇita*, p. 272, in Colebrooke, 1817).

Bhāskara's verse, which describes an analogy between the six aṅgas (parts) of the Vedas and those of the human body, is interesting (*SSI* (1981) *Grahagaṇitādhyāya*, Madhyamādhikāra, 10, p. 10):

शब्दशास्त्रं मुखं ज्यौतिषं चक्षुषी श्रोत्रमुक्तं निरुक्तं च कल्पः करौ ।
या तु शिक्षाऽस्य वेदस्य सा नासिका पादपद्मद्वयं छन्द आद्यैर्बुधैः ॥१०॥

This verse essentially means: It has been declared by the ancient scholars that Vyākaraṇa (Grammar) is the mouth, Jyotiṣa (Astronomy) is the eye, Nirukta (Etymology) is the ear, Kalpa (Ritual) is the pair of hands, Śikṣā (correct pronunciation) is the nose, and Chandas (Metrics) is the pair of lotus-like feet.

A very important attribute of Bhāskara's personality is that although Bhāskara was an outstanding scholar, he cared for the average and the "dull-witted" students as well. For example, he took great pains to make his writings charming, clear and easy for them. This fact is explicitly mentioned by Bhāskara in a few places in his writings. Some of the illustrations in this respect are: (i) the *Golādhyāya II* Praśnādhyāya, 62, *ASS 122*, p. 523 (see section 2.B. above); (ii) the *Bījagaṇita*, 207, p. 162 (see section 2.C. above); and (iii) the *Bījagaṇita*, 208, p. 162 (see below in the present section F.). Furthermore, similar facts are revealed also in the last verse of the *Bījagaṇita* (i.e. *BG*, 213, p. 163) where Bhāskara is making an appeal to the calculators for the study of his work:

गणक भणितिरम्यं बाललीलावगम्यं
सकलगणितसारं सोपपत्तिप्रकारम् ।
इति बहुगुणयुक्तं सर्वदोषैर्विमुक्तं पठ पठ
मतिवृद्ध्यै लघ्विदं प्रौढिसिद्ध्यै ॥२१३॥

Above all, Bhāskara is a true scholar who acknowledged most of the scholars whose works he used. For example, he mentions the algebraic works of Brahmagupta, Śrīdhara and Padmanābha (*BG*, 208, p. 162):

ब्रह्माह्वयश्रीधरपद्मनाभबीजानि यस्मादतिविस्तृतानि ।
आदाय तत्सारमकारि नूनं सद्युक्तियुक्तं लघु शिष्यतुष्ट्यै ॥२०८॥

Bhāskara declares here that the algebraic works of Brahmagupta, Śrīdhara and Padmanābha are too diffuse. Therefore, having extracted the essence and good points from their works, he has composed an easy compendium for the satisfaction of students.

G. *The Sources Used by Bhāskara*

Bhāskara seems to have studied the works of all of his predecessors. In particular, Bhāskara's sources include the works of all those astronomer-mathematicians whose names have been referred to by Bhāskara in his works, and those works which seem to have been used by Bhāskara without any specific mention of their authors' names. These mathematicians may be listed as follows:

Āryabhaṭa I (b. 476 A.D.), Varāhamihira (b. 505 A.D.), Bhāskara I (ca. 600 A.D.), Brahmagupta (b. 598 A.D.), Lalla (fl. eighth century A.D.), Caturveda Pṛthūdakasvāmin (fl. 864 A.D.), Śrīdhara (ca. eighth century A.D.), Mahāvīra (ca. 850 A.D.), Muñjāla (fl. 932 A.D.), Padmanābha, Āryabhaṭa II (fl. 950 A.D.), Mādhava (fl. tenth or eleventh century A.D.), Śrīpati (fl. 1039 A.D.), and Jayadeva (fl. before 1073 A.D.).

In his writings, Bhāskara generally alludes to these former scholars by employing the words पूर्वैः (by the predecessors), आद्यैः (by the first writers), पूर्वाचार्यैः (by the previous teachers) and आचार्यवर्यैः (by the best of teachers); though sometimes Bhāskara makes an explicit mention of some particular scholar. Some instances of Bhāskara's general allusion are: (i) *BG*, p. 22, 3-4, where Bhāskara proclaims एवं बुद्धिमतानुक्तमपि ज्ञायते इति पूर्वैर्नायमर्थो विस्तीर्योक्तः। बालावबोधार्थं तु मयोच्यते, with regard to the existence of the square-root of a given multinomial karaṇī-expression as well as in connection with the square-root of a square-karaṇī-expression which contains negative karaṇīs; (ii) *BG*, 162, p. 122 (quoted before, see the previous sub-section F. in which Bhāskara describes algebra; and (iii) *BG*, 12, pp. 5-6 concerning measures of the unknowns as colours, as follows:

यावत्तावत्कालको नीलकोऽन्यो वर्णः पीतो लोहितश्चैतदाद्याः।
अव्यक्तानां कल्पिता मानसंज्ञास्तत्सङ्ख्यानं कर्तुमाचार्यवर्यैः ॥१२॥

As far as the specific mention of the names of the authors by Bhāskara is concerned, one example is his *BG*, 208, p. 162 (quoted in the previous sub-section F.). Also several instances exist in his *Siddhāntaśiromaṇi*, such as *SSI* (1981) *Grahagaṇitādhyāya*, Madhyamādhikāra, 2, p. 6:

कृती जयति जिष्णुजो गणकचक्रचूडामणि-
जयन्ति ललितोक्तयः प्रथिततन्त्रसद्युक्तयः।
वराहमिहिरादयः समवलोक्य येषां कृतीः
कृती भवति मादृशोऽप्यतनुतन्त्रबन्धेऽल्पधीः ॥२॥

Here Bhāskara is praising the previous teachers Brahmagupta and Varāhamihira etc.

Next we elaborate on Bhāskara's specific use of the works of some of the astronomer-mathematicians which were listed before.

Bhāskara has used the *Āryabhaṭīya* of Āryabhaṭa I when he defines names of the places and the sum of the cubes of natural numbers in his *Līlāvatī*. Moreover, in his *Vāsanābhāṣya* on *SSI* (1981) *Golādhyāya*, Bhuvanakośa, 52, p. 360, Bhāskara quotes

Āryabhaṭa I as follows: अतोऽयुतद्वयव्यासे २०००० द्विकारन्यष्टयमर्तुमितः ६२८३ [*sic*] परिधिरार्यभटाद्यैरङ्गीकृतः । The second number should be ६२८३२ ।

This is a clear reference to Āryabhaṭa I's *AB* II, 10 (see e.g. Shukla & Sarma, 1976, p. 45):

चतुरधिकं शतमष्टगुणं द्वाषष्टिस्तथा सहस्राणाम्
अयुतद्वयविष्कम्भस्यासन्नो वृत्तपरिणाहः ॥१०॥

Here Āryabhaṭa I is proclaiming that 62,832 is the approximate measure of the circumference of a circle whose diameter is 20,000. (This indicates that Āryabhaṭa I takes $\pi = 3.1416$.)

Furthermore, Bhāskara's treatment of kuṭṭaka is based mainly on that of Āryabhaṭa II who essentially follows Āryabhaṭa I. Bhāskara I comments on the *Āryabhaṭīya* of Āryabhaṭa I and gives his own description of kuṭṭaka. Bhāskara I's idea, that the general solution of $by = ax + c$ would follow from that of $by = ax + 1$, was also used by Bhāskara II (see our commentary on kuṭṭaka, Chapter VI, sections 4.J. and 4.S.(b)).

Bhāskara has mentioned Brahmagupta both in his *Siddhāntaśiromaṇi* and *Vāsanābhāṣya* several times. As an illustration, see Bhāskara's commentary following his *SSI* (1981) *Golādhyāya*, Chedyakādhikāra, 36-37, p. 392. One of the citations included in it is: ब्रह्मगुप्तोऽत्र कारणमाह । त्रिज्याभक्तः परिधिः कर्णगुण इत्यादि । Thus it is clear that Bhāskara has considered the *Brāhmasphuṭasiddhānta* of Brahmagupta as the base for his *Siddhāntaśiromaṇi*. At the same time, Bhāskara has built on the astronomical works of Varāhamihira, Lalla, Pṛthūdakasvāmin, Muñjāla and Śrīpati. There exist some similarities also between Brahmagupta's *BSS* XVIII (which is Kuṭṭakādhyāya) and Bhāskara's *Bījagaṇita*. For example, Bhāskara's *BG*, 2, p. 1 (which was cited in the previous sub-section F.,) seems to be based on Brahmagupta's *BSS* XVIII, 1 (see Dvivedin, 1902, p. 294):

प्रायेण यतः प्रश्नाः कुट्टाकाराहृते न शक्यन्ते।
ज्ञातुं वक्ष्यामि ततः कुट्टाकारं सह प्रश्नैः ॥१॥

Bhāskara studied Lalla's astronomical work *SDVT* and wrote a commentary, called *Vivaraṇa*, on it before he wrote his astronomical work *SSI*. This is evidenced by the fact that Bhāskara refers to Lalla in his *SSI* and *VB*. For example, Bhāskara exposed Lalla's error pertaining to the area of the surface of a sphere in his *SSI* (1981) *Golādhyāya*, Bhuvanakośa, 53, p. 361 and *VB* on 54-57, p. 362 (see the next sub-section H.). An explicit reference to Lalla's *SDVT* appears in Bhāskara's *VB* on *SSI* (1981) *Grahagaṇitādhyāya*, Pātādhikāra, 11-14, p. 315 as follows: अत्र धीवृद्धिदपक्षे सूर्यापमादोजपदोद्भवादित्यादिलक्षणेन क्रान्तिसाम्याभावः। Furthermore, Bhāskara seems to have followed Lalla's *SDVT* as far as the choices of topics in his *Grahagaṇitādhyāya* and *Golādhyāya* are concerned.

Bhāskara made use of Caturveda Pṛthūdakasvāmin's gloss on Brahmagupta's *BSS*, because Bhāskara makes a reference to Caturveda Pṛthūdakasvāmin in his *VB* on *SSI* (1981) *Golādhyāya*, Golabandhādhikāra, 23-25a, p. 402 as follows:

तथा च ब्रह्मसिद्धान्तभाष्ये। ज्ञशुक्रयोः शीघ्रोच्चस्थाने यावान् विक्षेपस्तावानेव यत्र तत्रस्थस्यापि ग्रहस्य भवति। अत्रोपलब्धिरेव वासना नान्यत् कारणं वक्तुं शक्यत इति चतुर्वेदेनाप्यनध्यवसायोऽत्र कृतः।

As far as Śrīdhara, who wrote probably during the eighth century (Pingree, 1979, *GPV*, p. 889), is concerned, two instances of the influence of his works on Bhāskara's

Līlāvatī are given by the similarities between the following verses from Bhāskara's *Līlāvatī* and Śrīdhara's *Pāṭīgaṇita:*

(i) Bhāskara's *L I*, 19-20, *ASS 107*, p. 19

समद्विघातः कृतिरुच्यतेऽथ स्थाप्योऽन्त्यवर्गो द्विगुणान्त्यनिघ्नाः।
स्वस्वोपरिष्टाच्च तथाऽपरेऽङ्कास्त्यक्त्वाऽन्त्यमुत्सार्य पुनश्च राशिम् ॥१९॥

खण्डद्वयस्याभिहतिर्द्विनिघ्नी तत्खण्डवर्गैक्ययुता कृतिर्वा।
इष्टोनयुग्राशिवधः कृतिः स्यादिष्टस्य वर्गेण समन्वितो वा ॥२०॥

and Śrīdhara's *PG*, 23-24, p. 16 (see Shukla,1959)

कृत्वाऽन्त्यपदस्य कृतिं शेषपदैर्द्विगुणमन्त्यमभिहन्यात्।
उत्सार्य्योत्सार्य पदाच्छेषं चोत्सारयेत् कृतये ॥२३॥

सदृशद्विराशिघातो रूपादिद्विचयपदसमासो (वा)।
इष्टोनयुतपदवधो वा तदिष्टवर्गान्वितो वर्गः ॥२४॥

which contain rules for squaring of numbers. (See also our commentary on verse 7c-d of *Text Alpha.*)

(ii) Bhāskara's *L I*, 22, *ASS 107*, p. 21

त्यक्त्वाऽन्त्याद्विषमात्कृतिं द्विगुणयेन्मूलं समे तद्धृते
त्यक्त्वा लब्धकृतिं तदाद्यविषमाल्लब्धं द्विनिघ्नं न्यसेत्।
पङ्क्त्यां पङ्क्तिहृते समेऽन्यविषमात्त्यक्त्वाऽऽप्तवर्गं फलं
पङ्क्त्यां तद्विगुणं न्यसेदिति मुहुः पङ्क्तेर्दलं स्यात्पदम् ॥२२॥

and Śrīdhara's *PG*, 25-26, p. 18

विषमात् पदतस्त्यक्त्वा वर्गं स्थानच्युतेन मूलेन ।
द्विगुणेन भजेच्छेषं लब्धं विनिवेशयेत् पङ्क्तौ ॥२५॥

तद्वर्गं संशोध्य द्विगुणं कुर्वीत पूर्ववल्लब्धम् ।
उत्सार्य ततो विभजेच्छेषं द्विगुणीकृतं दलयेत् ॥२६॥

which contain the rule for finding the square-root of a number. (See also our commentary on verse 27c-28b of *Text Alpha.*)

Professor Shukla (1959, p. xiii) maintains that from Śrīdhara's algebra (which is now lost), the following rule for solving a quadratic equation $ax^2 + bx = c$, is quoted by Bhāskara in his commentary on the section dealing with quadratic equations in his *Bījagaṇita* :

चतुराहतवर्गसमै रूपैः पक्षद्वयं गुणयेत् ।
अव्यक्तवर्गरूपैर्युक्तौ पक्षौ ततो मूलम् ॥

Another instance is Bhāskara's use of Śrīdhara's rule for finding an approximate value of the square-root of a non-square number which he (Śrīdhara) gives in his *Triśatikā*, 46, p. 34 (see Dvivedin, 1899):

राशेरमूलदस्याहतस्य वर्गेण केनचिन्महता ।
मूलं शेषेण विना विभजेद्गुणवर्गमूलेन ॥४६॥

Bhāskara uses Śrīdhara's rule in order to find an approximate square-root of a fraction. Bhāskara's rule is *L*, 140, p. 280 (see Sarma, 1975, *VIS 66):*

वर्गेण महतेष्टेन हतान्छेदांशयोर्वधात्।
पदं गुणपदक्षुण्णाच्छिद्भक्तं निकटं भवेत् ॥१४०॥

For Bhāskara's example, the reader may refer to the approximate square-root of a non-square number in our commentary on the six-fold operation of the karaṇī, Chapter VI, section 3.D.(c)(ii).

The following reference to Śrīdhara exists in Bhāskara's *Vāsanābhāṣya* on *SSI* (1981) *Golādhyāya*, Bhuvanakośa, 52, p. 360:

यत् पुन श्रीधराचार्यब्रह्मगुप्तादिभिर्व्यासवर्गाद्दशगुणात् पदं परिधिः
स्थूलोऽप्यङ्गीकृतः स सुखार्थम्। नहि ते न जानन्तीति।

Here Bhāskara is saying that the rough formula for the circumference of a circle being the square-root of ten times the square of its diameter, was accepted by Śrīdhara, Brahmagupta, and so on, only for the sake of easiness, for it is certainly not the case that they did not know (that that was not exact).

Furthermore, there occur a few sūtras and udāharaṇas on geometry in Bhāskara's *Līlāvatī* which have been borrowed almost verbatim from Śrīdhara's *Gaṇitapañcaviṃśī*. One may see the similarities between the following:

(i) Bhāskara's *L*, 1-2, p. 103 (see Sharma, 1987)

इष्टो बाहुर्यः स्यात्तत्स्पर्द्धिन्यां दिशीतरो बाहुः।
त्र्यस्रे चतुरस्रे वा सा कोटिः कीर्त्तिता तज्ज्ञैः ॥१॥

तत्कृत्योर्योगपदं कर्णो दोः कर्णवर्गयोर्विवरात्।
मूलं कोटिः कोटिश्रुतिकृत्योरन्तरात्पदं बाहुः ॥२॥

and Śrīdhara's *GPV*, 27-28, p. 904 (see Pingree, 1979)

इष्टाब्दाहोर्यत् स्यात्
पार्श्वेऽन्यायां दिशीतरो बाहुः।
त्र्यस्रे चतुरस्रे वा
सा कोतिः कीर्त्तिता तज्ज्ञैः ॥२७॥

तत्कृत्योर्योगपदं
कर्णो दोः कर्णयोर्विवरात्।
मूलं कोटिः कोटि-
श्रुतिकृत्योरन्तरात् पदं बाहुः ॥२८॥

which describe the relationships between the sides and the calculation of the sides (including the hypotenuse or diagonal) of a right triangle or a rectangle.

(ii) Śrīdhara's *GPV*, 15, p. 904

कोटिश्चतुष्टयं यत्र
दोस्त्रयं तत्र का श्रुतिः।
कोटिं दोः कर्णतः कोटि-
श्रुतिभ्यां च भुजं वद ॥१५॥

and Bhāskara's *L* (1987), 1, p. 104, which is word for word the same, and contains an udāharaṇa involving calculation of the sides of a right triangle.

(iii) Śrīdhara's *GPV*, without number, p. 904

घनहस्ताः क्षेत्रफलं
खाते वेधेन ताडितम्।

and Bhāskara's *L* (1987), 52a, p. 170

क्षेत्रफलं वेधगुणं खाते घनहस्तसंख्या स्यात् ॥५२a॥

which contain the sūtra that in an excavation, the area of the (plane) figure multiplied by the depth is the number of solid cubits (i.e. cubic cubits), contained in the excavation.

(iv) Śrīdhara's *GPV*, without number to 19, p. 907

शङ्कुः प्रदीपतलशङ्कुतलान्तरघ्न-
श्छाया भवेद्विनरदीपशिखौच्च्यभक्तः ।

शङ्कुप्रदीपान्तरभूस्त्रिहस्ता
दीपोच्छ्रितिः सार्द्धकरत्रया चेत् ।
शङ्कोस्तदार्काङ्गुलसंमितस्य
तस्याः प्रभा स्यात् कियती वदाशु ॥१९॥

and Bhāskara's *L* (1987), 61a to 42, p. 186, which is word for word the same, and which contains a sūtra and an udāharaṇa respectively, involving calculation of the shadow of a gnomon.

(v) Śrīdhara's *GPV*, 31, p. 907

छायोद्धृते तु नरदीपतलान्तरघ्ने
शङ्कौ भवेन्नरयुते खलु दीपकौच्च्यम् ॥३१॥

and Bhāskara's *L* (1987), 61b, p. 187

छायाहृते तु नरदीपतलान्तरघ्ने
शङ्कौ भवेन्नरयुते खलु दीपकौच्च्यम् ॥६१b॥

which contain the rule to determine the height of light when the shadow of a gnomon is given.

(vi) Śrīdhara's *GPV*, 20, p. 907

प्रदीपशङ्क्वन्तरभूस्त्रिहस्ता
छायाङ्गुलैः षोडशभिः समा चेत् ॥
दीपोच्छ्रितिः स्यात् कियती तदास्याः
प्रदीपशङ्क्वन्तरमुच्यतां मे ॥२०॥

and Bhāskara's *L* (1987), 42, p. 187

प्रदीपशङ्क्वन्तरभूस्त्रिहस्ता छायाङ्गुलैः षोडशभिः समा चेत् ॥
दीपोच्छ्रितिः स्यात् कियती वदाऽऽशु प्रदीपशङ्क्वन्तरमुच्यतां मे ॥४२॥

which contain an exemplary problem, wherein the height of a light is to be determined when the shadow of a gnomon is given.

Thus Bhāskara expanded on the topics treated by Śrīdhara and others. In addition, he adorned them by his literary expertise.

Muñjāla composed a work entitled *Bṛhanmānasa* in 932 A.D. (Pingree, 1981a). After that he composed an (astronomical) work entitled *Laghumānasa* (*CESS A 4*, p. 435a). A reference to Muñjāla exists in Bhāskara's *SSI* (1981) *Golādhyāya*, Golabandhādhikāra, 18, p. 397:

अयनचलनं यदुक्तं मुञ्जालाद्यैः स एवायम्।
तत्पक्षे तद्भगणाः कल्पे गोऽङ्गर्तुनन्दगोचन्द्राः १९९६६९ ॥१८॥

The works of Padmanābha are non-existent, but his algebraic work has been referred to by Bhāskara in his *BG*, 208, p. 162 (see section 2.F. above).

The *Mahāsiddhānta* of Āryabhaṭa II was also one of the sources utilized by Bhāskara. In his *MS* XVI, 38, Āryabhaṭa II gives the rule for the area of the surface of a sphere as follows (Gupta, 1973, *ME 7*, p. 49):

परिधिघ्नो व्यासः स्यात् कन्दुकजालोपमं कुपृष्ठफलम् ॥३८॥

That is, the circumference (of a great circle) multiplied by its diameter; while Bhāskara gives the rule: area of a (great) circle multiplied by four (see the next sub-section H.). Moreover, there are several similarities in the treatments of kuṭṭaka by these two mathematicians. (See our commentary.)

Furthermore, Āryabhaṭa II has been cited by Bhāskara in his *VB* on *SSI* (1981) *Grahagaṇitādhyāya*, Spaṣṭādhikāra, 65, p. 135 as follows: अत एव आर्यभटादिभिः सूक्ष्मत्वार्थं दृक्काणोदयाः पठिताः। According to Professor Pingree (1992), here Bhāskara is referring to Āryabhaṭa II's *MS* IV, 38c-39b:

दृक्काणज्याः सर्वा मिथुनान्तद्युज्यया निघ्नाः।
स्वस्वद्युज्याभक्तास्तच्चापकला भवन्त्यसवः ॥३८c-३९b॥

because this process of dividing the quadrant into arcs of 10° and calculating the oblique ascentions is unique to Āryabhaṭa II. It has not been employed by anyone else before him. (*GB 14*, pp. 55-56)

Another source used by Bhāskara is the *Siddhāntacūḍāmaṇi* of Mādhava. Some references to this source exist in Bhāskara's *Mitākṣarā* as follows: (i) Bhāskara's *MK* on *SSI* (1981) *Golādhyāya*, Golabandhādhikāra, 23-25a, p. 402 reads तथा च माधवीये सिद्धान्तचूडामणौ पठिताः and (ii) Bhāskara's *MK* on *SSI* (1981) *Grahagaṇitādhyāya*, Pātādhikāra, 11-14, p. 314 contains (on p. 315)

रवेरोजपदक्रान्तेश्चन्द्रयुग्मपदोद्भवा ।
स्वल्पा चेन्न तयोः क्रान्त्योः साम्यं स्यादन्यथा भवेत् ॥

इति माधवोक्तसिद्धान्तचूडामणिलक्षणेनापि क्रान्तिसाम्याभावः ।

Finally, another mathematician, whose works have been used by Bhāskara, is Śrīpati. There is a close similarity between the language of many of the verses stated by these two astronomer-mathematicians; but somehow Bhāskara does not include Śrīpati's name in his acknowledgements in his *BG*, 208, p. 162. Majumdar (see Miśra, 1932, *Part I*) states that Bhāskara freely borrows from Śrīpati, but only occasionally remarks "Śekharokta-lakṣaṇena" ("by a definition told in the *Śekhara*"), which indicates that Śrīpati's *Siddhāntaśekhara* was a very widely known treatise at the time of Bhāskara (Introduction, p. xii). As an illustration, in his *VB* on *SSI* (1981) *Grahagaṇitādhyāya*, Pātādhikāra, 11-14, p. 314, Bhāskara says (on p. 315):

अत्र धीवृद्धिदपक्षे सूर्यापमादोजपदोद्भवादित्यादिलक्षणेन क्रान्तिसाम्याभावः । तथा ब्रह्मगुप्तपक्षेऽपि त्रिनवगृहेन्दुक्रान्तिरित्यादिना लक्षणेन तथा त्रिनवभवनजाता क्रान्तिरित्यादिना शेखरोक्तलक्षणेन ।

Furthermore, some similarities may be observed in the following: (i) Bhāskara's *L I*, 56, *ASS 107*, p. 54 and Śrīpati's *SSE* XIV, 13a-b (see Miśra, 1947, *Part II*, p. 103) about the concurrence-sūtra which have been cited in our textual commentary on the verses 33a-34d pertaining to karaṇī; (ii) Bhāskara's *BG*, 12a-c, pp. 5-6 and Śrīpati's *SSE* XIV, 2a-b (Miśra, 1947, *Part II*, p. 86) about measures of unknowns; (iii) Bhāskara's *BG*, 47c-d, p. 26 and Śrīpati's *SSE* XIV, 27a-b (Miśra, 1947, *Part II*, p. 122) about the reducer of two quantities which are mutually divided for the method of kuṭṭaka. The above three examples are all cited in our commentary.

Other interesting examples of similarity are: Bhāskara's *SSI* (1981) *Golādhyāya*, Bhuvanakośa, 41 and 42, pp. 350-351

ऐन्द्रं कशेरुशकलं किल ताम्रपर्णमन्यद्गभस्तिमदतश्च कुमारिकाख्यम् ।
नागं च सौम्यमिह वारुणमन्त्यखण्डं गान्धर्वसंज्ञमिति भारतवर्षमध्ये ॥४१॥

वर्णव्यवस्थितिरिहैव कुमारिकाख्ये शेषेषु चान्त्यजजना निवसन्ति सर्वे ।
माहेन्द्रशुक्तिमलयर्क्षकपारियात्राः सह्यः सविन्ध्य इह सप्त कुलाचलाख्याः ॥४२॥

and Śrīpati's *SSE* XIV, 46 and 47, (see Miśra, 1947, *Part II*, pp. 159-160)

ऐन्द्रं कशेरुकमतः खलु ताम्रपर्णी
    खण्डं गभस्तिदमनं च कुमारिकाख्यम् ।
सौम्यं च नागमथ वारुणनामधेयं
    गान्धर्वसंज्ञमिति भारतवर्षमध्ये ॥४६॥

कुलाचलाः सप्त महेन्द्रशुक्ति-
    सह्यर्क्षविन्ध्या मलयाचलश्च ।
सपारियात्रोऽत्र च कन्यकाख्ये
    वर्णव्यवस्था नहि सेतरत्र ॥४७॥

In these verses, Bhāskara's description of nine divisions of India and seven mountains in it is the same as that of Śrīpati.

Furthermore, Bhāskara's *SSI* (1981) *Grahagaṇitādhyāya*, Madhyamādhikāra, 10, p. 10 (section 2.F. above) is similar to Śrīpati's *SSE* I, 5 (see Miśra, 1932, *Part I*, p. 5):

छन्दः पादौ शब्दशास्त्रं च वक्त्रं
    कल्पः पाणी ज्यौतिषं चक्षुषी च ।
शिक्षा घ्राणं श्रोत्रमुक्तं निरुक्तं
    वेदस्याङ्गान्याहुरेतानि षट् च ॥५॥

The above list of the sources employed by Bhāskara is probably not exhaustive. As an illustration, Bhāskara might have used the mathematical treatise entitled *Gaṇitasārasaṅgraha* of Mahāvīra, in connection with division by zero, for instance. Mahāvīra's treatise was widely known and appreciated (especially) in Southern India as early as the eleventh century, though Bhāskara never mentions Mahāvīra in his works (Raṅgācārya, 1912, Preface, p. xi).

Nonetheless, Bhāskara acknowledges his indebtedness to Śrīdhara (see Bhāskara's *BG*, 208, p. 162). Śrīdhara's algebra is now lost but Professor Shukla (1959) has found some examples in Śrīdhara's *Pāṭīgaṇita* and *Triśatikā* which are almost literally the same as those in Mahāvīra's *Gaṇitasārasaṅgraha*. Also, according to Professor Shukla, Śrīdhara was posterior to Mahāvīra (Introduction, pp. xx-xxi). Since the *exact* date of Śrīdhara is not known, and the date to Mahāvīra's treatise has been assigned keeping in view the point that Mahāvīra wrote during the reign of Amoghavarṣa Nṛpatuṅga, who ruled over Mysore and other Kanarese regions from 814/815 A.D. to 877/878 A.D. (Raṅgācārya, 1912, Preface, pp. xi, xx), it is possible that Bhāskara is indebted to Mahāvīra at least by transitivity of indebtedness.

On the other hand, Professor Pingree (1979, *GPV*, pp. 888-889) argues strongly that Śrīdhara was anterior to Mahāvīra.

Another plausible argument in this regard is the following: Bhāskara freely borrows from Śrīpati, even though he only occasionally remarks "Śekharokta-lakṣaṇena". Śrīpati, in turn, seems to have looked at Mahāvīra's *Gaṇitasārasaṅgraha*, because Śrīpati's *Gaṇitatilaka* which does not survive in its entirety (Pingree, 1981b), contains a paribhāṣā (i.e. a technical term) which is modelled on that of Mahāvīra (p. 61). The argument of transitivity can now be applied.

As another illustration of a source (possibly employed), Bhāskara might have consulted the lost work of Jayadeva. Jayadeva wrote in the beginning of the eleventh century or earlier (Shukla, 1954). Quotations from this work exist in twenty stanzas in Udayadivākara's commentary entitled *Sundarī*, written in 1073 A.D., on the *Laghubhāskarīya* of Bhāskara I. Jayadeva's work contained, among other things, the cyclic method for finding the integral solutions to $Nx^2+c=y^2$, where c is positive or negative. Jayadeva's method, though not superior to, is different from that suggested by Brahmagupta. In his *Bījagaṇita*, Bhāskara discusses the cyclic method for solving the indeterminate equation of the type $Nx^2+1=y^2$, which has also been discussed by Jayadeva. Bhāskara calls this method *Cakravāla* and says that this name is due to previous writers: "चक्रवालमिदं जगुः" । The Cakravāla method does not exist in the works of Brahmagupta. The algebraic works of Śrīdhara and Padmanābha have not survived. Nor do we have any information about them. Thus it is very likely that Bhāskara has used the lost work of Jayadeva. Incidentally, the remarks of H. Hankel about the Cakravāla method are worth quoting: "It is above all praise; it is certainly the finest thing which was achieved in the theory of numbers before Lagrange." (*G 5*, pp. 1-4, 19-20)

Furthermore, the equation $Nx^2+1=y^2$, which has been (incorrectly) called Pell's equation by many modern mathematicians, has been called Jayadeva-Bhāskara equation by Selenius (1975). Selenius has investigated the rules of the Cakravāla method in detail. He concludes: "No European performances in the whole field of algebra at a time much later than Bhāskara's, nay nearly up to our times, equalled the marvellous complexity and ingenuity of Cakravāla." (*HM 2*, pp. 168, 180)

H. *Some of the Innovations Made by Bhāskara*

(i). Bhāskara was the first known Indian mathematician who added a *succinct* gloss to almost all of his works. Of course a great number of ancient Indian mathematical texts have been lost.

(ii). Bhāskara made use of unique vocative forms. In order to address some of his problems to his daughter Līlāvatī, he used (in his work *Līlāvatī*) such words as 'bāle,' 'aye bāle,' 'kānte,' 'sumate,' 'vatsa,' 'cañcalākṣi,' 'mṛgākṣi,' 'bāle bālakuraṅgalolanayane Līlāvati,' etc.

(iii). The introduction of the idea of infinity: Bhāskara gave the value of a 'khahara' quantity (which has zero as its divisor), as infinite ('ananta'). Furthermore, Bhāskara described this khahara quantity by comparing it with God Viṣṇu. (See *BG*, 11, p. 5 and Bhāskara's introduction to this verse.)

(iv). Bhāskara carried the idea of division by zero even further. This idea is contained in Bhāskara's *Līlāvatī* in the rule which may be written as $\frac{a \cdot 0}{0} = a$ (see Āpaṭe, 1937, *L I*, 45-46, *ASS 107*, p. 39). Clearly Bhāskara's rule is correct in the case of limiting processes where zero is considered as an infinitesimal quantity; but in the modern sense, $\frac{0}{0}$ is indeterminate. (See our textual commentary on Bhāskara's verses 10a-11d, Chapter VI, section 3.B.)

(v). Bhāskara gave a detailed and lengthy treatment of the six-fold operation of one and more than one colours. Bhāskara enunciates several verses (*BG*, 12a-23b, pp. 5-11) which contain both rules and examples. Brahmagupta treats this topic in only two verses (*BSS* XVIII, 41-42), and Śrīpati in only one (*SSE* XIV, 2).

(vi). Bhāskara also wrote a rigorous treatment of karaṇī. Brahmagupta and Śrīpati state only rules for this topic. Bhāskara provides several examples in addition to the rules.

An anonymous commentator of Brahmagupta's *BSS* XVIII, Kuṭṭakādhyāya provides some examples on the topic of karaṇī, all of which have also been treated by Bhāskara (see our commentary, Chapter VI, section 3.D.(a)(ii)).

The rule (3) about the sum and difference of two karaṇīs (*BG*, 23c-24b, pp. 11-12) seems to be Bhāskara's own innovation (see our commentary, Chapter VI, section 3.D.(a)(iii)).

(vii). Furthermore, Bhāskara provides a detailed treatment of the method of extracting the square-root of a karaṇī-expression. He states the specifics or limitations of this method which Brahmagupta and Śrīpati do not. Nor do the latter two mathematicians discuss how to deal with the negative karaṇīs in the square-root of a given (square) karaṇī-expression when this given expression contains negative karaṇīs, which the former does.

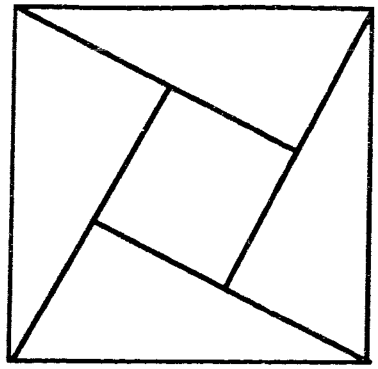
(viii). Bhāskara improved upon the rules concerning kuṭṭaka which he borrowed from his predecessors. For example, Bhāskara's result in *BG*, 52c, p. 27 is a refinement over that of Āryabhaṭa II, as will be explained in our commentary on the topic of kuṭṭaka.

(ix). Bhāskara discusses some cases in kuṭṭaka where the dividend or the divisor is negative (*BG*, 54a, p. 27; 59c-60b, p. 32; 60c-61a, pp. 33-34), which do not seem to have been considered by any of his predecessors.

(x). Bhāskara applied algebra to geometrical demonstrations (Colebrooke, 1817). For example, employing algebra, he gave two interesting proofs that the hypotenuse of a right triangle is the square-root of the sum of the squares of the legs (pp. xvi-xvii).

In the first, Bhāskara turns the triangle so as to make the (unknown) hypotenuse the base and produces a perpendicular upon it from the vertex above (Colebrooke, 1817). Using the rule of proportion the required result becomes almost self-evident (pp. 220-221).

More striking is the second demonstration where Bhāskara constructs his now famous figure using four copies of the given triangle,



and then uses areas to calculate the hypotenuse in terms of the given legs (Colebrooke, 1817, p. 222). Here, even though Bhāskara uses the values 15 and 20 for the lengths of the legs of the triangle, he makes it clear that the resultant hypotenuse length (= 25) is of secondary importance to the fact that the demonstration shows that it is equal to the square-root of the sum of the squares of the legs and that this method would apply in all cases, thus constituting a proof of the so-called Pythagorean Theorem.

(xi). Bhāskara invented an astronomical instrument called 'Phalaka' (Śāstrī, 1893, *JAS Bengal 62*, p. 225).

(xii). Bhāskara gave a close approximation for the length of an arc of a circle in terms of its chord and vice versa (Sarasvati, 1970, pp. 10-11).

(xiii). Bhāskara refuted some of the rules of his predecessors. For example, Bhāskara severely criticized Lalla's incorrect formula (contained in his *SDVT)* for the area of the surface of the earth, in his (Bhāskara's) *SSI* (1981) *Golādhyāya*, Bhuvanakośa, 53 (p. 361):

दुष्टं कन्दुकपृष्ठजालवदिलागोले फलं जल्पितं
लल्लेनास्य शतांशकोऽपि न भवेद्यस्मात् फलं वास्तवम्।
तत् प्रत्यक्षविरुद्धमुद्धतमिदं नैवास्तु वा वस्तु वा
हे प्रौढा गणका विचारयत तन्मध्यस्थबुद्ध्या भृशम् ॥५३॥

The verse containing Lalla's (incorrect) formula is (see Caturveda, 1981, p. 361, footnote 2):

नगशिलीमुखबाणभुजङ्गमज्वलनवह्निरसेषुगजाश्विनः २८५६३३८५५७ ।
कुवलयस्य बहिः परियोजनान्यथ जगुः खलु कन्दुकजालवत् ॥

Furthermore, in his *Vāsanābhāṣya* on *SSI* (1981) *Golādhyāya*, Bhuvanakośa, 54-57 (p. 362), Bhāskara quotes an incorrect rule from Lalla's *Pāṭīgaṇita* saying: तर्हि तेन लल्लेन -

वृत्तफलं परिधिघ्नं समंततो भवति गोलपृष्ठफलम्।

इति स्वगणिते कथं परिधिघ्नं कृतम्। किन्तु वृत्तफलं चतुर्घ्नमेव पृष्ठफलं भवति। अस्य लल्लोक्तस्य गणितस्य दुष्टत्वाद्भूपृष्ठफलमपि दुष्टमित्यर्थः।

Here according to Lalla, the area of the surface of a sphere (using the modern terminology) is $(\pi r^2)(2\pi r) = 2\pi^2 r^3$, but Bhāskara corrects this to $4\pi r^2$ providing also a derivation of this formula.

(xiv). In his *Līlāvatī*, Bhāskara gave the correct formula for the surface area and volume of a sphere as follows (see Sarasvati, 1979, p. 210):

वृत्तक्षेत्रे परिधिगुणितव्यासपादः फलं य-
त्क्षुण्णं वेदैरुपरिपरितः कन्दुकस्येव जालम्।
गोलस्यैवं तदपि च फलं पृष्ठजं व्यासनिघ्नम्
षड्भिर्भक्तं भवति नियतं गोलगर्भे घनाख्यम् ॥

That is, in a circle, one-fourth the diameter multiplied by the circumference is the area; which, multiplied by four is the surface area of the sphere like that of a net surrounding a ball. This surface area multiplied by the diameter and divided by six is called ghana (volume) inside the sphere.

I. *The Later Uses of Bhāskara's Works*

(i). Use by Bhāskara's immediate successors—For example, Nārāyaṇa Paṇḍita's (fl. 1356 A.D.) *Gaṇitakaumudī Part I*, 16, p. 5 (see Dvivedi, 1936, *PWSBT 57 I)* which describes the rule of division as follows:

भाज्यादन्त्याद् हारः
    शुध्यति येनाहतः फलं तत् स्यात्।
अपवर्त्य भाज्यहारौ
    केनापि समेन वा विभजेत् ॥१६॥

is virtually a rearrangement of the words in Bhāskara's *Līlāvatī*, 18, p. 18 (see Āpaṭe, 1937, *L I, ASS 107*,) which is:

भाज्याद्धरः शुध्यति यद्गुणः स्यादन्त्यात्फलं तत्खलु भागहारे।
समेन केनाप्यपवर्त्य हारभाज्यौ भजेद्वा सति संभवे तु ॥१८॥

Another example is Nārāyaṇa's *BGV*, 15a, p. 6:

अस्मिन् विकारः खहरे न राशावपि प्रविष्टेष्वपि निःसृतेषु ॥१५a॥

which is word for word Bhāskara's *BG*, 11a-b, p. 5. Furthermore, in his *Bījagaṇitāvataṃsa*, Nārāyaṇa treats some topics along lines parallel to those in

Bhāskara's *Bījagaṇita*. One such illustration is the treatment of 'one and more than one colours.' The reader can find several instances of Nārāyaṇa's borrowings from Bhāskara's *Bījagaṇita* in our commentary on the *Text Alpha*.

(ii). Allusions to Bhāskara's works by other mathematicians—There exist a few citations from, and allusions to, Bhāskara's *Bījagaṇita* in Jñānarāja's *SSU Bījādhyāya*. See, for example, the manuscript *Berlin 833* which has the following:

> अथ भास्करीयव्यक्ताव्यक्ते यदुक्तं खहरे राशौ विकारो नेति भिन्नांके व्यभिचरति। (f. 1v., 8-9)

> अयमर्थो भास्करीये बीजे तथा नारायणीयबीजे विस्तारेणोक्तः। अत्रास्माभिः सूचनामात्रं कृतम्। (f. 8v., 2-3)

> एतद्भास्करीयबीजोदितोदाहरणं तुल्यम्। (f. 17v., 3-4)

(iii). Use by scribes.

(iv). Use by editors, commentators and translators—For example, Bhāskara's *BG*, 11a-b, p. 5 has been cited by Sūryadāsa in his commentary on Bhāskara's *Līlāvatī* as follows (see the manuscript *GMK, Wai, PPM 9762*, f. 21v., 8-9): तदुक्तं बीजगणिते।

> अस्मिन् विकारः खहरे न राशावपि प्रविष्टेष्वपि निःसृतेष्विति।

(v). Future work for historians of Indian Mathematics and Astronomy—This includes studies, synopses, translations, notes, reprints, articles, papers, seminars and various analyses pertaining to Bhāskara's works by modern mathematicians such as Colebrooke (1765 – 1837 A.D.), Bhāu Dājī (fl. ca. 1865 A.D.), Bāpu Deva Śāstrī (d. 1890 A.D.), Sudhākara Dvivedin (b. 1855 A.D.), Sarada Kanta Ganguly (fl. ca. 1926

A.D.), Bibhutibhusan Datta (fl. ca. 1926 A.D.), Avadhesh Narayan Singh (fl. ca. 1927 A.D.) and T. S. Kuppanna Sastri (fl. ca. 1955 – 1985 A.D.). For short biographies of some of these writers, see Jośī, 1988, *Golādhyāya,* Adhyayana, pp. 74-91.

(vi). Use by various institutions—Bhāskara's works were used as text books by teachers and students for at least five centuries.

(vii). Use by Bhāskarācārya Pratiṣṭhāna—This foundation was established in 1976 at Poona (see Jumde in Abhyankar, 1980). It provides research facilities for the study of Mathematics, in addition to acquainting the common readers with Bhāskara's works as well as the works of other Indian mathematicians. It has a collection of rare books, journals and research papers in Mathematics. It holds summer programmes on an all India basis (pp. 4-5). Incidentally, the establishment of the Bhāskarācārya Pratiṣṭhāna bears a testimony to the esteem in which Bhāskara's name is held even in modern India. The remarkable achievements of this genius—particularly in the realm of Mathematics, Astronomy and Astrology—are still a source of inspiration to scholars in the field of the History of Science.

3. *Description of the Manuscripts and Stemma*

A. *Overview of the Manuscripts*

Twelve manuscripts, out of some twenty-four known to be or to have been extant, have been collated in order to establish a critical edition of the first three chapters of Sūryadāsa's commentary, the *Sūryaprakāśa*. Upper case letters of the Latin alphabet are used to denote the above manuscripts, while Greek letters are used to denote hypothetical manuscripts.

The study shows that the available manuscripts of the portion of Sūryadāsa's commentary edited here belong to two recensions, which we have named A and β. Recension A consists of manuscript A and its descendants N and R, while recension β consists of the remaining nine manuscripts B, L, D, T, W, I, M, S and H. The text which is reconstructed from A and β is here called *Text Alpha*. It does not represent exactly the manuscript α. It represents the text of manuscript α with corrections (see Section 3.D. of this chapter). The reconstructed text is probably not the precise original text of Sūryadāsa's *Sūryaprakāśa* but must be very close to it (as will be argued).

The twelve available manuscripts are:

1. A. Oriental Institute, Baroda. 1424. Ff. 3-9, 11-25, 27-30, 38-44, 46-48, 50-73. Monday March 14, A.D. 1552.
2. B. Oriental Institute, Baroda. 9281. Ff. 119.
3. D. India Office, London. 2823. Ff. 73.
4. H. British Museum, London. 447. Ff. 46. Nineteenth century.
5. I. India Office, London. 2824 (1891). Ff. 71.
6. L. Akhila Bhāratīya Saṃskṛta Pariṣad, Lucknow. 4514. Ff. 1, 3-25, 27-99, 104-105, 108. Tuesday October 9, A.D. 1688.
7. M. Asiatic Society of Bombay. 279. Ff. 46.
8. N. India Office, London. 2825 (789). Ff. 1-62, 62b-73, 73b-86, 88-133.
9. R. Stadtsbibliothek, Berlin. 832. Ff. 1-15, 20-30, 32-129.
10. S. British Museum, London. 448. Ff. 40. Nineteenth century.
11. T. Wellcome Institute, London. β 589. Ff. 109. May 2, A.D. 1882.

12. W. Prājña Pāṭhaśālā Maṇḍala, Wai. 9777. Ff. 100. Friday October 11, A.D. 1813.

The manuscripts which were not available are the following:

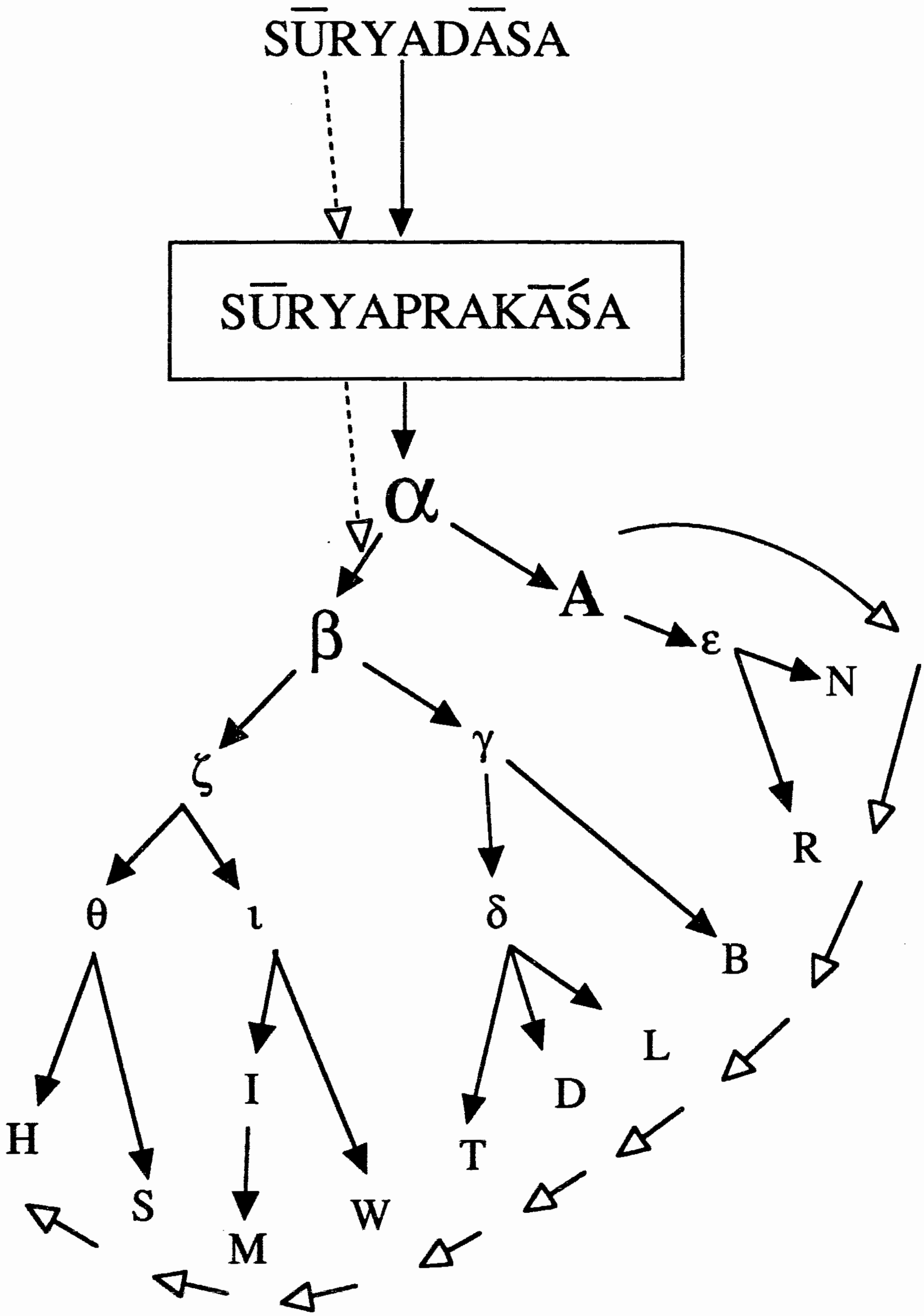
1. Oriental Research Institute, Alwar. 2597. 135ff. A.D. 1851.
2. Sarasvatībhavana Library, Benares. 37105. Ff. 1-55 and 55b-122. Saṃ. 1889 = A.D. 1832.
3. Asiatic Society of Bengal. I.B. 3.
4. Anup Sanskrit Library, Bikaner. 4906. 23ff. Incomplete.
5. Sanskrit College, Calcutta. Jyotiṣa 183. 74ff.
6. Cāndā, Central Provinces. Kielhorn XXIII 94. 149ff. Property of Balīrāma Subhājī.
7. Ranbir Sanskrit Research Institute, Jammu and Kashmir. 3061. 7ff.
8. India Office Library, London. 2826 (2290).[3]
9. Akhila Bhāratīya Saṃskṛta Pariṣad, Lucknow. 4529. 32ff. Incomplete.
10. Government Oriental Manuscripts Library, Madras. D. 13462. 122 pp.
11. Mahinathpur, P.O. Deodha, Darbhanga. Mithila III 216. 93ff. Incomplete. Property of Paṇḍita Rāmacandra Jhā.
12. Scindia Oriental Institute, Ujjayinī. 9350 (?).

B. *The Stemma*

On the basis of a comparison of the readings of the twelve manuscripts (which were available) the following family-tree or stemma has been constructed via an Apparatus Criticus:

3. 2826 (2290) is a reproduction of India Office 2825 (789), which is slightly corrected by the copyist. (Eggeling, Julius. Ed. 1896. *Catalogue of the Sanskrit manuscripts in the Library of the India Office. Part V.* London. P. 1011.)

The Stemma



In the stemma, the arrows indicate the inter-relationships of the manuscripts. The downward (filled) arrows indicate the descending of a manuscript from the one above it, by way of scribes' copying. The (unfilled) arrows around the perimeter in the clock-wise direction indicate the general order in which a choice has been made between the readings from the various manuscripts when the readings of A are not available or the readings of the two classes A and β disagree with each other. Thus, these arrows indicate the general heirarchy of authority of the manuscripts. The dotted arrows in our stemma denote the hypothesized return of Sūryadāsa to revise the *Sūryaprakāśa*.

The subsequent section describes the relationships among the various manuscripts and the specific characteristics of the manuscripts falling under the various classes, starting from the top of the stemma and working down, from right to left. Since these relationships and characteristics depend on the readings of these manuscripts relative to the *Text Alpha*, it is pertinent to state here that Chapter II, the *Text Alpha*, has its own numbering (from 1 to 70) printed in bold Devanāgarī numerals (१, २, ३, . . .) at the bottom centre of each page (except the title page). References throughout the thesis to pages of the *Text Alpha* (including especially references in the present section on manuscripts and in Chapter III, Apparatus Criticus) correspond to these Devanāgarī numbers even when these references are typeset with common numerals (1, 2, 3, . . .). For example the notation "p. 39, 1" means "page ३९, line 1". This form will be used generally for Sanskrit texts as well as the *Text Alpha*. The pages of Chapter II also include the continuous numbering of the thesis in the upper right hand corner and these numbers are used only in the Table of Contents.

As many sentences in the ensuing sections contain English, Greek and Sanskrit texts, one further convention we make is to end a sentence with a period (.) or a daṇḍa (।) depending on whether the last character of the sentence is in Latin/Greek or Devanāgarī script respectively.

C. *Relationships Between the Archetypes and Descendants*

(a). *Copy α and Its Relation to Sūryadāsa's Original Text*

α is an early copy of Sūryadāsa. It was written between the date of the composition of the *Sūryaprakāśa* (1538 A.D.) and that of the copy A (1552 A.D.). It can essentially be reconstructed from A and β. It may not be entirely correct because some of the readings shared by A and β are corrupt. For example, in the Apparatus Criticus we read: (i) p. 2, 10. °गतव्यक्ता° α, (ii) p. 48, 11. निमित्तभूतौं° α. These incorrect readings have been corrected by us in the *Text Alpha*. Moreover, it can not be determined whether some of the apparently genuine passages in β that are omitted by the writer of A were in α or were acquired by the writer of β from some other source (see our comments in section 3.D. of the present chapter).

Certainly, α was not complete, but exhibited some lacunae. For example, the introduction to, lemma or mūla of, and explanation of verse 67 (see *Text Alpha*, p. 69), are missing from α, though the solution to the problem contained in this verse is in the text. Another place where there is a lacuna is after p. 53, 17. Here the introduction to, mūla of, and explanation of verse 53a are missing, but there is a reference to this verse (beginning with हरतष्टे) on p. 54, 9.

(b). *Class A*

This class consists of manuscripts A, N and R. Occasionally, the readings of manuscript L which are deviant from those of class β (to which L belongs), share the readings of class A. For example: (i) *Text Alpha*, p. 26, 19-20. करणीति ... तथा om. β (except L; °रणीति ... तथा in the bottom margin W^1 (the superscript '1' refers to a later scholar who wrote in the margin of W)); (ii) p. 24, 18. मूलं om. Nγ (except L); (iii) Also p. 63, 15, class A and manuscript L share the wrong reading ५ | १३ (instead of १३ | ५, which is correct. Note that here N omits ५).

(i). *Manuscript A*. Oriental Institute, Baroda. 1424.[4] Ff. 3-9, 11-25, 27-30, 38-44, 46-48, 50-73. Paper: Dimensions unknown. Number of lines per page: 12-13. Number of 'akṣaras' (syllables) per line: 46-51, or about 48 on average.

Colophon on folio 73 verso: इति श्रीमद्दैवज्ञज्ञानराजश्रुतपंडितसूर्यदासविरचिते सूर्यप्रकाशनाम्नि भास्करीयबीजभाष्ये भावितकबीजं संपूर्णं ॥ छ ॥ छ

Post-colophon: सूर्यप्रकाशभाष्यं वियदंगपुरंदरैर्मिते शाके ॥ श्रीगौडसूर्यनाम्ना लिखितं स्वपरोपकाराय ॥ छ॥ संवत १६०९ ॥ वर्षे शाके १४७३ प्रमालपक्वदेशे श्रीसलेमसाहराज्ये ॥ अवंतीनगरे लक्षितं ॥ सूर्यप्राकास संपूर्णा स्मासः ॥ फागुणमासे कृश्नापक्षे तिथिपंचमीदीने ॥ सोमवासरे ॥ लषितं पोस्तिका पठनार्थं ॥ लेषकपाठिकययो स्तुः ॥ कल्यणांमस्तुः ॥ श्रीरस्तुः ॥ कल्पमस्तुः ॥ माहामांगल्यश्रीः

According to this information manuscript A was copied by Sūrya Gauḍa in the city of Avantī (modern Ujjayinī), in the Mālava region, during the reign of Salema Sāha, i.e., Islāma Śāha, who ruled from 1545 until 1553 A.D.; and the copying was completed on Monday March 14, 1552 A.D. (see Pillai, 1922, *Vol. V*, p. 306). Thus, manuscript A was copied in the lifetime of Sūryadāsa.

As manuscript A is about 440 years old, the writing on many of its folios is not only blurred but has also faded away completely in places, in the microfilm. In most such instances, the akṣaras from the folios which precede or follow a particular folio, appear on that particular folio. Since the microfilm of A is illegible at several places, and some of the folios of A are missing, to fill the gaps one must resort to the readings of the descendants N and R, if their readings exist, coincide and make sense; if not, the readings of the β-class become the sole witness of α. (See section 4.A. below, Principles of the Edition.)

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4. Nambiyar, Raghavan. (Ed.). (1950). *GOS No CXIV. Vol. II. An alphabetical list of manuscripts in the Oriental Institute Baroda*. Baroda: Oriental Institute. Pp. 1212-1213.

Some of the unique characteristics of manuscript A are the following:

A includes the use of the letter छ to mark the end of an idea, a sub-section, or a section. Such a use of the letter छ is found in some of the writings of the sixteenth century as we can see from Aryan (1989, p. 36).

Occasionally, 'pūrvamātrā' is used in A. For example, (i) folio 17 verso, 12, i.e., *Text Alpha*, p. 52, 2. °गुणाको for °गुणाकौ; (ii) folio 21 recto, 2 and 4, i.e., p. 62, 19 and 23. वियाग° for वियोग°; (iii) folio 21 recto, 6, i.e., p. 63, 1. हारेण for हारेण।

Moreover, when the long – ā mātrā is omitted, a slanting stroke is put over the consonant. For example, f. 12v., 1, i.e., p. 33, 15. °द्विध for °द्विधा।

Quite often तु is written as नु (apparently intentionally); see f. 21v., 8, i.e., p. 65, 5, *Text Alpha*.

The copyist of manuscript A has replaced the 'visarga' by ष् before the gutturals क or ख (as on p. 27, 5. गुणाकष्कल्पित:) or before the labials प or फ (as on p. 15, 3. ततष्पीतो). Normally it remains a visarga in these cases, for instance, in the manuscripts of the other recension, β, of the *Sūryaprakāśa*.

On p. 5, 24, i.e., f. 3r., 12, manuscript A reads तद्वेदेत्यण: instead of the correct तद्वेदेत्यण् found in Pāṇini's rule 4, 2, 59 (see e.g. Böhtlingk, 1977, p. 176), which says: तदधीते तद्वेद, with Pāṇini's rule 4, 1, 83 (Böhtlingk, 1977, p. 159), which says: प्राग्दीव्यतोऽण्।

Also on p. 6, 9, manuscript A has the grammatically incorrect reading ज्ञानयत instead of the correct reading ज्ञायत। This reading has been corrected by ε.

An example of a mathematically incorrect reading contained in A is the reading ५।१३ on p. 63, 15, which was also mentioned under section (b), Class A.

Rarely, a 'nyāsa' (setting out) is not placed correctly in A. As an illustration, हार: क १८ क ३ (*Text Alpha* p. 33, 4-5) is misplaced on f. 12r., 8 after °क्रमेण (p. 33, 5), whereas it should have occurred before अथ on f. 12r., 8 (p. 33, 5). This misplacement is followed by ε.

Manuscript A has a few marginalia, some of which seem to be in a hand different from that of Sūrya Gauḍa, who is the copyist of manuscript A. For example, f. 9r. has = त्वेनो = in the bottom margin, while the text already has this word in line 12, though it is not very clear. Usually, the words to be inserted have the sign × to their left and right, and the line number (from the top or bottom) or the syllable number to their right (see e.g. f. 18r., 10, which has × रा × ४ in the left margin). Incidentally, this marginalium indicates that manuscript A has been studied by some later scholar.

(ii). *Relation of Manuscript A to Manuscript α.* Given the brief time between the date of the composition of the *Sūryaprakāśa* (1538 A.D.) and that of the copying of manuscript A (1552 A.D.), manuscript α is probably a direct copy of Sūrya's original and manuscript A is a direct copy of manuscript α. A is not entirely correct. Also, there were passages in α that the copyist of A missed (e.g. (i) p. 9, 5-8. भवति ... °शेषस्य and (ii) p. 25, 8-9. भवति ... च: both missed due to homoeoteleuton).

(iii). *Relation of Manuscript A to the Sūryaprakāśa.* Manuscript A is apparently a copy of a copy of the original *Sūryaprakāśa*; no traces of possible contamination are found in it. But since A contains errors and omissions which Sūryadāsa, who was an expert in the Sanskrit language, would never have made, A does not represent the original *Sūryaprakāśa* with entire fidelity.

(iv). *Manuscript ε.* The omissions of manuscript A are generally (but not always) shared by its descendants N and R, as far as the portion of the commentary which is being edited is concerned. For example consider (i) p. 6, 20. पूर्वं and (ii) p. 9, 5-8. भवति ... °शेषस्य: both of them are omitted in A, N and R. But there exist some errors, omissions and corrections on which only N and R (but not A) agree; for example on p. 6, 9 the reading of A is ज्ञानयत but that of N and R is ज्ञायत; on p. 17, 9-10. तद्भवति ...

गुणाने is omitted in N and R (due to homoeoteleuton) but not in manuscript A; on p. 5, 14 for the correct reading एकतर°, A, N and R have एकत° in their texts, while only manuscript A has ×र× in the right margin. These instances reveal that there exists some manuscript ε between manuscript A and its two descendants N and R. It is the writer of ε who has made the errors, omissions and corrections. N and R are copied from ε and hence they are the immediate descendants of ε. Manuscript ε does not seem to have been written very carefully because N and R have several spelling errors. Nonetheless, ε plays an important role in the reconstruction of α because the readings of ε are taken into account (provided they make sense), whenever manuscript A is illegible or skips a folio.

(v). *Manuscripts N and R*. N and R are independent copies of ε because in addition to their common omissions and errors (which they share with manuscripts A and/or ε), they have individual errors and omissions such as: (i) p. 18, 6-7. इति ... स्पष्टैत्र om. N; (ii) p. 25, 16-19. एवं ... ४ om. R; (iii) p. 22, 14. २ं om. R, २ां N.

The scribes of N and R do not seem to be making any additions, alterations or corrections besides those already made by ε, as far as the portion of the text, which has been edited, is concerned. This, however, does not preclude the possibility that N and R might have access to the manuscripts or sources other than ε.

A peculiarity of N and R is the use of न् for त्। This use is frequent in N (see, e.g., f. 2r., 1) but rare in R (see f. 29r., 5).

There exist some similarities between N, R and B. One of them is the use of letters for numerals. For example, N has the letter उ for numeral ३, while R and B have the letter ङ।

Manuscript T shares some of its readings with N and R as follows: (i) p. 14, 14. °संवादो RBT; (ii) p. 25, 15-16. जातः ... सन् om. εT.

The separate particulars of manuscripts N and R are discussed below.

(vi). *Manuscript N.* India Office Library, London. 2825 (789).[5] Ff. 1-62, 62b-73, 73b-86, 88-133 (the catalogue has foll. 134. Counted 133, 87 passed over). Paper: 9 $\frac{3}{4}$ in. × 4 in. (i.e. 24.77 cm. × 10.16 cm.). Number of lines per page: 10 (sometimes 9 or 11). Number of akṣaras per line: 31-37, or about 33 on average.

Colophon on folio 133 recto: इति श्रीमद्दैवज्ञज्ञानराजसुतपंडितसूर्यदासविरचिते सूर्यप्रकाशनाम्नि भास्करीयभाष्ये भावितारव्यकवीजं सं समाप्तं छछ छछ छछछ ||

Post-colophon: Missing.

There is no mention of the name of the scribe, place or date at which the copying was completed.

It was given to the library by Henry Thomas Colebrooke, as is mentioned in the beginning (i.e. on folio 1 recto) of this manuscript. Colebrooke (1765 – 1837 A.D.) was the son of Sir George Colebrooke, Chairman of the East India Company's Directors in 1769 (Buckland, 1968). Colebrooke was a mathematician, astronomer and a profound scholar of Sanskrit. He held a variety of offices in India—from an Assistant Collector to the Head of a famous court, the Sadr Diwani Adalat. He wrote on various subjects such as algebra, astronomy, Sanskrit grammar, botany, geology, comparative philosophy, agriculture, commerce, Hindu law and the Vedas. Colebrooke was a member of several literary academies—both within and outside India. Furthermore, he was President of the Asiatic Society of Bengal from 1807 to 1814. Also, Colebrooke was Director of the Royal Asiatic Society which he helped to found in 1823. He donated his collection of Sanskrit manuscripts to the East India Company's library in 1818. (Pp.87b - 88b)

The special features of manuscript N are the following:

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5. Eggeling, Julius. (Ed.). 1896. *Catalogue of the Sanskrit manuscripts in the Library of the India Office. Part V.* London. P. 1011.

There exist many repetitions and omissions in N. Moreover, f. 33v., 6 - f. 34v., 5 have some text which is not supposed to be there. The mistakes in N include many omissions of or wrong uses of vowels and consonants, such as (i) f. 13r., 8. कारणी° for करणी°; (ii) f. 11v., 4. अ for अथ।

Furthermore, N has lots of corrections all over the text, and plenty of marginalia which include notes, remarks and corrections made most probably by Colebrooke.

(vii). *Manuscript R.* Stadtsbibliothek, Berlin. 832 (Chambers 348).[6] Ff. 1-15, 20-30, 32-129. (Number 9 was not covered in the counting of the folios). Paper: Dimensions unknown. Number of lines per page: 8-10. Number of akṣaras per line: 35-46, or about 39 on average.

Colophon on ff. 128v. - 129v.: इति श्रीमद्दैवज्ञज्ञानराजसुतपंडितसूर्यदासविरचिते सूर्यप्रकाशानाम्नी भास्करीयवीजभाष्ये भावितकवीजं संपूर्णम् ॥ रा

Post-colophon: Missing.

In the microfilm, the same folio is numbered 8 and 9 but no text is lost. Also, folios 16-19 are missing but no text is lost. However, text is lost because of folio 31 which is missing. There is no mention of the name of the scribe, date or place. In the beginning (before f. 1r.), R has Chambers 348, vol 1st. This indicates that R comes from the collection of Sanskrit manuscripts belonging to Sir Robert Chambers (1737 – 1803 A.D.)—a judge who joined the Calcutta Supreme Court in 1744 (Buckland, 1968, p. 78a).

R contains several instances of omissions of or wrong use of vowels and consonants. For example, (i) f. 29r., 3. ऊक्तं for उक्तं; (ii) f. 30 r., 2. स व for सम एव; (iii) f. 25v., 4. इतिमा for इमाः। Out of all of the twelve available manuscripts of the *Sūryaprakāśa*, manuscript R has the maximum number of such errors.

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6. Weber, A. (Ed.). (1853). *Die Handschriften-Verzeichnisse der Königlichen Bibliothek. Erster Band. Verzeichniss der Sanskrit-Handschriften.* Berlin. P. 231.

R has several omissions but very few repetitions. The dot or sign for a negative number is usually ignored in R (for an exception, see f. 24r., 2).

The scribe of R does not seem to be making any additions from other sources as he is copying. There are no insertions and no marginalia.

(c). *Class β*

This class, consisting of the nine manuscripts B, L, D, T, W, I, M, S and H, is also important for the reconstruction of α (and *Text Alpha*), because some portions of the text of the *Sūryaprakāśa* exist only in this class. For example, (i) p. 8, 14-16. यद्युभे ... स्पष्टम् and (ii) p. 10, 15-16. तथा[3] ... स्यात् which exist in class β, are missing from class A. Thus class β comes to the rescue of α under the following situations: (i) When the reading of class A is missing; (ii) When manuscript A is illegible or omits a folio and the reading of ε does not make sense or N and R disagree on a particular reading; (iii) When the reading of manuscript A is incorrect.

As far as the date of composition of manuscript β is concerned, some estimate can be drawn as follows. Out of all of the available manuscripts of class β, the one which has the earliest date mentioned in it is the manuscript L. It was written in 1688 A.D. Class β (and hence manuscript β) shows contamination by Kṛṣṇa's commentary, the *Bījapallava* on Bhāskara's *Bījagaṇita*. The *Bījapallava* was written in about 1600 A.D. Therefore β was written sometime between 1600 A.D. and 1688 A.D. So we assign ca. 1620 A.D. to the copy β.

The special features of class β are the following:

There exist apparently genuine passages or words in class β that are omitted in class A. An illustration of this fact is a portion (p. 29, 15 - p. 31, 4. अथ ... बुद्धिमता) of Sūryadāsa's explanation to verse 27c-28b. This text has been supplied from class β in order to reconstruct *Text Alpha*. Presumably Sūryadāsa came back at some point and revised his *Sūryaprakāśa*. The writer of β possessed a copy of this revised version, and incorporated the revisions made by Sūryadāsa in his copy. This copy of the revised version stands between α and β in the stemma.

One instance of evidence for Sūryadāsa's revising the *Sūryaprakāśa* is that a verse, which appears at the end of both the *Sūryaprakāśa* and the *Gaṇitāmṛtakūpikā*, and which lists the first eight of Sūryadāsa's works mentioned by Dikshit (Sarma, 1950, *SB VIS 2*, p. 222), is absent from the very early copy A (and its descendants), though this very verse exists in all of the manuscripts belonging to recension β (see Chapter I, sections 1.B. and 1.E. above). This verse, which appears to be Sūryadāsa's own creation, was thus added to the *Sūryaprakāśa* at a later date. Quite conceivably other revisions could have been made by Sūryadāsa as well at that time. One instance of other revision is the addition of another verse to the *Sūryaprakāśa* after the one just mentioned. This last verse (which is stated below) also exists in the manuscripts of the *Sūryaprakāśa* belonging to only the β–recension:

परोपकारार्थमुदाहृतेन मद्वाग्विलासेन रसोज्वलेन ।
शब्दाभिधं ब्रह्मयतः प्रवृत्तं सः प्रीयतां यादवराजसिंह ॥८॥

The Appendices and Apparatus Criticus for the *Text Alpha* show that there exist instances of contamination of the text of class β by Kṛṣṇa's commentary *Bījapallava* on Bhāskara's *Bījagaṇita*, Sūryadāsa's commentary *Gaṇitāmṛtakūpikā* on Bhāskara's *Līlāvatī*, and Bhāskara's *Bījagaṇita* as follows: (i) The text pertaining to the explanation of verses 37c-38d, which we have put in the Appendix #9, seems to have been borrowed by the writer of β, with some additions, from the *Bījapallava* (see Radhakrishna Sastri, 1958, *Madras GOS 67*, p. 77, 5-6). (ii) The text pertaining to the explanation of verses 64c-65d, which we have put in the Appendix #17, has been borrowed by the writer of β from the *Gaṇitāmṛtakūpikā* (see the manuscript *Wai, PPM 9762*, f. 121r., 5-8). (iii) Part of the text pertaining to the artha of verse 43b-44a, which has been put in the Appendix

#12, has been taken by the writer of class β from Bhāskara's *Bījagaṇita* (see Vidyāsāgara, 1878, p. 24, 17).

Class β has some disorders or misplacements of sentences. For example, the text which belongs to the solution of the first example under verse 30c-31b is out of place. Therefore it has been placed in the Appendix #5. The corresponding text of class A, which has been chosen, is different for it omits most of the text contained in class β.

At some places, class β has repetitions and is inconsistent. For example, the text which has been put in the Appendix #3 should have come before (and not after) verse 16c-d because it explains the previous verse 15c-16b. It is omitted in class A. Also it is a repetition (with omissions and additions) of the explanation which is already given under verse 15c-16b.

Class β contains a few errors. As an illustration, some part of the text related to the solution of the problems given in verse 36c-37b, makes no sense in relation to the context. This β-text had to be put in the Appendix #8. It differs from its counterpart which is in the A-text.

At some places, class β contains detailed explanations where class A has short summaries. Some illustrations to this effect are as follows. (i) The detailed explanation of class β pertaining to verse 39b has been utilized for the reconstruction of *Text Alpha*, on the observation that Sūryadāsa generally provides detailed explanations to the verses involving 'sūtras' (rules). On the other hand, the one-sentence explanation of A has been placed in the Appendix #10. (ii) Similar is the situation pertaining to the meaning and demonstration of verse 19a-d where recension A has only a one-line hint (which goes to the Appendix #4). The detailed text of class β has been chosen. (iii) In the demonstration part of verses 33a-34d, class A has one short-sentence text which has been placed in the Apparatus Criticus; the corresponding text from class β has been utilized. (iv) The brief explanation of verse 40b-41a provided by the manuscripts of class A goes to the Appendix #11 and the corresponding detailed explanation from the manuscripts of class β has been supplied for the purpose of reconstructing the *Text Alpha.*

The omissions in class β include parts of the text such as p. 11, 20. अत्रो° ... ज्ञेया । Another notable omission of class β pertains to the colophons, such as those at the conclusion of karaṇī (p. 46, 9-10. इति ... °गमत्) and kuṭṭaka (p. 70, 12-13. इति ... °मगात्).

The above details, repetitions, insertions or alterations in class β give rise to several possibilities which need to be explored, (which will be stated later in a separate section D. of this chapter,) which include, among others, revision of his text by Sūryadāsa himself and results of students' copying etc.

(i). *Differences of Class β From Class A.* The major differences include the following:

(1). Class β consists of nine manuscripts, while class A consists of three manuscripts; A being an early copy while β is a later descendant of α. Of course, this proposition might change if further manuscripts of the *Sūryaprakāśa* can be located.

(2). Class A is not contaminated by external sources, whereas class β is.

(3). Since it cannot be determined whether some of the apparently genuine passages in class β that are omitted by class A were in α or were acquired by β from some other source such as Sūryadāsa's revision of α, the readings of class β which are not shared by class A are less reliable than those of class A which are not shared by class β.

(4). Class A summarizes the text in a few places but class β does not.

(5). Class β has some repetitions, while class A does not.

(6). Class β omits the colophons which class A contains.

(ii). *Relation of Manuscript β to Manuscript α.* β is descended from Sūryadāsa's revision of α. It was copied about 70 years later than the copy A. On the one hand, it omits certain portions of α, for example, the colophons. On the other hand, β has several additions or alternative explanations, some of which seem to be the genuine creations of Sūryadāsa, while some others are clearly due to its being contaminated by sources other than the *Sūryaprakāśa*. Those alterations which did not seem to be genuine have not been incorporated in the *Text Alpha* which we have reconstructed. Such alternative texts have been placed either in the *Apparatus Criticus* or in the *Appendices*.

Nonetheless, copies β and α have a large portion of the text of Sūryadāsa's commentary in common (at least 90%). Since β fills the omissions of A and replaces the short summaries of A, the reconstructed text would have been less organized and clear without the text of β.

(iii). *Relation of Manuscript β to the Sūryaprakāśa*. β is a contaminated descendant of a revised version of the *Sūryaprakāśa*. The authenticity of some of the alterations in β is questionable, for it is not clear who has provided those alterations. Consequently, it cannot be said with certainty how close β is to the original *Sūryaprakāśa*.

(d). *Class γ*

This class consists of the manuscripts B, L, D and T. This group has similar readings at most places. Here are three examples: (i) p. 32, 10. वि^{o2} om. γ; (ii) p. 36, 7-8. योगे क्रियमाणे] योगं कृत्वा वर्ग संस्थाप्य ता एकादिसंकलितमिताः करण्यः स्युः। ननु यत्र द्वित्रिचतुरादिस्थानस्थितानां तुल्यकरणीनां वर्गे क्रियमाणे ये सहजा वर्गराशयस्तावतामेव मूलैक्यं रूपाणि प्रकल्पयेदित्यर्थः। निमित्तजास्तु करण्य एव कल्प्याः। तासां यथासंभवं योगे क्रियमाणे γ; (iii) p. 24, 18. मूलं om. Nγ (except L). (Recall that occasionally, the deviant readings of L coincide with the readings of class A. See section C.(b), Class A).

The readings of class γ are significant in the construction of manuscript β and hence in the reconstruction of the *Text Alpha*. The important manuscripts of class γ are B and L. We do not know the dates of B and D. The copying of L was completed in 1688 A.D., and that of T in 1882 A.D. Manuscript B seems to have been copied directly from γ ; whereas L, D and T are copied from δ which is a copy of γ. (As mentioned before, T shares some of its readings with N and R. See section C.(b)(v), Manuscripts N and R).

Furthermore, B, L, D and T are copied independently of each other, as is evidenced by the following: (i) p. 2, 17-18. विश्लेषे ... पूरयन् om. B; (ii) p. 11, 19-21. धनांक° ... मूलमिति om. L; (iii) p. 8, 9-10. संबंधः ... युति° om. D; and (iv) p. 10, 3. अन्य° ... स्यात् om. T. Note that none of these manuscripts repeats the omissions of the other.

(i). *Manuscript γ.* This manuscript is a copy of β. It seems that γ was not written very carefully because its direct descendant B has many errors and empty spaces. Certainly, γ was written sometime between the date of copying of β (ca. 1620 A.D.) and that of L (1688 A.D.).

(ii). *Differences of Class γ From Class ζ.*

(1). Class γ consists of manuscripts B, L, D and T while class ζ consists of manuscripts W, I, M, S and H where γ and ζ are the respective ancestors of these two classes.

(2). Generally, the readings of classes γ and ζ coincide, but there exist some dissimilarities as well. For example, (i) p. 25, 15. जातः करणी° om. ζ; (ii) p. 25, 16. °वर्गेणा° ... सन्] °वर्गेण हतः सन्नंतरमिदं ४ ζ. In these places, class γ has the correct text. This indicates that the classes γ and ζ have a common distant ancestor β but they have different immediate ancestors, namely, γ and ζ respectively.

(3). Furthermore, manuscript W was written in 1813 A.D.; the dates of I and M are unknown but M is a copy of I; S and H were copied in the nineteenth century. Thus class ζ consists of modern manuscripts. Therefore, the readings of class γ are likely more authentic than those of class ζ. Note that γ was written between ca. 1620 A.D. and 1688 A.D. and ζ was written between ca. 1620 A.D. and 1813 A.D.

(iii). *Manuscript B*. Oriental Institute, Baroda. 9281.[7] Ff. 119. Paper: Dimensions unknown. Number of lines per page: 9. Number of akṣaras per line: 38-47, or about 43 on average.

Colophon on f. 119v.: ॥ शुभमस्तु ॥

Post-colophon on f. 119v.: धर्माधिकारिदेवोपनाकमाहाद्देवसुतकृष्णस्येदं पुस्तक काश्यां लिखितमिदं स्वार्थं परार्थं च ॥

According to this information, B was copied in Kāśī by Kṛṣṇa, the son of Mahādeva who has the surname Dharmādhikārideva. There is no mention of the date at which the copying was completed. On folio 1 recto, the manuscript has something which is illegible.

The peculiarities of manuscript B are the following:

B has a few language errors. These include the wrong use of vowels and consonants as follows: (i) P. 23, 12 or f. 12r., 5 has गुणाान° for गुणान° and स्वरुपं for स्वरूपं। (ii) In a few places, the wrong uses of the consonants ल् for न् and ज् for य् are found, such as f. 16v., 5. °माले for °माने and f. 10v., 8. यातं for जातं। (iii) Likewise f. 11v., 5 and 8 have the wrong use of letters for digits such as ट for ८, ड़ for ३ and द for ६ ।

B has several repetitions of words or lines.

B has several omissions. A peculiarity of B is that it has many *blank spaces* in the text, which correspond to the omissions. Perhaps its ancestor was corrupt or illegible at these places. Sometimes there are omissions in B, but no blank spaces are left to indicate

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7. Nambiyar, Raghavan. (Ed.). (1950). *GOS No CXIV. Vol. II. An alphabetical list of manuscripts in the Oriental Institute Baroda* Baroda: Oriental Institute. Pp. 1212-1213.

this fact; for example, f. 18v., 2 has अ in place of अथ and f. 28v., 3 has ष्टं for दृष्टं, but there are no blank spaces for the missing letters.

Quite often, a nyāsa is not placed correctly in B. For instance, क २५ं क २७ (see *Text Alpha* p. 30, 18) is misplaced on f. 17r., 1 after गुणिते (p. 30, 19), whereas it should have occurred before अस्मिन् on f. 16v., 9 (p. 30, 19). Similarly क ५४ क ९ (p. 33, 15) is misplaced on f. 19r., 1 after याव° (p. 33, 16), whereas it should have occurred before अत्रापि on f. 19r., 1 (p. 33, 16).

Finally, manuscript B has marginalia which consist of insertions to be made; see, e.g. f. 21v., 3 and f. 118r., 6.

(e). *Class δ*

This class consists of manuscripts L, D and T. These are independent copies of δ (see section C.(d), Class γ). Manuscript δ seems to have been written between the dates of copying of the manuscripts γ and L (1688 A.D.).

Clearly, B is not a copy of δ. The following three examples support our claim: (i) p. 2, 17-18. विश्लेषे ... पूरयन् om. B; (ii) p. 2, 8-11. याव° ... भजे om. LD; and (iii) p. 12, 15. यथावत्स्थित° B, यथावस्थित° ANLD, यथास्थित° RTζ. (In the third example, manuscripts A, N, L and D have the correct text. Note that T is a later manuscript and its writer seems to have had access to manuscripts other than δ.) These three examples suggest that L and D copy what δ has. On the other hand, the many similarities of B with L, D and T (see section C.(d), Class γ), and the unique way in B of showing omissions through corresponding blank spaces suggest that B and δ descend independently from γ.

(i). *Differences of Class δ From Manuscript B.*

(1). Manuscripts of class δ have fewer mistakes, repetitions and omissions than does manuscript B.

(2). Manuscript B has blank spaces for omissions, while L, D and T do not.

(3). Manuscript δ contains a more correct and complete text than does B; it was probably copied earlier from γ, before the latter became illegible in places. Nonetheless, the writer of B seems to have tried very hard to leave the text as close to its archetype as possible. Hence, the authority of B is to be considered greater than that of δ as far as reconstructing β goes.

(ii). *Manuscript L.* Akhila Bhāratīya Saṃskṛta Pariṣad, Lucknow. 4514.[8] Ff. 1, 3-25, 27-99, 104-105, 108. Paper: Dimensions unknown. Number of lines per page: 11-12. Number of akṣaras per line: 35-43, or about 39 on average.

Colophon: Missing.

Post-colophon on f. 108v.: ।। संवत् १७४५ वर्षे आश्वनमासे कृष्णापक्षे तिथौ द्वादश्यां भौमवासरे लिखितेयं वीजगणितटीका नथमल्लपठनार्थः ।।

According to this information, L was copied for Nathamalla and the copying was completed on Tuesday October 9 in 1688 A.D. (see Pillai, 1922, *Vol. VI*, p. 179). The place is unknown.

As far as the physical condition of L is concerned, since this manuscript is over 300 years old, the writing on a few of the folios which are at the end of this manuscript is blurred. Also, there seems to be discolouration of the entire manuscript. It has also been eaten by moth-larvae in places, though it is legible.

The distinguishing characteristics of manuscript L are the following:

The scribe of L makes intelligent corrections, changes or additions as he copies from its archetype δ. For example: (i) He has tried to correct the solution of the problem pertaining to verse 36c-37b, perhaps using (with alterations) Bhāskara's

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8. Iyer, K. A. Subramania et al. (Eds.). (1963). *A catalogue of manuscripts in the Akhila Bharatiya Sanskrit Parishad Lucknow*. Lucknow. (See Pingree, 1970b, *CESS A 1*, p. 26a.)

Bījagaṇita text, p. 21 (see apparatus criticus to Appendix #8). (ii) P. 47, 20. रूढः (see *Text Alpha)* has been changed to दृढ, perhaps following Bhāskara's verse 48b. (iii) P. 9, 9. अथात्र ... रूपत्रयमिति (which contains the introduction to verse 3c-4b), has been replaced by the corresponding introduction in the *Bījapallava*, p. 11, 1. अथोक्तेऽर्थे शिष्य-बोधार्थमुदाहरणचतुष्टयमुपजातिकयाह । (iv) In order to explain verse 3c-4b, some text has been added on page 9 between lines 13 and 14. This text has been borrowed word for word from the *Bījapallava*, p. 11, 6-8 (see the Apparatus Criticus).

The scribe of manuscript L seems to have had access to a manuscript of class A (see section C.(b), Class A for illustrations to this effect).

L contains the first 35 complete verses from Bhāskara's *Bījagaṇita* and adds इति to the end of almost every verse. Of course, L omits the lemmas when it contains a complete verse (see, e.g., p. 9, 9. रूपत्रयमिति om. LS).

The scribe of L employs sandhi wherever it is possible. In fact, out of the twelve available manuscripts of the *Sūryaprakāśa*, L has the maximum use of the rules of sandhi.

L has very few repetitions.

There exist a few omissions in L. For instance, p. 23, 11-13. °वद्[1] ... निरूप° om. L. This omission seems to be a consequence of homoeoteleuton.

Rarely, L has numerals above the words to correct their order. For example, f. 22v., 7 has न यदि where there is a "२" over न and a "१" over यदि, which indicates that this text is to be read as यदि न ।

Words or letters to be inserted in the text are usually above the line in manuscript L (see e.g., f. 4v., 3. धन), though sometimes below the line (see f. 6r., 12. ता) or in the margin (see f. 7v., 6. कं). Some of these insertions seem to be in a hand different from that of the original scribe, which indicates that L has been studied by some later scholar.

(iii). *Manuscript D*. India Office Library, London. 2823 (1533a).[9] Ff. 73. Paper: 9 in. × 6¼ in. (i.e. 22.86 cm. × 15.88 cm.); size royal 8 vo. Number of lines per page: 26-28, sometimes 24. Number of akṣaras per line: 22-28, or about 24 on average.

Colophon on f. 73r.: शुभमस्तु ॥

Post-colophon: Missing.

There is no mention of the name of the scribe, or place or date at which the copying was completed. In the beginning (i.e. on f. 1r.), D has: Presented by H. T. Colebrooke, Esq. (Recall that manuscript N was also presented by Colebrooke).

The particular features of manuscript D are:

D has very few repetitions. D has omissions. The dot representing a negative number is almost always omitted in D (for an exception, see f. 8r., 1).

Like manuscript L, D too has numerals above some words to indicate their correct order. See, e.g. f. 14r., 25. The complete word in the text is स्वर्गास्वगा with a " १ " over (the second) स्व and a " २ " over र्गा । This means that the word is to be read as स्वस्वर्गागा । Another illustration is the word °युछेक्द° with a " १ " over क् and a " २ " over छे (see f. 17v., 15) meaning thereby that the reading in the correct order is °युक्छेद° ।

A peculiarity of D is that it has a symbol somewhat like " २ " over the place where a long vowel is intended, as in f. 2r., 2, we find शैस्त्र, indicating that शास्त्र is the correct reading. These marks may have been inserted by some later scholars.

Words to be inserted are written in the margin, see, e.g. f. 2v.

If a complete word does not fit in a line, the sign |\ is used at the end of the line to indicate this fact (see f. 3v., 11).

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9. Eggeling, Julius. (Ed.). 1896. *Catalogue of the Sanskrit manuscripts in the Library of the India Office. Part V.* London. Pp. 1009-1010.

(iv). *Manuscript T.* Wellcome Institute for the History of Medicine, London. β 589.[10] Ff. 109. Paper: Dimensions unknown. Number of lines per page: 12, 13, or 15. Number of akṣaras per line: 35-47, or about 40 on average. No catalogue listing except as noted in the footnote.

Colophon on f. 109v.: शुभंमस्तु सर्वजगताम् ॥

Post-colophon on f. 109v.: संवत् १९। ३९ ॥ वैसाख दी° १

This information indicates that the copying of the manuscript was completed on May 2, 1882 A.D. (see Pillai, 1989). The places and names of the scribes are unknown.

The following are the peculiarities of manuscript T:

T has been copied by two scribes—the first twenty-two folios are copied by the first scribe and the rest of the copying is done by the second scribe.

The portion after the first twenty-two folios contains more scribal errors (i.e. omissions and repetitions etc.) than do the first twenty-two folios.

The scribe, who copies the latter portion of the text, uses very frequently the sign  at the end of a line to indicate that the particular word is incomplete.

Both scribes write न् for ह्न (see, e.g. ff. 4r., 5 and 24r., 12, 13 etc.). Words to be inserted are in the left or right margins (see, e.g. ff. 1v., 15 and 20v., 15 etc.).

T does not contribute much towards the reconstruction of *Text Alpha*, due to the fact that its scribes seem to be using some of the other manuscripts as well as δ. There are several similarities between the deviant readings of T and the readings of manuscripts not belonging to class δ. For example: (i) p. 23, 10-11. अथ ... प्रकल्पयेत् om. NT; (ii) p. 69, 7. च om. γ (except T); and (iii) p. 69, 9. °संज्ञकः Tζ. Also see section C.(b)(v), Manuscripts N and R, and section C.(e), Class δ.

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10. Raghavan, V. *Catalogue of Sanskrit manuscripts in the Wellcome Historical Medical Research Library*. Typed. London. (See Pingree, 1970b, *CESS A 1*, p. 32b.)

(f). *Class ζ*

As mentioned before, this class consists of the five manuscripts W, I, M, S and H. For further information see section C.(d)(ii) above. Occasionally, the readings of class ζ are shared by manuscript T. For example, (i) p. 49, 11. अत्रोपांतिमेत्य°] अत्रोपांतिमेनेत्य° Tζ; (ii) p. 69, 9. °संज्ञकः Tζ.

The five manuscripts constituting class ζ agree on most of their readings. However, there exist some readings on which only W, I and M agree, while S and H have a different (common) reading. Since the readings common to W, I and M are different from those common to S and H, it is evident that the immediate ancestor of the first group is different from that of the second group. Using ι for the immediate ancestor of W, I and M, and θ for that of S and H, we have: ι and θ are copies of ζ. Therefore ζ can be reconstructed from ι and θ according to principles similar to those which will be found in Chapter I, section 4.A. below.

The following are the examples of the different readings in classes ι and θ: (i) p. 12, 14. स्वयोगे ... स्यात् om. θ; (ii) p. 45, 20. चत्वारिंशदिति om. θ, चत्वारिंशदशीतिर्द्विशती तुल्या इति ι; (iii) p. 62, 24-25. °र्लब्धिः ... भाज्य° om. ι | °ब्धिः ... भाज्यहा° om. S | द्वं ... भाज्य° om. H.

(g). *Class ι*

As far as the dates of the manuscripts W, I and M (which form class ι) are concerned, the copying of W was completed in 1813 A.D., but I and M have no dates of completion. W and I are independent copies of manuscript ι, while M is a copy of I. The following illustrations support this claim: (i) p. 41, 14. वर्गं° ... तदेकादि om. W; (ii) p. 12, 11. एवं ... शून्यषड्विधं om. IM; (iii) p. 20, 4-5. छेदस्थि° ... तथा च om. IM; (iv) p. 65, 22-23. गुणास्तु ... हारः om. M | द्वादश ... हारः om. ζ (द्वादश भाज्यः कुदिनानि हारः add. W^1 in the bottom margin). (Here W^1 refers to the later scholar who made the insertion in W). (v) In the Appendix #15, line 24, both I and M have the meaningless माहानाया।।स for महानायासः which indicates that the scribe of M imitates I without much understanding.

(i). *Differences Between Class ι and Class θ.*

(1). Class ι consists of the copies W, I and M but class θ consists of the copies S and H.

(2). Class ι has only lemmas whereas class θ has complete verses from Bhāskara's *Bījagaṇita* throughout the entire part of the commentary which is being edited. (S generally omits the lemmas but H has the lemmas as well as the complete verses).

(3). S and H have additions from Kṛṣṇa's *Bījapallava* and Bhāskara's *Bījagaṇita*, respectively. On the other hand, the copies W, I and M contain no such insertions except for an insertion or two in W by some later scholar; see for example, f. 20v., 7 (or Appendix #9 to the *Text Alpha*). Here the न in the left margin of W, seems to have been borrowed by some later scholar from the *Bījapallava* (see Radhakrishna Sastri, 1958, *Madras GOS 67*, p. 77, 5).

(ii). *Manuscript W*. Prājña Pāṭhaśālā Maṇḍaḷa, Wai. 9777 L. No. $\frac{11\text{-}12}{551}$.[11] Ff. 100. Paper: 24.9 cm. × 10.3 cm. Number of lines per page: 10 (but sometimes 9 or 11). Number of akṣaras per line: 42-49, or about 45 on average. Extent C (i.e. complete manuscript).

Colophon on f. 99v.: इति श्रीदैवज्ञज्ञानराजात्मजसूर्यदासविरचितं सूर्यप्रकाशाख्यं बीजभाष्यं समाप्तं ॥ श्रीरस्तु ॥ छ

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11. Joshi, Laxmanshastri. (Ed.). (1970). *Descriptive catalogue of Sanskrit manuscripts. Part II.* Wai: Prājña Pāṭhaśālā Maṇḍaḷa. Pp. 1242-1243.

Post-colophon on f. 100r.: शके १७३५ श्रीमुखनामसंवत्सरे आश्विनकृष्णाषष्ट्यां भृगुवासरे इदं पुस्तकं श्रीमत्र्यंबकसूनुना केशवेन लिखितं श्रीशतुष्टयेस्तु ॥ ॥ छ ॥ श्रीः ॥ छ ॥ शुभमस्तु ॥ छ ॥

According to this information, W was copied by Keśava, the son of Tryaṃbaka; and the copying was completed on Friday October 11, 1813 A.D. (see Pillai, 1989). The place is unknown. The manuscript does not contain folio 1 recto but there is no loss of text.

The special features of manuscript W are the following:

There are no repetitions in W.

There are very few omissions in W (see, for example, p. 17, 15. भागादिकमिति om. LW (add. W^1 in the top margin)).

W has numerals above some words to indicate their right order, as do the manuscripts L and D. For example, f. 6r., 8 has च स्य with a "२" over च and a "१" over स्य, which implies that the correct reading should be स्य च।

W has marginalia, some of which seem to be in a hand different from that of Keśava. (For example, see the left margins of ff. 20v., 7. न and 20v., 6. य; the text already has य, though it is not very clear). Words to be inserted are in any of the four margins. If they are in the top or bottom margins, they usually have the line numbers (e.g. f. 10r., 9 has वर्गराशौ ये २, in the bottom margin, indicating that the words are to be inserted in the second to the last line).

The amount of marginalia in W is more than that in any other manuscript of class ζ. This indicates that W has most definitely been studied by later scholars.

(iii). *Manuscript I.* India Office Library, London. 2824 (1891).[12] Ff. 71. Paper: $12\frac{1}{4}$ in. × 4 in. (i.e. 31.12 cm. × 10.16 cm.). European paper. Number of lines per page: 11-13. (But the catalogue has 24). Number of akṣaras per line: 50-62, or about 56 on average.

Colophon on f. 71r. - f. 71v.: ॥ शुभं भवतु ॥ ॥ श्रीसदाशिवार्यणमस्तु ॥ ॥ छ ॥ ॥ छ ॥ ॥ छ ॥ ॥ छ ॥ ॥ छ ॥ ॥ छ ॥ ॥ छ ॥

बीजभाष्यं समाप्तिमगात् ॥ श्रीगणपति ॥

Post-colophon: Missing.

According to the information given on folio 1 recto, this manuscript was bequeathed by Dr. John Taylor to the Hon. Court of Directors of the East India Company through William Erskine, executor, in Bombay, on April 20, 1822. William Erskine lived in Bombay from 1803 – 04 to 1823 A.D. (Buckland, 1968). He held various legal offices. Moreover, he was the Secretary and Vice-President to the Literary Society. (p. 139b)

There is no mention of the name of the scribe, or place or date on which the copying was completed. Folio 1 recto also has: बीजभाष्य प्रारंभोयं ॥ श्रीगजानन प्र०

Manuscript I has the following peculiarities:

There exist a few repetitions in manuscript I, such as on f. 20r., 5 (i.e. p. 61, 13) the word शुद्धे is written twice (instead of once). This repetition is followed by M as well (see f. 14r., 17-18).

Manuscript I contains a few instances of misplacements of nyāsas as well as of the results obtained on performing various mathematical operations. For example हारः क १८ क ३ (*Text Alpha* p. 33, 4-5) is misplaced on f. 11v., 5 after ऋणात्वं (p. 33, 6), whereas it should have occurred on f. 11v., 5 before अथ (p. 33, 5). Likewise क २५ क २७ (*Text Alpha* p. 30, 18) is misplaced on f. 10v., 10 after जातं (p. 30, 19),

12. Eggeling, Julius. (Ed.). 1896. *Catalogue of the Sanskrit manuscripts in the Library of the India Office. Part V.* London. P. 1011.

whereas it should have occurred on f. 10v., 9 before अस्मिन् (p. 30, 19). These misplacements are followed in M on f. 8r. and f. 7v. respectively.

Manuscript I has very few marginalia; they consist of insertions to be made in the text (see e.g. f. 2v., 11; f. 8r., 11 etc.). Sometimes the numerical place where a letter in the margin is to be inserted in a word is written to the letter's right. As an illustration, f. 4r., 9 has वा ३ in the left margin and लाघर्थं in the text, meaning thereby that the intended reading of this word is लाघवार्थं। Furthermore, the sign m appears at some places on top of a letter, as on f. 5v., 1, we find वदीदौ। This means that the द् is to be replaced by the next consonant ध्। Here M has the correct reading वधादौ (f. 4r., 2); perhaps its scribe understood the symbol. It is likely that some of these marginalia have come from the pen of a later scholar.

(iv). *Manuscript M*. Royal Asiatic Society, Bombay. 279.[13] Ff. 46. Paper: $15\frac{1}{2}$ in. × $9\frac{1}{2}$ in. (i.e. 39.37 cm. × 24.13 cm.). Number of lines per page: 17-20. (The catalogue has 20). Number of akṣaras per line: 47-57, or about 53 on average.

Colophon on f. 46v.: ॥ शुभं भवतु ॥ छ ॥ छ ॥ श्रीः ॥ छ ॥

Post-colophon : Missing.

On folio 1 recto, manuscript M has ॥ अथ बीजभाष्य प्रारंभः ॥ श्रीः ॥ २९ ॥ २५; here the २९ ॥ २५ is an old library shelf-mark but the extra mark ` above श्रीः makes no sense. Two folios (i.e. 31v., 32r., 37v. and 38r.) are missing from the microfilm. Therefore the text is lost. There is no mention of the name of the scribe or place or date at which the copying was completed. This manuscript comes from the collection of the Bombay Branch of the Royal Asiatic Society.

The following are the peculiarities of manuscript M:

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13. Velankar, H. D. (Ed.). (1926). *A descriptive catalogue of Sanskṛta and Prākṛta manuscripts, Vol. 1*. Bombay: Royal Asiatic Society. P. 92.

There exist several instances which reveal that M is a slavish imitation of I. These include: (i) p. 4, 11. घट°] घष्ट° IM; (ii) p. 56, 22. न्यासः + भाज्यः हारः २२१ १९५ क्षे ६५ IM; (iii) p. 68, 25-26. °मित्यल्पमति°] ९वमल्पमिति° IM; (iv) p. 45, 6. मूलं + रा° IM (रा° in the right margin of ms. I, f. 15v., 10 but in the middle of a line of ms. M, f. 11r., 12); (v) p. 29, 17. °ल° om. IM. In example (v), the scribe of I inadvertently dropped the syllable ल at the end of a folio (f. 10r., 13) and the scribe of M slavishly copies this omission in the middle of a line (f. 7v., 3).

Rarely, both I and M have numerals above the letters to correct their order. For example, on p. 10, 13 they have घेव (ms. I, f. 4v., 9 and ms. M, f. 3v., 3) where there is a " २ " over घे, which indicates that the correct order is वधे ।

The repetitions present in the manuscript I have also been copied by the scribe of M almost all the time. An exception to this is the omission in M (f. 15r., 10) of the repetition तद्विकल्पा° ... शेषं^{2} of I (f. 21r., 9-10. For this text, see Appendix #18, lines 4-5).

The dot representing a negative number is generally ignored in M as is done in R and D. For exceptions in M, see f. 3r., 13-14.

M has very few marginalia. Words to be inserted are written in the margin or in the next line exactly under the insertion sign (v). In the latter case, the symbol "×" is put on the left and right of the akṣaras to be inserted; as on f. 3r., 7 we find × युति ×.

(h). *Class θ*

This class consists of the two manuscripts S and H, both of which were written in the nineteenth century. Both belonged to Major T. B. Jervis, and are written on European paper.

Both manuscripts contain complete verses from Bhāskara's *Bījagaṇita.* Manuscript S generally omits the lemmas corresponding to the completely quoted verses (as does L, see p. 9, 9. रूपत्रयमिति om. LS), while H contains the lemmas as well.

Moreover, unlike L, the manuscripts S and H do not have इति added at the end of a complete verse.

S and H are considered to be independent copies of θ due to several reasons. They differ in the placement and wording of some of the verses. Also they have different omissions. The following examples may be considered in view of these differences: (i) p. 18, 17-18. Verse 17c-d. H; but L and S transposed to after verse 17a-b. (ii) p. 19, 10-11. सन्त्स्वेषु H, सन् S. (iii) p. 5, 24. तदधीते तद्वेदेत्यण् om. H. (iv) p. 9, 9. रूपत्रयमिति om. LS. (v) p. 28, 20-p. 29, 1. अनयो° ... रू १ om. H.

The scribes of S and H have made unique augmentations into the text of the commentary from Kṛṣṇa's *Bījapallava* and Bhāskara's *Bījagaṇita*, respectively. The following are two illustrations to this effect: (i) p. 13, 18. अनंत इति] अनंतो राशिः खहर उच्यत इति (from Kṛṣṇa's *BP*, p. 28, 11-12) S; अनंत इति + अनंतो राशिः खहर इत्युच्यते (from Bhāskara's *BG*, p. 5, 11) H. (ii) p. 45, 7-8. वर्गो ... °नेयम्] Appendix #13. β (from Bhāskara's *BG*, p. 25). The apparatus criticus to this appendix shows that S contains the augmentation एवंविधे वर्गे करणीनामासन्नमूलकरणेन मूलान्यानीय रूपेषु प्रक्षिप्य मूलं वाच्यमिति तद्रूपसंख्याकाः करण्यो मूलमित्यर्थः which has been borrowed, word for word, from Kṛṣṇa's *BP*, p. 82, 20-22. On the other hand, H contains the augmentation एवंविधेषु वर्गेषु करणीनामासन्नमूलकरणेन मूलान्यानीय रूपेषु प्रक्षिप्य मूलं वाच्यं अत्र महती रूपाणीत्युपलक्षणं ततः क्वचिदल्पापि which has been borrowed, with slight modifications, from Bhāskara's *BG*, p. 25, 12-15.

S and H have very few marginalia and the marginalia are in their original scribes' own hands.

(i). *Manuscript S*. British Museum, London. 448 (Add. 14,361a).[14] Ff. 40 (The catalogue has 41, but f. 41 is missing from the microfilm and therefore text is lost). Paper: European, folio. Dimensions unknown. Number of lines per page: 22. Number of akṣaras per line: 48-59, or about 55 on average. Was copied in the nineteenth century. Belonged to Major T. B. Jervis.

Colophon: Folio missing.

Post-colophon: Folio missing.

The name of the scribe, or place, or date at which the copying was completed are unknown. In the beginning (before f. 1r.) S has: Purchased of Major T. B. Jervis, July 1843.

As far as the special features of manuscript S are concerned, some of them have already been described in section C.(h), Class θ. The following may be added to them:

Occasionally, S has नृ in place of हृ (e.g., f. 9v., 9. नृपाणि in place of हृपाणि).

Quite often, the dot for a negative number is omitted in S. S has very few repetitions. Words to be inserted are in the margins (see e.g. f. 6v., 22; and f. 9v., 11). They are in the hand of the original scribe.

(ii). *Manuscript H*. British Museum, London. 447 (Add. 14,358c).[15] Ff. 46 (now 74-119). Paper: European, sm. folio. Dimensions unknown. Number of lines per page: 21-22, or 27 (21 in the catalogue). Number of akṣaras per line: 52-67, or about 62 on average. Was copied in the nineteenth century. Belonged to Major T. B. Jervis.

Colophon: Missing.

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14. Bendall, Cecil. (Ed.). (1902). *Catalogue of the Sanskrit manuscripts in the British Museum*. London. P. 185.

15. Bendall, Cecil. (Ed.). (1902). *Catalogue of the Sanskrit manuscripts in the British Museum*. London. P. 185.

Post-colophon: Missing.

The name of the scribe, place or date at which the copying was completed are unknown. Note that the numbers 74 through 119 which appear on the folios are not the number of folios which H has. These are the folio numbers of BM Add. 14,358.

The important characteristics of manuscript H are the following:

The scribe of H corrects as he copies. For example: (i) p. 9, 2. °स्त्रींश्च] °स्त्रांश्च N, °त्रयाश्वं R, °स्त्रयश्च ζ (except H); (ii) p. 62, 24-25. °र्लब्धिः ... भाज्य° om. ι | °ब्धिः ... भाज्यहा° om. S | ढं ... भाज्य° om. H; (iii) p. 38, 19. शोध्य A (स्योध्य R) LDζ (संशोध्य H). Note that शोध्य is grammatically incorrect because it needs a prefix. Out of all of the manuscripts of class ζ, only H contains the correction. B and T have विशोध्य, which we have chosen for our *Text Alpha*.

The writer of H seems to have access to other manuscripts of the commentary also. See, e.g. p. 60, 8-9. भा ... १३ om. ζ (except H).

Manuscript H has several additions from Bhāskara's (commentary in his) *Bījagaṇita*, some of which are as follows: (i) p. 16, 8. सर्वं स्पष्टार्थम्] न्यासः या २ या ६ं रू ८ शोधिते जातं या ८ रू ८ं H (from Bhāskara's *BG*, p. 7, 7-8); (ii) p. 35, 11. पदानि + न्यासः क २ क ३ क ५ क ३ क २ क ६ क ५ क ३ । क २ क १८ क ८ क २ स्छाप्योंत्यवर्गो द्विगुणांत्यनिघ्ना इति कृते जाता यथाक्रमं वर्गाः रू १० क २४ क ४० क ६० रू ५ क २४ रू १६ क १२० क ७२ क ४८ क ६० क ४० क २४ अत्रापि यथासंभवं करणीनां योगं कृत्वा वर्गवर्गमूले कर्तव्ये क १८ क ८ क २ योगे जातं करणी ७२ अस्य वर्गः रू ७२ इति मूलं अथ टीका H (from Bhāskara's *BG* pp. 17-18, with slight modifications); (iii) p. 60, 19-22. अत्र ... °लब्धी] एते स्वतक्षणाभ्यामाभ्यां १३।६० शुद्धे जाते ऋणभाज्ये धनक्षेपे २।९ं । एते स्वतक्षणाभ्यां शुद्धे जाते H (from Bhāskara's *BG*, parts of p. 32 lines 18, 20, 21 and p. 33 lines 1, 4, 6).

The sign for a negative number is omitted in H only very rarely. There are no repetitions. Words to be inserted are written usually in the left margin. They seem to be in the original scribe's own hand and have the sign * above them; see e.g. f. 6v., 18. *तर्हि and f. 12v., 5. *गुणा ।

D. *Possibilities to be Explored*

Our *Text Alpha* does not represent exactly the manuscript α, because there are mistakes shared by both A and β, which therefore must have occurred in manuscript α. We have corrected these mistakes.

Now a problem arises in that there are passages in karaṇī and kuṭṭaka where β presents complete explanations and A presents abbreviated (or defected) explanations. There are the following possible ways to explain this:

(a). The copyist of text A has abbreviated the readings of α while the copyist of β has copied them out in full. In this case, *Text Alpha* represents the corrected text of manuscript α.

(b). The copyist of text A has correctly copied out α while the copyist of β has introduced the longer and more correct and/or more complete explanations from some other source. There are two possible sources: (i). Corrections subsequent to the writing of manuscript α, introduced by Sūryadāsa himself. (ii). The copyist of β may have found these explanations in another commentary which is lost, and rewritten those passages to conform to Sūryadāsa's style.

Among (a), (b)(i) and (b)(ii), we think (b)(i) is the most likely explanation, and (b)(ii) seems to us to be the least possible hypothesis, because it requires the copyist of β to be a reviser of the text rather than simply a scribe.

We cannot prove any one of these explanations to be the correct one, so that our use of the term *Text Alpha* has to be understood as being subject to modifications in accordance with whichever of these three hypotheses, (a), (b)(i) or (b)(ii), is correct.

In view of the differences between the texts of the two recensions A and β, some of the possibilities which need to be investigated may be stated as follows:

(i). Do the two recensions represent two occasions on which Sūryadāsa iectured on the *Bījagaṇita*?

(ii). Do the texts of manuscripts A and β differ because they were copied down by two different students at the same series of lectures?

(iii). Did the owner/scribe of manuscript β possess more than one manuscript of the *Sūryaprakāśa*?

(iv). Did the writer of β use sources other than Sūryadāsa's *Gaṇitāmṛtakūpikā*, Kṛṣṇa's *Bījapallava*, and Bhāskara's *Bījagaṇita* to expand on the *Sūryaprakāśa*?

(v). Did, contrary to our hypothesis, the writer of β *himself* provide all of the alterations and details which are not found in text A?

The subsequent section discusses the principles and conventions of the edition.

4. *Principles and Conventions of the Edition*

A. *Principles of the Edition*

In the section 3.C.(c) above, it was mentioned with respect to the texts A and β that:

(1). The readings of manuscript A are not available at some places due to its illegibility or loss of folios.

(2). There exist apparently genuine passages or words in text β that are omitted in text A (due to homoeoteleuton or otherwise).

(3). Text β has detailed explanations where text A has one-sentence summaries or short passages. In other cases, the explanations in text β differ from those in text A only in their wordings.

(4). There exist alternative explanations or additions from other sources in text β.

These cases have been dealt with as follows:

In *case (1)* above, when the reading of manuscript A is not available, the readings of its descendants N and R have been resorted to if their readings exist, coincide and make sense with reference to the context. Otherwise the readings of class β have also been taken into account along with those of N and R, and the common reading is chosen for *Alpha*. In some cases the readings of class β become the sole witness for *Alpha*.

Furthermore, if all manuscripts of class β do not have the same reading, then, at first, the reading of class γ is considered. If its descendants B and δ disagree on a reading, then the reading which makes contextual sense with that of N or R has been chosen. If neither B has the correct reading nor all manuscripts of class δ contain the same reading, then a choice has been made out of the readings of δ's descendants L, D and T in order; and so on, moving through the stemma from right to left.

The following examples from the Apparatus Criticus may explain the above rules of editing belonging to case (1):

(a). P. 2, 18. The reading °नुरक्त° has been chosen for the *Text Alpha* even though manuscript A omits the relevant folio, the readings of N and R are not identical and B omits the relevant line; but N and L share the chosen reading.

(b). P. 2, 17. Here again manuscript A omits the relevant folio, and B omits the particular line. The reading shared by N and L is समुल्हासयन् which has been discarded because it does not seem to be contextually correct. So the reading समुल्लासयन् which is common to R, D, T and ζ has been chosen for the *Text Alpha.*

(c). P. 4, 8-9. The reading °धरादेरु° ... कवयः। य° has been chosen for the *Text Alpha* because it is shared by R and β. Manuscript A skips the pertinent folio and N omits this reading.

(d). P. 5, 1. जनितु° is in agreement with the context. In order to locate this reading, manuscript H from class ζ had to be resorted to; because the relevant folio is missing from manuscript A, and the reading विनेतु° which is shared by ε and γ had to be discarded. The reading of class ζ (except H) is जनेतु° which is an irregular formation and, therefore, had to be rejected.

In *case (2)*, the apparently genuine passages of text β have been included in the *Text Alpha* if they are needed in view of their relevance to the context, or to maintain continuity and completeness. For example:

(a). P. 4, 14. °द्घटव° of β is needed with reference to the context. Manuscript A omits the relevant folio at this point and ε omits this reading.

(b). P. 29, 1-2. अस्य ... अंतरम् of β provides continuity to the solution of the problem. It is missing from the manuscripts of class A. Here is an example of our use of the assumption that Sūryadāsa came back at some point and revised his *Sūryaprakāśa* and the writer of β seems to have been in possession of a copy of the revised version (see section 3.C.(c), Class β).

(c). P. 36, 2 - p. 37, 6. एवमत्र ... सर्वत्र of β is needed for the *Text Alpha* because it contains a detailed explanation of the sahajā and nimittajā quantities in the squaring of a karaṇī-expression, which is not given anywhere else in the section dealing with karaṇī. Also, part of this text discusses the solution of the fourth problem given by the verse, and thus completes the solution. Here, again, Sūryadāsa provided these details in a later copy of the *Sūryaprakāśa* which was used by the writer of β (see our commentary). This text is missing from the manuscripts of class A.

In *case (3)*, a choice between texts A and β has been made considering the context, order, clarity etc. Note that either complete text A or complete text β has been chosen. The discarded text goes either to the *Appendices* (Chapter IV, section 1.) or to the *Apparatus Criticus* (Chapter III). For example:

(a). P. 19, 15-22. भाजयितुं ... °पपन्नम् of β replaces a one-sentence summary contained in the manuscripts of class A. The text A has gone to Appendix #4.

(b). P. 45, 20 - p. 46, 4. अथान्य° ... क २ || For this text, the arrangement in the A-recension is chosen. The β-recension contains this text before p. 45, 9 तथा, which creates a disorder and discontinuity of the discussion of approximate square-root.

In *case (4)*, text A has been chosen for the *Text Alpha* and text β has been placed in the *Appendices*. For example:

(a). Appendix #1. यद्वा ... °पन्नम् is only in β. The explanation contained in it is alternative to that in p. 10, 1-4 of the *Text Alpha*. The latter explanation is contained in both A and β.

(b). For additions from other sources in text β, see the section 3.C.(c) of this chapter.

In conclusion, *Text Alpha*, which has been edited, is certain where the readings of texts A and β agree and are correct. On the other hand, *Text Alpha* is uncertain to varying degrees where (i) the readings of A and β disagree, or (ii) the readings shared by A and β are corrupt, or (iii) the readings of one of A and β are not available.

As mentioned earlier, manuscript A seems to be a direct copy of manuscript α, which in turn seems to be a direct copy of Sūrya's original. Manuscript A is superb in the sense that no traces of possible contamination are found in it. It is written in a very neat hand. Though it is written carefully, there do exist a few errors, omissions and expandable short summaries in A. Hence the *Text Alpha,* which we have reconstructed, is very close to but probably not exactly Sūryadāsa's (revised) original *Sūryaprakāśa* which is, in principle, impossible to attain.

B. *Principles Used in the Reporting of Variants in the Apparatus Criticus*

Mainly those variants which help in the classification of the manuscripts have been reported. That is to say, they include the most persuasive parts from all manuscripts: Insertions or additions from any other source (e.g. Appendix #17 from the *Gaṇitāmṛtakūpikā*) have been reported; significant grammatical errors (e.g. p. 6, 9. ज्ञानयत A (except ε)) have been reported; significant variant readings from a class or classes have been reported even if those readings occur in later manuscripts (e.g. p. 21, 22. व्यवकलनं om. LT and p. 29, 21. ६२५ DM). Finally, any specially significant individual variant from a single manuscript has been recorded (e.g. p. 15, 2. °वर्यैरिति L indicates that L adds इति at the end of a complete verse of the mūla).

What has not been reported includes: Any insignificant variants or omissions of a word or two, if a manuscript is later or has no descendants; variants in a repetition, if one of the two expressions of the repetition has the correct reading; insignificant spelling errors within the same words in different manuscripts.

If a variant reading has been reported from one manuscript, then the corresponding *variants* from all other manuscripts have also been reported, whether or not these variants are meaningful.

More specifically, the following criteria have been employed in reporting or recording the variants:

(1). If some parts of the text are omitted by more than one manuscript such that the omitted parts of the text have the same beginning but the omitted texts have different lengths in different manuscripts, then the longest omission has been recorded first. All omissions are recorded using the symbol 'om.' As an illustration, for page 8, the recording for lines 9 and 10 is done as follows: 9 – 10. संबंध: ... युति° om. D ॥ 9. संबंध: ... तयो: om. B ।

(2). The lemma (i.e. mūla or the correct reading) has been reported only when it is necessary for clarity. For example, p. 8, 16. तत्र] अथ यदि β.

(3). The little circles at the beginning and/or end of a framing line of a word indicate that only a part of the word has been recorded for the sake of brevity.

(4). A variant has been recorded under the name of the ancestor if the variant is shared by all or almost all of its descendants. However, variant readings within the *same* class have been enclosed by parentheses. For example: (i) p. 31, 5. °हारे β (°हार L, °हारो T, °हरे IM); (ii) p. 69, 7. °ैक्यं + स्यात् (स्यात् om. L) स (स om. BLD, व T) β.

(5). Variants shared by manuscripts belonging to *different* classes are followed by the names of those manuscripts, for example, p. 61, 15. ४ NBDM, १४ R.

(6). If all manuscripts of a particular class have the same variant reading, except some manuscript which has the *correct* reading, then the reporting is done using 'except' as follows: (i) p. 13, 12. संसृतिपदं β (except L); (ii) p. 68, 14. २ εβ (except LT); (iii) p. 60, 25. °दाहरणांतमाह A (except ε).

(7). If a later hand corrects the reading of a manuscript, this fact is reported using 'corr.', as for example, p. 10, 20. अन्य Tι (corr. W^1 in the left margin) S. The superscript '1' refers to this later scholar who wrote in the margin of W. As another illustration, p. 69, 8. क्षेपं] शेषं β (except LD, corr. W^1 in the text of W).

(8). Any illegible letters in a manuscript, other than in manuscript A, have been replaced by × in the Apparatus Criticus. As an illustration, p. 15, 15. °वर्गाणां om. D, °वर्णानां εL, °वर्गा× T, °स्य Bζ. If the whole word cannot be read, then 'illegible' or 'illeg.' is written with that word. For example, p. 60, 18. २ illeg. in B, ७ δ (corr. L[1] in the text of L).

(9). The variants common to A and β have been recorded under α. This indicates that the text is being corrected by the present writer (e.g. p. 2, 10. °गतव्यक्ता° α).

(10). If the correct reading appears only in the later manuscripts, the word 'add.' is used to record it. For instance, p. 21, 10. द्विनिघ्नीं om. α, add. ιS. This word 'add.' is used also when an original scribe of any manuscript or a later scholar adds (usually in a margin) the text which was omitted in the manuscript. For instance: (i) p. 70, 2 – 3. भा ... ६३ add. A in the bottom margin (except ε); (ii) p. 18, 5. °रसमयो° om. Bζ (°र्वा corr. W^1 to र in the text of W, and समयोर्वा° add. W^1 in the top margin); (iii) p. 10, 20. श्रन्य Tι (था add. W^1 in the left margin) S.

(11). The same variant of A, N and R has been recorded under the name of their ancestor A (e.g. p. 58, 20. ० om. A (except R)).

(12). Since N and R have numerous scribal errors, only those variants which indicate relations between N, R, ε and A have been recorded. An exception is when such a variant must be recorded because of corresponding variants in other manuscripts being recorded. Thus here the criteria are:

(a). Any variant (even if it is an error), on which both N and R agree, has been reported.

(b). If N and R are the *only manuscripts* which have variant readings which do not coincide, those readings have not been recorded.

(c). If *only one* of N and R has a variant reading, but all other manuscripts have the correct reading, then such a variant has been ignored.

(d). The *insignificant* variants in N and R resulting from the (wrong) use of न् for त् or from the omission of the anusvāra ṃ, (ं), have been ignored.

(13). The variants belonging to the same line of *Text Alpha* are separated by a daṇḍa, (।), while those belonging to different lines are separated by a double daṇḍa, (॥). As an illustration, for p. 64, 2-3, we report: 2. एवं ... °दानीं] अथ β । °कुट्टकसि° β ॥ 3. क्षेपं ... °मिति om. θ, क्षेपं विशुद्धिमिति γι ॥

C. *Conventions Followed in Preparing Text Alpha and the Appendices*

In the manuscripts of the *Sūryaprakāśa*, as is normal in the Indian tradition, the text is continuous. There are no headings, titles or sub-titles, but only colophons (and sometimes post-colophons as well) indicating the ends of chapters and sections.

In *Text Alpha*, the headings, paragraphs, indentations, displays, sentence dividers—daṇḍas (।) or question marks—and paragraph dividers—double daṇḍas (॥)—are supplied by the editor. Though many manuscripts have daṇḍas, they are irregularly and inconsistently employed. The avagraha sign (ऽ), often omitted in the manuscripts, is supplied by the editor where needed without comment. The letter छ, which is employed to mark the end of sections or sub-sections in manuscript A, has been retained. Extra space before a paragraph, which introduces a new idea, has been used.

The symbols ⌊ ... ⌋ indicate illegible portions of the text of manuscript A. These gaps have been filled by the suitably chosen portions of the text from the other manuscripts as described in the sub-section entitled Principles of the Edition.

The parts of the mūla (i.e. text of Bhāskara's *Bījagaṇita*) which appear in our *Text Alpha* are in bold face Devanāgarī script, but the remaining part of Sūryadāsa's commentary is in plain Devanāgarī type. For example, the verses which exist in manuscripts L, S and H are parts of the mūla. Therefore, they are in bold type, have a number and, since they have not been given by Sūryadāsa, they are enclosed within angle brackets, '<' and '>', as explained below. On the other hand, any verses given by Sūryadāsa, whether composed by Sūrya himself or taken by him from any text other than the mūla, or from that part of the mūla the commentary on which is not being edited, are in plain type, are not necessarily numbered and are indented.

If there are any discrepancies among the wordings of a verse of the mūla (contained in S, H and L), then the words used by Sūrya in his commentary on that verse have been taken into consideration.

The numbering given to the verses belonging to the mūla corresponds to that in the *Bījagaṇita: A treatise on algebra by Bhāskarācārya*, which is edited by Jībānanda Vidyāsāgara, 1878, Calcutta. This text has been chosen by us because Vidyāsāgara's numbering does not seem to be based on the numbering belonging to any particular commentary. The manuscripts of the *Sūryaprakāśa* have no numbering for the verses of the mūla. Thus the verses pertaining to the mūla in our *Text Alpha* follow Sūryadāsa's order but Vidyāsāgara's numbering. The natural order is disturbed only in verses 53 and 54.

The angle brackets '<' and '>' have been used in the following situations:

(i). To enclose the verses (from the mūla) which exist in manuscripts L, S and H and which have not been given by Sūryadāsa, as mentioned before.

(ii). To enclose any suggested headings or sub-headings.

(iii). To enclose all suggested (i.e. corrected) readings in the text. For example, on page 2, 10 of the *Text Alpha* we read °गत<ा>व्यक्ता°, whereas all of the available readings from our manuscripts are °गतव्यक्ता°; manuscript A omits the relevant folio, and L and D omit the whole verse which contains this reading.

The sandhi given in the various manuscripts has been preserved. New sandhi has also been introduced at places, for example, when a word ending in a vowel is followed by a word beginning with a vowel (see e.g. *Text Alpha*, p. 9, 14. यान्यृणगतानि). However, the grammatically incorrect use of ṣ, (ष्), in place of a visarga ḥ, (:), as in manuscript A, has been eliminated.

On the other hand, the words शोध्य (e.g. *Text Alpha*, p. 44, 11) and योज्य (e.g. Appendix #19, lines 2 and 4) have not been replaced by विशोध्य and वियोज्य

respectively; because Sūryadāsa and others seem to be following some convention which was prevalent at their time of writing.

To facilitate understanding, the word or words of a verse which are being commented on by Sūryadāsa, have been underlined.

Any citation from the mūla, which is made by Sūryadāsa in his commentary on a verse, is either enclosed within double quotes or is written as an indented verse in plain Devanāgarī type. Likewise for any cited sūtra which is taken by Sūryadāsa from any text other than the mūla. But if the wording by Sūryadāsa of such a citation differs from its actual wording, it is set apart within single quotes.

Any part of the text A or text β, which is placed in the *Appendices* to *Text Alpha*, has been assigned a number. The section *Appendices* is followed by its own *Apparatus Criticus*.

The next chapter is composed of *Text Alpha* as it has been edited.

CHAPTER II

THE TEXT ALPHA

श्रीमद्दैवज्ञपंडितसूर्यदासविरचितः

सूर्यप्रकाशः

(भास्करीयबीजभाष्यम्)

< 1. प्रथमोऽध्यायः >

< उपोद्घातः >

[श्रीगणेशाय नमः ॥ श्रीसरस्वत्यै नमः ॥ श्रीगुरुभ्यो नमः ॥

भाले प्रालेयरश्मिः सुनयनयुगलोन्मीलने सिद्धयोऽष्टौ
कंठे श्रीकंठसूनोर्द्युमणिफणिफणासन्मणीनां प्रकाशः।
आस्ते ब्रह्मादिमौलिस्थलमधुपगणो यत्पदांभोजपीठ-
प्रांतेऽनंतप्रभावं गणपतिरिति यज्ज्योतिरव्यादिहास्मान् ॥१॥

यावत्कालकनीलवर्णरुचिरं पीतं सितं लोहितं
वासो हासकराधरैरिव गुणं हारप्रवर्गं दधत्।
सन्मूलं समशोधनैरधिगत<ा>व्यक्ताभिधं वा धिया
वेद्यं योगविशेषतः किमपि तत्कृष्णेति बीजं भजे ॥२॥

यत्पादांबुरुहप्रसादकणिकासंजातबोधादहं
पाटीकुट्टकबीजतंत्रजलधिप्रोत्तुंगपारं गतः।
छंदोऽलंकृतिकाव्यनाटकमहासंगीतशास्त्रार्थवित्
तं वंदे निजतातमुत्तमगुणं श्रीज्ञानराजं गुरुम् ॥३॥

चक्रे चक्रविहंगयोरिव युतिं निघ्नन्विमोहक्षपां
विश्लेषे गणितार्थयोः कविमुखांभोजं समुल्लासयन्।
श्रृंगारादिरसानुरक्तविबुधेंद्राशामसौ पूरयन्
शश्वद्विष्णुपदस्थिरो विजयते सद्बोधसूर्योदयः ॥४॥

स्फुरद्बीजांभोधौ विविधरचनागाधसलिले
तितीर्षूणां माभूदपगतमतीनां श्रमभरः।

विमुग्धानामित्यादृतसदयचेताः परिमिता-
मिमां व्याख्यानावं सपदि रचये सूर्यगणकः ॥५॥

बीजाक्षरार्थः प्रथमं दुरापः
किं वासना तत्र विचारणीया।
तथापि सूर्योऽहमुदारबुद्धि-
र्बीजं सबीजं विशदीकरोमि ॥६॥

तत्र श्रीमन्मार्तंडमंडलाखण्डप्रचंडकरनिकरविस्फारविधूतविततध्वांतकांतप्रदेश-विशेषरूपोपकल्पितब्रह्मांडमंडपांतर्विविधविधिलीलाविलसितत्रिलोकीमध्यवर्त्तिसकल-लोकानुग्रहगृहीतविग्रहेण ब्रह्मणा तदैहिकामुष्मिकफलसमर्पककर्मसाद्गुण्यप्रयोजना-भिलाषिणा निर्मितं ज्योतिःशास्त्रं निखिलागमांगमौलिम्। कलिकालमहिम्ना लुप्तमिदमुद्धर्तुमज्ञानतमोनिहतं जगदुपकर्तुमाविष्कृततनुरसौ भास्करः प्रकृतोपक्रमानुसारं व्यक्तं गणितं विरचय्य तद्बीजभूतमिदमतिगहनमव्यक्तगणितं विवक्षुरादावारब्धसमाप्तिकामनया तत्प्रतिबंधकविघ्नोत्सारणासाधारणकारणविशिष्ट-शिष्टाचारानुमितश्रुतिबोधितकर्त्तव्यताकं शिष्टत्वसाधर्म्यादिष्टदेवतानमस्काररूपं मंगलं स्वयमनुष्ठितम्। छात्रोपयोगमात्रेणानेकार्थैः पदैः संश्लिष्योपेंद्रवज्रावृत्तेन ग्रंथतोऽनुबध्नाति। उत्पादकमिति ॥

<उत्पादकं यत्प्रवदन्ति बुद्धे-
रधिष्ठितं सत्पुरुषेण सांख्याः।
व्यक्तस्य कृत्स्नस्य तदेकबीज-
मव्यक्तमीशं गणितं च वन्दे ॥१॥>

अहं बुद्धेरीशं वंदे इति क्रियाकारकयोजना। बुद्ध्यधीशं गणाधिपतिमहं वंदे। नमस्करोमीत्यर्थः। अत्र सिद्धिबुद्ध्योराधिपत्यमस्यागमशास्त्रप्रामाण्यतः सिद्धमेव। नन्वत्र बुद्ध्यधिपतेरेव नमस्करणे किं कारणमिति चेन्न। अस्याव्यक्तशास्त्रस्य बुद्ध्येकसाध्यतया तस्यास्माभिरमुष्मादेवाभ्यर्थनीयत्वादेतदेवाग्रतोऽभिसंधायाचार्योऽपि वक्ष्यति ॥

बीजं मतिर्विविधवर्णसहायिनी हि
मंदावबोधविधये विबुधैर्निजाद्यैः।
विस्तारिता गणकतामरसांशुमद्भि-
र्या सैव बीजगणिताह्वयतामुपेतेत्यादि ॥

नन्वेतदभीष्टदैवतप्रणामादर्शने कोऽसौ किमाकार इत्याद्यज्ञानात्तन्नमस्करणमप्रामाणिकमित्याशंक्य विशेषणोक्त्या प्रमाणं सूचयत्युत्पादकमिति। किंभूतं तम् ? कृत्स्नस्य व्यक्तस्योत्पादकम्। कृत्स्नस्य समस्तस्य व्यक्तस्य स्थूलस्य कार्यस्य भूभूधरादेरुत्पादकं कर्तारमिति च। परं संख्यावान्पंडितः कविरित्यभिधानात् सांख्याः कवयः। यदव्यक्तं तत्तेन सत्पुरुषेणाधिष्ठितं प्रवदन्ति अव्यक्तममूर्तं व्योमादि येन सत्पुरुषेणाधिष्ठितं व्याप्तम्। अस्यायमर्थः। जायमानं कार्यं कर्तारमाक्षिपतीति न्यायः। तथा चैतदपि कार्यत्वेन हेतुना घटदृष्टांतेन सकर्तृकं पर्यवस्यति। तत्र कर्ता च परेश एव सिद्ध्यति। स एव कार्यवशेन विघ्नेशोपाधित्वमुपगतोऽस्माभिरुपास्यत इति। तत्रास्य सकर्तृकत्वे प्रयोगोऽपि न्यायशास्त्रे प्रसिद्धः। स यथा क्षित्यादिकं सकर्तृकं कार्यत्वाद्घटवदित्यादि। अव्यक्तमधिष्ठितमित्यादिना कालादीन् व्याप्नुवतोऽस्य विभुत्वमपि सूचितं भवति। तच्च नित्यत्वसहचरितमेवेत्यर्थान्नित्यत्वमपि। कथंभूतं गणितम्। अगणो गणः संजात इति गणितः। तं गणितम्। स्वयमीशोऽपि कार्यविशेषवशान्महेशोपाधित्वेन विकुर्वाणोऽन्येषां प्रवृत्त्युत्पादनार्थं स्वरूपविशेषे महत्वद्योतनार्थं गणाधीशोपाधिना गणाध्यक्षत्वमालंब्यैकत्वेऽपि सेव्यानुचरभावः स्वीकृत इति भावः। पुनः किंभूतमित्याह एकबीजमिति। एकं बीजमक्षरं यस्य सः। तथा एकाक्षरगणपतिमंत्राभिप्रायेणैतदुक्तमिति ध्येयम्। तथा च गणपतिः परः पद्यार्थः ॥

अथ

यस्य देवे परा भक्तिर्यथा देवे तथा गुरौ।
तस्यैते सकला ह्यर्थाः प्रकाशंते महात्मनः ॥

इत्यादि स्मृतिबलादेतदव्यक्तशास्त्रोपदेष्टारं गुरुं महेश्वरं स्वजनितारमप्यनेनैव पद्येन प्रणमत्युत्पादकमिति। तत्राहमुत्पादकं वंदे इति संबंधः। उत्पादयतीत्युत्पादकः पिता। तमुत्पादकं पितरं वंदे नमस्करोमीत्यर्थः। नन्वत्र गृणात्युपदिशति स गुरुरिति

व्युत्पत्त्या जनितुरेव गुरुपदाभिधेयत्वेन तस्यैव स्मृत्या नमस्कारबोधनात्। इह तु पितुरुत्पादकस्य प्रणतिर्युक्ता स्नेहादिव प्रतिभातीत्याशंक्याह बुद्धेरिति। कथंभूतमुत्पादकम्। बुद्धेश्चीशम्। बुद्धेरिति पंचम्यर्थबलाद्वशत्वोपस्थितौ ज्ञानवशादपीशमिति। तथा च ज्ञानहेतुतया गुरुत्वे व्यवस्थिते नतिरपि तस्य युक्तेति। एतदेवाचार्येणाभिसंधाय ग्रंथोपसंहारावसरे पितुरेव गुरुत्वमभिव्यक्तीकरिष्यते यथा।

आसीन्महेश्वर इति प्रथितः पृथिव्या-
माचार्यवर्यपदवीं विदुषां प्रयातः।
लब्धावबोधकलिकां तत एव चक्रे
तज्जेन बीजगणितं लघु भास्करेणेति ॥

अथाव्यक्तगणितप्रकथनावसरे तन्नमस्कारे कोऽतिशय इत्याशंक्य द्वितीयान्वयव्याजेन तस्यातिशयमेव प्रकाशयति सांख्या इति। संख्या परिसंख्यानं गणनम्। तच्छीलाः सांख्या ज्यौतिषिकाः। यदव्यक्तगणितं बीजाख्यं तत्तेन सत्पुरुषेणाधिष्ठितं प्रवदंति। तथा च यदुद्दिश्य ग्रंथकृत्प्रवृत्तिस्तत्राधिष्ठातृत्वेनावश्यमेव तस्य संभावनीयत्वादिति भावः। अव्यक्तगणितनैपुण्यद्योतनेनैतदाशंकानुदयेऽप्येकतर-विधावन्यतरनिषेध एवेति मन्यमानमंदानामस्य व्यक्तेः कौशलमासीन्न वेति संशयापनोदार्थमव्यक्तं विशिनष्टि। व्यक्तस्येति। किंभूतमव्यक्तम्। कृत्स्नस्य व्यक्तस्यैकबीजम्। व्यक्तस्य व्यक्तगणितस्य पाटीगणितापरपर्यायस्यैकबीजमुप-जीव्यमिति यावत्। एतदुपजीव्यैव सुधीभिस्तद्रचितमिति भावः ॥छ॥

एवमस्य पद्यस्य गुरुपरतयाप्यन्योऽर्थः। अथ भक्त्यतिशयविशेषेण स्वस्य सांख्यदर्शनदर्शित्वं सूचयन् श्लेषद्वारा चातुरीमभिव्यंजयन्नव्यक्तादिपदैरर्थौ संश्लिष्य शास्त्रदेवतामपि स्वाभीष्टामनेनैव पद्येनाभिवादयत्युत्पादकमिति। तदव्यक्ताख्यं तत्त्वमव्याकृतगुणसाम्यकारणाद्यपरपर्यायं वंद इति योजना। तत्किमित्याह यदिति। यदव्यक्तं पुरुषेणाधिष्ठितं सत् बुद्धेर्महत्तत्त्वस्योत्पादकं सांख्याः प्रवदंति। सांख्यं चतुर्विंशतितत्त्वप्रतिपादकं शास्त्रं तद्वदंति ते सांख्या इति। तदधीते तद्वेदेत्यण् ॥

अयमर्थः। सांख्यास्तावत्परमा[त्मनोऽ]व्यक्तापरपर्यायप्रकृतिसंबंधाद्बुध्यादि-तत्त्वोत्पत्तिद्वारेण सर्ग इति मन्यंते। तथा भास्कराचार्यैः स्वकृतसिद्धांतशिरोमणावुक्तम्।

यस्मात्क्षुब्धप्रकृतिपुरुषाभ्यां महानस्य गर्भे।
अहंकारोऽभूदित्यादि ॥

तथा सिद्धांतसुंदरेऽस्मत्पितुश्चरणैरपि।

प्रकृतिपुरुषयोगाद्बुद्धितत्त्वमित्यादिना चेति ॥

नन्वव्यक्तमस्तीत्यत्र किं प्रमाणमित्याशंक्याह व्यक्तस्येति। किंभूतमव्यक्तम्। व्यक्तस्य कृत्स्नस्यैकबीजमिति। व्यक्तस्य व्यक्तिं प्राप्तस्य पृथिव्यादेरेकं बीजं कारणमिति। व्यक्त[स्य कृत्स्नस्य समस्तस्य] कारणत्वेनाव्यक्तं ज्ञायत इत्यर्थः। पुनः किंभूतम्। ईशम्। समर्थमेतादृक्कार्यसंपादनादित्यर्थः ॥छ॥

अथ गणितस्य प्रस्तुतत्वादीश्वररूपत्वाच्चैतैरेव पदैः श्लेषोक्त्या गणितमप्यभिवादयत्युत्पादकमिति। तदव्यक्ताख्यं बीजापरपर्यायं गणितं वंद इति योजना। अथ तत्किमित्याह यदिति। सत्पुरुषेणाधिष्ठितं बुद्धेरुत्पादकं सांख्याः प्रवदंति। सत्पुरुषेणाभ्यासादिगुणवता पुरुषेणाधिष्ठितमाश्रितं बुद्धेरुत्पादकम्। सांख्याः संख्यां गणनां कुर्वंति। ते सांख्या गणकाः प्रवदंतीत्यर्थः। आतश्चोपसर्ग इति कः। पुनः किंभूतम्। व्यक्तस्य कृत्स्नस्यैकबीजम्। व्यक्तस्य पाटीगणितस्यैकबीजं कारणमिति। पुनः किंभूतम्। ईशम्। अप्रतिहतेच्छा यस्मिन्नित्यर्थः संपन्नः ॥छ॥ ॥छ॥

एवं प्रथमपद्येनाभीष्टदेवतानमस्कारलक्षणं मंगलं विधा[ये]दानीं ग्रंथमारभमाण आचार्यस्तदारंभप्रयोजनकथनव्याजेन बीजं प्रशंसन्नेकया शालिन्याह। पूर्वं प्रोक्तमिति ॥

<पूर्वं प्रोक्तं व्यक्तमव्यक्तबीजं
प्रायः प्रश्ना नो विनाव्यक्तयुक्त्या।

ज्ञातुं शक्या मन्दधीभिर्नितान्तं
यस्मात्तस्माद्वच्मि बीजक्रियां च ॥२॥>

पूर्वं व्यक्तं प्रोक्तम्। अथाहं तस्माद्बीजक्रियां वच्मीत्यन्वयः। तस्मात्कुत इत्याह यस्मादिति। यस्मात्कारणात्प्रायो मंदधीभिरल्पबुद्धिभिरव्यक्तयुक्त्या विना प्रश्नाः नितांतं ज्ञातुं न शक्याः। अत्यंतं दुरवगमा इत्यर्थः। किंभूतं व्यक्तम्। अव्यक्तबीजम्। अव्यक्तं बीजं यस्य तदव्यक्तबीजम्। व्यक्तस्य कारणीभूतमव्यक्तगणितमित्यर्थः ॥ छ ॥

< 2. द्वितीयोऽध्यायः >

< षड्विधं प्रकरणम् >

< A. धनर्णषड्विधम् >

अथ ग्रंथ⌊प्रति⌋पाद्य निरूपणप्रसंगेन सर्वगुणनभजनाद्यपेक्षयास्य प्राथमिकत्वाद्धनर्णसंकलनव्यवकलनमुपेंद्रवज्रावृत्तार्धेनाह। योगे युतिः स्यादिति ॥

<योगे युतिः स्यात्क्षययोः स्वयोर्वा
धनर्णयोरंतरमेव योगः ॥३a-b॥>

क्षययोः स्वयोर्वा योगे युतिः स्यात्। तथा धनर्णयोर्योगेऽंतरमेव ⌊स्या⌋दिति संबंधः। क्षयश्च क्षयश्च क्षयौ। तयोः क्षययोर्ऋणगतयोस्तथा स्वयोर्द्धनगतयोश्च योगे क्रियमाणे युतिर्योग एव स्यात् तयोर्मिथःसमजातीयत्वात्। तथात्र स्वयोर्योगः स्वमृणयोर्योग ऋणमेवेति ज्ञेयम्। अथ धनर्णयोरंकयोर्योगेऽंतरमेव स्यात्तयोर्विषमजातीयत्वात् ॥

अत्रोपपत्तिः। यथा ग्रहगणिते रविस्पष्टीकरणार्थं चरोदयांतरसंस्कारे क्रियमाणे यद्युभे ऋणगते स्यातां तदा रवेः सकाशात्प्रथममुदयांतरं शोध्यं ततश्चरं च शोध्यमिति प्राप्तम्। अथ लाघवार्थमुभयोर्योगे शोधितेऽपि फलं तुल्यमेव स्यात्। अत ऋणयोर्योग ऋणमेव तथैव धनयोर्योगो युतिरिति स्पष्टम्। तत्र चरमृणमुदयांतरं च धनं दृष्टम्। तथा च प्रथमं रवावुदयांतरे धनत्वाद्योजिते सति पश्चाद्यदा चरमृणत्वाच्छोध्यते तदोभयोर्विषमजातीयत्वादंतरमेवावशिष्यते कर्पूराग्न्योरिवेत्युपपन्नम् ॥छ॥

एवं धनर्णयोगे कृते सति यदवशेषं तत्र धनत्वमृणत्वं च बह्वंकसदृशं ज्ञेयम्। उक्तं च।

स्वयोर्योगे स्वमेव स्यादस्वयोरस्वमेव च।
धनर्णयोः समायोगे बह्वंकसदृशं भवेदिति ॥

अत्रोपपत्तिः। तत्र धनयोर्योगो धनमिति स्पष्टम्। अथ दशभ्यः सकाशाच्चतुस्त्रींश्च शोधयेति केनचित्पृष्टे सति यदा दशभ्यः प्रथमं चत्वारः शोध्यंते तदावशेषं षरिमतं भवति। तस्मादपि पुनस्त्रयः शोध्यंते। तदा त्रिमितं शेषं भवति। अथ लाघवार्थं चतुर्णां त्रयाणां च योगो यदा शोध्यते तदापि त्रयमेवावशेषं भवतीति कृत्वर्णगतयोर्द्वयोर्योगे ऋणत्वमेव भवति। अथ बह्वंकसदृशं भवेदिति धनर्णयोर्योगे यदंतरमवशेषत्वेन ज्ञातं तत्कस्यावशेषं धनस्यर्णस्य वेति ज्ञातव्यम्। यस्यावशेषं तत्सदृशमेव। यथा ग्रहगणिते शरक्रांत्योरन्यदिशि वियोगे कृतेऽवशेषं स्फुटा क्रांतिः। तत्र यदधिकं तस्य या दिक् सैवावशेषस्य भवतीत्युपपन्नम् ॥छ॥

अथात्र छात्रावबोधार्थमुदाहरणं पूर्ववृत्तेनैवाह। रूपत्रयमिति ॥

<रूपत्रयं रूपचतुष्टयं च
क्षयं धनं वा सहितं वदाशु ॥३॥
स्वर्णं क्षयस्वं च पृथक् पृथङ्मे
धनर्णयोः संकलनामवैषि ॥>

अत्र यानि धन[गता]नि तानि यथावस्थितान्येव। तथा "यान्यृणगतानि तान्यूर्द्धबिंदूनी"त्येवमंकानां धनर्णत्वसंज्ञां विधाय योगांतरं कुर्यात्। तथाकृते प्रकृते न्यासः ३̇।४̇ । अत्र "योगे युतिः स्यात् क्षययोः स्वयोर्वे"ति सूत्रक्रमेण जातो योगः ७̇ । अथ पुनर्न्यासः ३।४ । योगे जातं ७ । पुनर्न्यासः ३।४̇ । योगे जातं १̇ । पुनर्न्यासः ३̇।४ । योगे जातं १ ॥

एवं धनर्णसंकलनमुक्त्वेदानीं धनर्णव्यवकलनमाह। संशोध्यमानमिति ॥

<संशोध्यमानं स्वमृणत्वमेति
स्वत्वं क्षयस्तद्युतिरुक्तवच्च ॥४॥>

संशोध्यमानं स्वं धनं ऋणत्वं एति। तथा संशोध्यमानः क्षयो [ऋणगतं स्वत्वं धनत्वमेति प्राप्नोति। ततस्तद्युतिरुक्तवत्। "योगे युतिः स्यादि"त्यादिवत् स्यादित्यर्थः ॥

अत्रोपपत्तिः। तत्र संशोध्यमानत्वमृ⌋णगतत्वं चेति पर्यायः। अतः शोध्यमानस्य धनस्यर्णत्वं सुकरमेव। अथर्णगतस्यर्णत्वे ⌊क्रियमाणेऽभावाभावे⌋ भावनियम इति न्यायेन परिशेषाद्धनत्वमेव भवति। अन्यथर्णयोर्योगे युतिर्न ⌊स्यात्⌋। अतः संशोध्यमानः क्षयः स्व⌋त्वमेतीत्युपपन्नम्⌋ ॥

अत्रोदाहरणमाह। त्रयाद्द्वयमिति ॥

<त्रयाद्द्वयं स्वात्स्वमृणादृणं च
व्यस्तं च संशोध्य वदाशु शेषम् ॥>

सर्वं स्पष्टार्थं ग्रंथतश्चावबुध्यते ॥

⌊एवं धनर्णसंकलनं व्यव⌋कलनं चोक्त्वाधुना धनर्णगुणने करणसूत्रमाह। स्वयोरस्वयोः स्वमिति ॥

<स्वयोरस्वयोः स्वं वधे स्वर्णघाते
क्षयो भागहारेऽपि चैवं निरुक्तम् ॥५॥>

स्वयोरस्वयोर्वा वधे ⌊स्वं भवति। त⌋था स्वर्णघाते क्षयो भवति। च परम्⌋। भागहारेऽपि एवमेव निरुक्तमिति संबंधः। स्वयोर्द्धनगतयोर्गुणने ⌊धनम्⌋। तथास्वयोर्⌋ऋणगतयोर्वधेऽपि धनम्। तथा धनर्णयो⌊र्गुणने ऋणं⌋ स्यादित्यर्थः। तथा स्वयोरस्वयोर्वा भागहारे फलं धनमेव। तथा धनर्णयोर्भागहारे फलमृणं स्यात् ॥

अत्रोपपत्तिः। तत्र स्वयोर्धनयो⌊र्व⌋धे ⌊स्वं धनमित्युचितमेव⌋। अथर्णयोरपि वधे स्वमेव स्यादि⌊त्यत्रर्णगतेन धनगते भाज्ये लब्ध⌋मृणग⌊तमे⌋व स्या⌊त्⌋। ततः पुनश्च⌋र्णगतलब्धस्य तथर्णगत⌊हारस्य च⌋ वधे क्रि⌊यमाणे धनगत एव भाज्यः स्यात्। अन्यथा भागहरणे ऋणयोः सम⌋जातीयत्वेन भागेऽपि योग एव स्यात् ॥

अत्रोदाह⌊रणे तु⌋भाज्यो ६ भा⌊जकः ३̇ । अत्र⌋ भागहारेऽपि ⌊चैवं⌋ नि⌊रुक्तमित्यादिना भा⌋गहरणे ⌊लब्धं २̇ । अथानेनर्णलब्धेन पुनर्भाजकेऽस्मिन् ३̇⌋ गुणिते फलं पूर्वभाज्य ⌊एवा⌋यं ⌊६⌋ जात इत्यु⌊पपन्नम्⌋ ॥

⌊अथ भागहारेऽपि चैवं निरुक्तमित्यत्रोपपत्तिः। तत्रर्णगतभाज्यस्यर्णगतहारेणै⌋व

भागे ह्रियमाणे "यद्गुणो हारो ⌊भाज्याच्छुद्ध्यति तत्फलमि"ति सूत्रक्रमादेन हारो गुणितः सन्⌋ भाज्याच्छोधितः स तावद्धनग⌊त⌋ आसीत्। तथा तदेव ⌊च भागहरणे⌋ फलं ⌊स्यादित्युपपन्नम्⌋ ॥

⌊अत्रोदाहरणमाह। धनं धनेनर्ण⌉मिति ॥

<धनं धनेनर्णमृणेन निघ्नं
द्वयं त्रयेण स्वमृणेन किं स्यात् ॥>

तथा रूपाष्टकं ⌊रूपचतुष्टये<न> चेति⌋ ॥

<रूपाष्टकं रूपचतुष्टयेन
धनं धनेनर्णमृणेन भक्तम् ॥६॥
ऋणं धनेन स्वमृणेन किं स्याद्
द्रुतं वदेदं यदि बोबुधीषि ॥>

⌊स्पष्टार्थम्। उपपत्तावुदाहृतमपि⌋ ॥

एवं ⌊धनर्णगुणनभजनानंतरं क्रमप्राप्तं धनर्णवर्गार्थं सूत्रमाह⌋। ⌊कृतिः स्वर्णयोरिति⌋ ॥

<कृतिः स्वर्णयोः स्वं स्वमूले धनर्णे
न मूलं क्षयस्यास्ति तस्याकृतित्वात् ॥७॥>

⌊स्वर्णयोः कृतिः स्वं भ⌈व⌊ती⌉ति योजना। स्वर्णयो⌈रित्यत्र⌋ स्वयोर्ऋणयोरिति ज्ञेयम्। अ⌊थ जा⌈तानां वर्गाणां मूलेषु ⌊गृह्यमा⌈णेषु ⌊धनर्णव्यवस्था⌈माह। स्वमूल इति। धनांकवर्गस्य मूलं धनमृणस्यर्णमित्यर्थः ॥

⌊अत्रोपपत्तिस्तु धनर्णगुणनोपपत्तिवदेव ज्ञेया⌋। अथर्णगतवर्गस्य मूलव्यवस्थामाह ⌊न मूलमिति। क्षयस्य मूलं नास्ति। क्षयस्यर्णगतवर्गस्य⌋ मूलाभावः। कुतो हेतोरित्यत आह त⌊स्या⌈कृतित्वादिति। ⌊तस्य⌈र्णगतवर्ग⌈स्⌊य वर्गलक्षणानाक्रांत⌊त्वा⌋<दित्य>⌊यमर्थः ॥

त्र्य ऋणं तथा त्रयो धनं ३।३ । उभयोर्गुणनेऽसमानत्वाद्वर्गाभावः। "समद्विघातः कृतिरि"ति वर्गलक्षणायोगादि[ति भावः] ॥छ॥

अत्रोदाहरणमाह। धनस्य रूपेति ॥

<धनस्य रूपत्रितयस्य वर्गं
क्षयस्य च ब्रूहि सखे ममाशु।
धनात्मकानामधनात्मकानां
मूलं नवानां च पृथग्वदाशु ॥८॥>

स्पष्टम् ॥

इति धनर्णषड्विधम् ॥छ॥

< B. शून्यषड्विधम् >

एवं धनर्णषड्विधमुक्त्वेदानीं शून्यषड्विधं निरूपयति। खयोग इति ॥

<खयोगे वियोगे धनर्णं तथैव
च्युतं शून्यतस्तद्विपर्यासमेति ॥>

खयोगे वि[योगे च] धनर्णं तथैव स्यात्। खेन शून्येन योगे तथा वियोगे च क्रियमाणे सति धनर्णं तथैव यथावस्थितमेवे[त्यर्थः]। यतो[यस्य कस्याप्यंकस्य शून्ययोगे वियोगे च स्वं रूपं न विकरोति। तथा शून्यतश्च्यु[तं सत्त]द्विपर्यासमेति। शून्या[च्छोधितं स]द्धनर्णं वैपरीत्यं प्राप्नोतीत्यर्थः "संशोध्यमानं स्वमृणत्वमेती"त्युक्तत्वात् ॥

अत्रोदाहरणमाह। रूपत्रयं स्वमिति ॥

<रूपत्रयं स्वं क्षयगं च खं च
किं स्यात्खयुक्तं वद खच्युतं च ॥९॥>

स्पष्टार्थम् ॥

अथ शून्यगुणनमाह। वधादाविति ॥

<वधादौ वियत्खस्य खं खेन घाते
खहारो भवेत्खेन भक्तश्च राशिः ॥>

खस्य शून्यस्य वधादौ [खं] शून्यं भवति। येन केनाप्यंकेन शून्ये [गुणिते शून्यं स्या]दिति यतः शून्यगुणितोऽंकः शून्यम्। तस्य स्वातंत्र्येण संख्याविषयत्वाभावादिति भावः। अत्रादिशब्देन भजनवर्गवर्गमूलानि तथैवेति ज्ञेयम्। एवमेतत्प्रसंगेन साहित्योक्त्या नारायणोऽपि स्वकृतबीजे निरूपयां चकार यथा ॥

शून्याभ्यास[वशात्खता]मुपगतो राशिः पुनः खोद्धृतो-
ऽप्यावृत्तिं पुनरेव तन्मयतया न प्राक्तनीं गच्छति।
आत्माभ्यासवशादनन्यममलं चिद्रूपमानंददं
प्राप्य ब्रह्मपदं न संसृतिपथं योगी गरीयानिवेति ॥

तथा खेन भक्तो राशिः खहरो भवे[त्] ॥

अत्रोदाहरणमाह। द्विघ्नमिति ॥

<द्विघ्नं त्रिहृत्खं खहृतं त्रयं च
शून्यस्य वर्गं वद मे पदं च ॥१०॥>

स्पष्टार्थम् ॥

अथ गणितशास्त्रे खहारस्यांकस्य संज्ञांतरमस्तीति प्रकटयति। अनंत इति। अथैतस्यानंतत्वं युक्त्या निरूपयति। अस्मिन्निति ॥

<अस्मिन् विकारः खहरे न राशा-
वपि प्रविष्टेष्वपि निःसृतेषु।
बहुष्वपि स्याल्लयसृष्टिकाले-
ऽनन्तेऽच्युते भूतगणेषु यद्वत् ॥११॥>

अस्मिन्खहरे राशौ बहुष्वंकेषु प्रविष्टेषु निःसृतेष्वपि विकारो न स्यात्। यस्य शून्यं हरस्तेन समच्छिदा योज्यमाने भिन्नांके छेदांशयोः शून्यत्वमेव भवतीत्यर्थः। नन्वत्र खहरे राशावेकद्वित्र्यादिभिन्नांकसंयोगादौ विकृतित्वदर्शनात् कथमुक्तं विकारो नास्तीति चेत्सत्यं खहरराशौ खहरत्वाविकार इति पदार्थस्यानुगमात्। अथवांकेष्वित्यत्राभिन्नेष्विति ज्ञेयम् ॥

अथ खहरस्यानंतत्वमनंतत्वसाधर्म्याद्विष्णुदृष्टांतेन दृढयन् स्वकवि⌊ताचमत्कारं⌋ दर्शयति ⌊<u>यद्वदि</u>ति। यद्व⌋<u>ल्लयसृष्टिकालेऽनंतेऽच्युते भूतगणेषु बहुषु प्रविष्टेषु निःसृतेषु</u> च सत्सु <u>विकारो ना</u>स्ति तद्वदिति। लयकाले भूतेषु विष्णौ प्रविष्टेषु तथा सर्गकाले विष्णोः सकाशान्निःसृतेषु च तस्यानंतत्वाद्यथा विकारो नास्तीति भावः। तदुक्तं भारते शांतिपर्वणि ⌊भीष्म⌋युधिष्ठिरसंवादे ॥

⌊यतः⌋ सर्वाणि भूतानि भवंत्यादि युगागमे।
⌊यस्मिंश्च प्रलयं⌋ यांति ⌊पुनरेव⌋ युगक्षये ⌊इ⌋ति ॥छ॥

इति शून्यषड्विधम् ॥छ॥

< C. <u>एकानेकवर्णषड्विधम्</u> >

एवं शून्यषड्विधमुक्त्वेदानीमस्मिन्नव्यक्तकर्मण्यव्यक्तवर्णापेक्षायां तत्षड्विधं विवक्षुरादावव्यक्तानां वर्णत्वाकारेण कल्पितानि नामान्याह। यावत्तावदिति ॥

<यावत्तावत्कालको नीलकोऽन्यो
वर्णः पीतो लोहितश्चैतदाद्याः।

अव्यक्तानां कल्पिता मानसंज्ञा-
स्तत्संख्यानं कर्तुमाचार्यवर्यैः ॥१२॥>

प्रथमं यावत्तावत् ततः कालकोऽनंतरं नीलकस्ततः पीतो लोहितश्चेति। ननु कालकादीनां वर्णानां प्रसिद्धत्वात्तेषामव्यक्तनामकल्पनमुचितम्। परं तु यावत्तावदव्य[क्त]स्याव्यक्तनामकल्पने किं कारणमिति चेन्न तस्य मानापरपर्यायत्वात्। "यावत्तावच्च साकल्येऽवधौ मानेऽवधारणे" इत्यमरोक्तेः। अथ किं कर्तुमित्याह तदिति। तत्संख्यानं कर्तुम्। तच्छब्देनाव्यक्ताः। तेषां संख्यानं गणनां कर्तुमित्यर्थः ॥

अथाव्यक्तसंकलनं व्यवकलनं चाह। योगोऽंत[रमि]ति ॥

<योगोऽन्तरं तेषु समानजात्यो-
र्विभिन्नजात्योश्च पृथक्स्थितिश्च ॥>

तेषु समानजात्योर्योगोऽंतरं वा कार्यम्। तेषु वर्णेषु मध्ये समानजातीयवर्णानां परस्परं योगस्तथांतरं कार्यमित्यर्थः। तथा विभिन्नजात्योः पृथक्स्थितिरेव कार्या। वर्णानां रूपैः साकं योगे क्रियमाणे तत्र रूपाणां पृथक्स्थितिरेव कार्या। [त]था अव्यक्तवर्गाणां केवलाव्यक्तैः साकं योगेऽपि पृथक्स्थितिरेवेति सुगमम् ॥

अत्रोदाहरणमाह। स्वमव्यक्तमिति ॥

<स्वमव्यक्तमेकं सखे सैकरूपं
धनाव्यक्तयुग्मं विरूपाष्टकं च ॥१३॥
युतौ पक्षयोरेतयोः किं धनर्णे
विपर्यस्य चैक्ये भवेत्किं वदाशु ॥>

स्पष्टार्थम् ॥

अथाव्यक्तवर्गाणां केवलाव्यक्तानां च योगे पृथक्स्थितिरेवेति ज्ञापनार्थमुदाहरणमाह। धनाव्यक्तवर्गत्रयमिति ॥

<धनाव्यक्तवर्गत्रयं सत्रिरूपं
क्षयाव्यक्तयुग्मेन युक्तं च किं स्यात् ॥१४॥>

तथा छात्रशिक्षायै पुनर्दृढीकुर्वन्नाह। धनाव्यक्तयुग्मादिति ॥

<धनाव्यक्तयुग्मादृणाव्यक्तषट्कं
सरूपाष्टकं प्रोह्य शेषं वदाशु ॥>

सर्वं स्पष्टार्थम् ॥

एवमव्यक्तयोगवियोगा⌊वुक्त्वाधुना⌋ अव्यक्तगुणने विशेषमाह। स्याद्रूपवर्णेति ॥

<स्याद्रूपवर्णाभिहतौ तु वर्णो
द्वित्र्यादिकानां समजातिकानाम् ॥१५॥
वधे तु तद्वर्गघनादयः स्यु-
स्तद्भावितं चासमजातिघाते ॥>

रूपवर्णाभिहतौ वर्णः स्यात्। अत्र रूपं व्यक्तांको वर्णश्चाव्यक्तः। तयोर्गुणने ह्यव्यक्त एव स्यात्। ⌊नन्वत्र व्य⌋क्ताव्यक्तगुणने ह्यव्यक्त एव स्यादिति नियमे किं कारणमिति ⌊चेत्तत्र⌋ श्रूयतामव्यक्तस्य व्यक्तापेक्षया ⌊मूलभूतत्वे सति⌋ बहुत्वात् यतोऽव्यक्तमेव व्यक्तीक्रियते न परं तु ⌊व्यक्तमव्यक्ती⌋क्रियते व्यक्तस्य ⌊स्वतःसिद्धत्वादेव। तथा च यद्बहुतरं तत्सदृशमेव⌋ भवतीति ॥

अत्रोपपत्तिरपि। तत्र केवला⌊व्यक्तेन रूपेषु गुणितेष्वव्यक्तो जातः। तस्य⌋ पुनः केवलाव्यक्तेन ⌊भागे ह्रियमाणे⌋ लब्धं रूपाण्येव भवति यतो "यैर्यैर्वर्णैः संगुणो यैश्च रूपै"रिति ⌊वक्ष्यमाणभागहरण⌋सूत्रक्रमात्केव⌊लाव्यक्त⌋च्छेदो यदि वर्णैर्गुण्यते तदा अव्यक्तवर्गो भवति। स तु केवलाव्यक्तलक्षणभाज्यान्न शुद्ध्यतीति कृत्वा ⌊रूपैर्गुणितः⌋

शोध्यः। त⌊था च यद्गुणो हरो⌋ भाज्याच्छुद्ध्यति तदेव फलम्⌋। अतः प्रकृते रूपाण्येव फलं भवंति। अथ ⌊तानि च⌋ पुनर्यदि ⌊केवलाव्य⌋क्तेन गु⌊ण्यंते तर्हि पुनरव्य⌋क्त एव भवतीत्युपपन्नम्⌋। एवं "स्याद्रूपवर्णाभिहतौ तु वर्णः" इत्येतस्यार्थः संपन्नः ॥

अथ द्वित्र्यादिकानां सम⌊जातिकानां वधे त⌊द्वर्गघनादयः स्युः। द्वौ त्रयश्चादयो येषां त एवंभूताश्चतुष्पंचादयः। तेषां क्रमेण गुणने वर्गघनादयो भवंति य⌊तः समयोर्द्वयोर्घाते⌋ वर्गस्तथा त्रयाणां घाते घ⌊न इ⌊ति प्रसिद्धमेव। अत्रोभयत्रादिशब्देन चतुष्पंचादीनां समानां घाते वर्ग⌊वर्गा⌋स्तथा घनघना इत्यादि भवंतीत्य⌊र्थः। त⌋थासमजातिकानां वधे तद्भवति भावितं च भवति। असमजातिकानां ⌊यावत्तावत्कालकनीलका⌋दीनां परस्परं गुणने तद्भवति। अत्र तच्छब्देन येन गुणित⌊स्तन्नामा⌋क्षरं तथा गुण्यनामाक्ष⌊रं तथा भावितं चेति वर्णत्रयं भ⌋वति। तत्र भिन्नवर्णगुणनोपलक्षणत्वेन संभावितः संज्ञाविशेषो भावितमित्युच्यते। अयमर्थः। ⌊यावत्तावता⌋ कालकेषु गुणितेषु याकाभेति भवति। तथा कालकेन नीलके गुणिते ⌊कानीभेति⌋ भवतीति। एवं ⌊गुणकाक्षरमाद्यं⌋ कृत्वा ⌊लेख्यमिति भावः⌋॥

⌊अथाव्यक्तकर्मणि भिन्नां⌋केषूत्पन्नेषु तत्र विशेषमाह। भागादिकमिति ॥

<भागादिकं रूपवदेव शेषं
व्यक्ते यदुक्तं गणिते तदत्र ॥१६॥>

भागादिकं कर्म रूपवदेव ⌊ज्ञेयम्⌋। व्यक्तांक⌊वत्करणीय⌋मित्यर्थः। तथा शेषमु⌊र्वरितं⌋कर्म व्यक्ते पाटीगणिते यदुक्तं तदे⌊वात्र ज्ञे⌋यम्। ⌊वर्गघनसमच्छेद⌋त्रैराशिकश्रेढीक्षेत्रादि सर्वमपि पाटीगणिताभिप्रायेणैव सिद्ध्यतीत्यर्थः ॥

एवं <भागादौ> विशेषं निरूप्येदानीं गुणने करणसूत्रमाह। गुण्यः पृथगिति ॥

<गुण्यः पृथग् गुणकखण्डसमो निवेश्य-
स्तैः खण्डकैः क्रमहतः सहितो यथोक्त्या ॥>

गुण्यः पृथग् गुणकखण्डसमो यथा भवति तथा निवेश्यः स्थापनीयः। ततो य[त्खण्डसमो] निवेशितस्तैरेव खण्डकैः क्रमेण हतो गुणितस्तदनंतरं यथोक्त्या सहि[तः का]र्यः। अत्र यथोक्त्येत्यनेन समयोरसमयोर्वा व्यक्ताव्यक्तयोर्धनर्णयोर्वोक्तवत्प्रकारेण योगः कार्य इति सूचितम्। तथाकृते सति गुण[ने] फलं भवतीत्यर्थः ॥

अत्रोपपत्तिः [स्पष्टैव तथापि मुग्ध]च्छात्रबुद्धिवृद्ध्यर्थमुच्यते। तत्र गुणनं नाम [गुण्यस्य] गुणकांकसंख्याप्रमितावृत्तिपूर्वको योगविशेषः। तथा भजनं नाम भाज्याद् भाजकांकसंख्याप्रमितावृत्तिपूर्वकोऽंतरविशेषः। एवमत्राव्यक्तगुणने यो हि गुण्यस्तत्र यावत्तावदादयो भिन्नवर्णाः संति। तथैव गुणकेऽपीति कृत्वात्र खण्डगुणनं प्रवर्तते। तत्र यावंति गुणकखण्डानि तावंत्येव गुण्यखंडानि कृत्वा प्रत्येकं गुण[ने कृते] उक्तवत् सहिते फलं लभ्यते। यथा द्वादशभिर्द्वादशसु गुणितेषु चतुश्चत्वारिंशदधिकं शतं भवति। अथ द्वादशखण्डानि [कृत्वा] प्रत्येकं द्वादशभिः संगुण्य यावदेकीक्रियते तावत्तदेव फलं लभ्यत इत्युपपन्नम् ॥

अथ प्रसंगेन वक्ष्यमाणकरणी[गुणने त]थाव्यक्तवर्गे विशेषमाह। अव्यक्तवर्गेति ॥

<अव्यक्तवर्गकरणीगुणनासु चिन्त्यो
व्यक्तोक्तखण्डगुणनाविधिरेवमत्र ॥१७॥>

अव्यक्तवर्गकरणीगुणनास्वत्र व्यक्तोक्तखण्डगुणनाविधि[रेव चिंत्यः। अ]व्यक्ता यावत्तावदादयः। तेषां वर्गे तथा करणीगुणने च क्रियमाणे

गुण्यस्त्वधोऽधो गुणखण्डतुल्यस्तैः।
खण्डकैः [संगुणितो युतो वे]ति ॥

पाटीगणितोक्तसूत्रक्रमेण विधिः कर्तव्य इत्यर्थः ॥

अथ गुणनोदाहरणमाह। यावत्तावत्पंचकमिति ॥

<यावत्तावत्पञ्चकं व्येकरूपं
यावत्तावद्भिस्त्रिभिः सद्विरूपैः।
संगुण्य द्राग्ब्रूहि गुण्यं गुणं वा
व्यस्तं स्वर्णं कल्पयित्वा तु विद्वन् ॥१८॥>

⌊एवमत्र गुण्यगु⌋णकौ या ⌊५ रू⌋ १̇ । या ३ रू २ । अथ "गुण्यः पृथग् गुणकखण्डसमो निवेश्य" इति सूत्रक्रमेण ⌊गुणनाज्जा⌋तं फलं या ⌊व १५⌋ या ⌊७⌋ रू २̇ ॥

अथ ⌊भागहरणे⌋ करणसूत्रमाह। भाज्याच्छेद इति ॥

<भाज्याच्छेदः शुद्ध्यति प्रच्युतः सन्
स्वेषु स्वेषु स्थानकेषु क्रमेण।
यैर्यैर्वर्णैः संगुणो यैश्च रूपै-
र्भागाहारे लब्धयस्ताः स्युरत्र ॥१९॥>

छेदो यैर्यैर्व⌊र्णै⌋र्यै⌊श्च रू⌋पैः संगुणो ⌊भाज्यात्प्रच्युतः⌋ सन् ⌊शुद्ध्यति। ता ए⌋वात्र भागहरणे लब्धयः स्युरिति संबंधः। भाजयितुं योग्यो भाज्यः। तस्मात्छेदो हरो यैर्वर्णै रूपैर्वा संगुणः शुद्ध्यति तावल्लब्धयः स्युरित्यर्थः ॥

अत्रोपपत्तिः। पूर्वं गुणनसूत्रे गुणकखण्डतुल्यानि गुण्यस्य खण्डानि स्थाप्य तेषां गुणकखंडैर्गुणितानां योगो गुणनफलं जातमासीत्। इदानीं गुणनफलमेव भाज्यः कल्पितम्। गुणक एव छेदः कृतः। स च पूर्वगुण्यस्थितैरेव वर्णैर्गुणितो भाज्याच्छेदः शुद्ध्यति। अतो यैर्यैर्वर्णैः संगुणितः सन् शोधितः। तदेव फलं तच्च वर्णात्मकं भवति। अथ पूर्वरूपगुणोऽपि गुण्यो योजितोऽभूत्। इदानीं स एव रूपगुणोऽपि शोध्यत इत्युपपन्नम् ॥

अ⌊थेदं⌋ छात्रावबोधार्थमुदाहरणत्वेन स्प⌊ष्टं⌋ व्याख्यायते। तथाहि अत्र पूर्वगुणनफलमेव भाज्योऽयं या व १५ ⌊या ७ रू⌋ २ं । तथा ⌊च गुणक एव⌋ हारोऽयं या ३ रू २ । एवमयं छेदो यैर्यैर्वर्णै रूपैर्वा संगुणः सन् भाज्याच्छुद्ध्यति ता एव लब्धयो भवंति। तथा ⌊चात्र⌋ छे⌊दस्थिता⌋स्त्रयः पंचभिर्गुणिताः संतो भाज्याच्छुद्ध्यंति। तथा च न्यासः। या व १५ या ७ रू २ं । या ३ रू २ । अत्र पंचगुणेषु त्रिषु शोधितेषु ⌊लब्धं या ५ । शेषं⌋ या ७ रू २ं । अथ छेदस्य वर्णरूपात्मकतया स्थानद्वयस्थितत्वाद्वर्णैर्भागे हृतेऽपि रू⌊पैर्ह⌋र्तव्यः। अतो यद्गुणैरेव वर्णैर्भागो ⌊हृत⌋स्तद्गुणैरेव रूपैरिति कारणात्पंचगुणमेव छेदगतं रूपद्वयमुपरिष्ठाच्छोध्यम्। तथाकृतेऽधस्ताद्दश भवंति। तेषां ⌊च शोध्य⌋त्वात् 'संशोध्यमानं धनमृणं भवती'ति सूत्रक्रमाच्छोध्याः। तथा च ते ऋणम्। अथ शोध्यभाज्ययोर्दशसप्तमितयोरुभयोर्धनर्णत्वादंतरे कृते जातं

या ३ं रू २ं ।

या ३ रू २ ।

अथ पुनरुत्सारणे कृते रूप⌊स्थान⌋भाज्यतया यै रूपैश्छेदो गुणितः ⌊सन्⌋ भाज्याच्छुद्ध्यति। तथाकृते जातं १ं । रूपगुण एव छेदः शुद्ध्यति। अत ⌊ऋ⌋णगतरूपेण छेदे गुणिते "⌊स्व⌋र्णघाते क्षय" इति धनीभूतस्य छेदस्यर्णत्वे सति जातं

या ३ं रू २ं ।

या ३ं रू २ं ।

एवमत्र 'संशोध्यमानमृणं धनत्वमेती'ति "धनर्णयोरंतरमेवे"ति ⌊कृते ल⌋ब्धो गुणयः। या ५ रू १ं । एवं सर्वत्र ॥छ॥

अथाव्यक्तवर्गोदाहरणमाह। रूपैः षड्भिर्वर्जितानामिति ॥

<रूपैः षड्भिर्वर्जितानां चतुर्णा-
मव्यक्तानां ब्रूहि वर्गं सखे मे ॥>

अत्र

अ[व्य]क्तवर्गकरणीगुणनासु चिन्त्यो।
व्यक्तोक्तखण्डगुणनाविधिरिति ॥

सूत्रेणाव्यक्तवर्गः सिद्ध्यति ॥छ॥

अथ वर्गमूले करणसूत्रमाह। कृतिभ्य आदायेति ॥

<कृतिभ्य आदाय पदानि तेषां
द्वयोर्द्वयोश्चाभिहतिं द्विनिघ्नीम् ॥२०॥
शेषात्त्यजेद्रूपपदं गृहीत्वा
चेत्सन्ति रूपाणि तथैव शेषम् ॥>

कृतिभ्यः पदान्यादाय द्वयोर्द्वयोरभिहतिं द्विनिघ्नीं शेषात्त्यजेदिति संबंधः। वर्गराशौ ये वर्गाः सन्ति तेभ्यः पदानि गृहीत्वा तत्र द्वयोर्द्वयोः पदयोरभिहतिं द्विगुणामुर्वरितांकाच्छोधयेदित्यर्थः। अथ किं कृत्वेत्याह चेत्संतीति। वर्गराशौ चेद्रूपाणि संति तदा पदार्थं तद्रूपपदं गृहीत्वेति। पदार्थं वर्गराशिमूलार्थमित्यर्थः ॥

अत्रोपपत्तिः। यथाव्यक्तवर्गे क्रियमाणे प्रकृते त्वव्यक्तराशौ खण्डद्वयं विद्यते। एवं स्थानद्वये खण्डद्वयेन गुणने प्राप्ते प्रथमांको येनाव्यक्तखण्डेन गुणितस्तत्र तस्यैव वर्गो जातः। तथा तेनैव खण्डेन रूपेषु गुणितेषु वर्णो भवति। अथ द्वितीयरूपात्मकखण्डकेन द्वितीयस्थानस्थितांके गुणिते तत्र रूपवर्गो भवति। तथा यावत्तावद्भवति। एवं वर्गराशौ एकोऽव्यक्तवर्गो जातः। द्वितीयो रूपवर्ग इति वर्गद्वयं जातम्। अतः कृतिभ्य आदाय पदानीति युक्तम्। अथ द्वयोर्द्वयोरिति वर्गराशौ यो हि या[वत्ता]वत्संज्ञकः स स्थानद्वयेऽपि मूलद्वयेन गुणितः पुनः स्वेनैव युतत्वाद्द्विगुण आसीत्। अतो द्वयोर्द्वयोश्चाभिहतिं द्विनिघ्नीं शेषात्त्यजेदित्युपपन्नम् ॥छ॥

एवमेकवर्णषड्विधमुक्त्वेदानीमनेकवर्णषड्विधं निरूपयिषुस्तावत्संकलनं व्यवकलनं चाह। यावत्तावत्कालकेति ॥

<यावत्तावत्कालक-
नीलकवर्णास्त्रिपञ्चसप्तधनम् ॥२१॥
द्वित्र्येकमितैः क्षयगैः
सहिता रहिताः कति स्युस्ते ॥>

⌊अत्र⌋

योगोऽंतरं ⌊तेषु समान⌋जात्यो-
र्विभिन्नजात्योश्च पृथक्स्थितिश्चेति ॥

सूत्रक्रमेण योगोऽंतरं वा कार्यमित्यर्थः। शेषं स्पष्टम् ॥

अथ तद्गुणनार्थमुदाहरणमाह। यावत्तावत्त्रयमिति ॥

<यावत्तावत्त्रयमृणमृणं कालकौ नीलकः स्वं
रूपेणाढ्या द्विगुणितमितैस्ते तु तैरेव निघ्नाः ॥२२॥
किं स्यात्तेषां गुणनजफलं गुण्यभक्तं च किं स्या-
द्गुण्यस्याथ प्रकथय कृतिं मूलमस्याः कृतेश्च ॥>

अत्र गुण्यः या ३̇ का २̇ नी १ रू १ । अथ गुणको गुण्यापेक्षया द्विगुणोऽयं या ६̇ ⌊का ४̇⌋ नी २ ⌊रू २⌋ । अथ "गुण्यः पृथगि"त्यादिना गुण्यः स्थान⌊च⌋तुष्टये स्थाप्यः। चतुर्भिर्गुणकखण्डैर्गुणयेत्। तत्र

समजातीनां गुणने ⌊तत्तद्वर्गा⌋स्तथा
⌊भिन्नजाती⌋नां गुणने ⌊तद्भा⌋वितं चेति ॥

पूर्वकथितवद्गुणने यथास्थानं योगे च जातं गुणनात्फलं ⌊या व १८ का व ८ नी व २ या का भा २४ या नी भा १२̇⌋ का नी भा ८̇ या १२̇ ⌊का ८̇⌋ नी ४ रू २ । एवमुक्तवद्भजनवर्गवर्गमूलानि ज्ञेयानि। शेषं स्पष्टं ग्रंथत⌊श्चावबुध्यते⌋ ॥

⌊इति वर्णषड्विधम्⌋ ॥छ॥

< D. करणीषड्विधम् >

एवमव्यक्तषड्विधं निरूप्येदानीं करणीषड्विधं विवक्षुरादौ तत्संकलनं व्यवकलनं चाह। ⌊योगं करण्योरिति⌋॥

<योगं करण्योर्महतीं प्रकल्प्य
वधस्य मूलं द्विगुणं लघुं च ॥२३॥
योगान्तरे रूपवदेतयोः स्तो
वर्गेण वर्गं गुणयेद्भजेच्च ॥>

⌊करण्योर्योगं म⌋हतीं प्रकल्प्य तथा करण्योर्वधस्य मूलं द्विगुणं लघुं च प्रकल्प्य तयोर्योगांतरे रूपवत्स्त इति संबंधः। उद्दे⌊शकालापे ये कर⌋ण्यौ स्यातां तयोर्योगे महतीमिति संज्ञां प्रकल्पयेत्। अथ करण्योर्वधस्य यन्मूलं तद्विगुणं कृत्वा तस्मिन् लघुमिति च संज्ञां प्रकल्पयेत्। ततस्तयोर्महतीलघ्व्यो रूपवद् व्यक्तांकवद् योगमंतरं वा कुर्यादित्यर्थः। अथैतस्याः करण्या गुणनविधिकथ⌊नव्या⌋जेन स्वरूपं निरूपयन्नाह वर्गेणेति। वर्गेण वर्गांकेण वर्गं गुणयेत् तथा वर्गेणैव वर्गं भजेत्। न परं तु रूपेण वर्गं गुणये⌊द्भ⌋जेद्वेत्यर्थः। अनेन करणीत्वं नाम वर्गत्वाभिमतांकत्वं सूचितं भवति। तदुक्तं नारायणेन।

मूलं ग्राह्यं राशेः यस्य।
तु करणीति नाम तस्य स्यादिति ॥

अथ प्रकारांतरेण करणीयोगवियोगावाह। लघ्व्या हताया इति ॥

<लघ्व्या हतायास्तु पदं महत्याः
सैकं निरेकं स्वहतं लघुघ्नम् ॥२४॥
योगान्तरे स्तः क्रमशस्तयोर्वा
पृथक्स्थितिः स्याद्यदि नास्ति मूलम् ॥>

लघ्व्या हताया महत्याः पदं ग्राह्यम्। तद्द्विधा स्थाप्यैकत्र सैकमपरत्र निरेकमुभयत्र स्वहतं लघुघ्नं च कृतं सद् योगांतरे भवत इति ⌊संबंधः⌋। स्वहतं वर्गीकृतम्। तथात्र लघुत्वं महत्वं च न्यूनाधिकसंख्याकत्वं न तु पूर्ववदित्य⌊र्थः⌋। अथ योगेऽं⌊तरे⌋ वा ⌊क्रियमाणे यदि लघ्व्या हताया⌋ महत्या मूलं न संभवति तदा किं कार्यमित्याह ⌊पृथक्स्थितिः स्याद्यदि नास्ति⌋ मूलमि⌊ति⌋ ॥

⌊ननु⌋ "योगं ⌊करण्योर्महतीं⌋ प्रकल्प्ये"त्यादिना यद्योगोऽंतरं वा साध्यते तत्रोक्तवद्गुणनादिसाम्येऽपि योगवियोगयोरेव कथं विप्रतिपत्तिरिति चेत्तत्रोपपत्ति⌊रुच्यते⌋। अत्र वक्ष्यमाणद्विकाष्टमित्योः करण्योरुक्तवद्योगांतरे १८।२ । अत्र द्वयमष्टौ चेति वर्गौ कल्पितौ। अनयोर्मूलैक्यवर्गो यावान्भवति तत्प्रमितयोगेन भाव्यम्। तथा चोक्तवद्योगे क्रियमाणे मूलैक्यवर्गप्रमित एवांको जायते। तथाहि करणी २ । अस्या मूलं १।२५ । तथा करणी ८ । अस्या मूलं २।५१ । अनयोरैक्यं ४।१६ । अस्य वर्गः १८।<१>२ । अयमेव करणीयोगः। अथ मूलयोरंतरं १।२६ । अस्य वर्गोऽयं २।३ । इदमेवांतरमित्युपपन्नम् ।।छ।।

अथ "योगं करण्योरि"त्यादितः करणीसूत्रोपपत्तिः। तत्र मूलयोर्वर्गयोगो मूलवधेन द्विगुणेन युतः सन् मूलैक्यवर्गो भवति। एवं प्रकृते तु मूलयोरज्ञाने कथं वा तदैक्यवर्गो ज्ञायत इत्याचार्यो भंग्यंतरेण मूलैक्यवर्गानयनार्थं सूत्रं रचितवान् "योगं करण्योरि"ति। अत्र मूलाज्ञानेऽपि मूलवर्गौ करणीत्वाकारेण ज्ञातौ। तयोर्योगो मूलयोर्वर्गयोगो भवतीत्यत उक्तं "योगं करण्योर्महतीमि"ति। अथ तयोर्वर्गयोर्वधस्य मूलं मूलवधेन समं भवति। तेन द्विगुणेन वर्गयोगो युतः सन्मूलैक्यवर्गो भवतीत्यत उक्तं "वधस्य मूलं द्विगुणं लघुं चे"त्यादि। एवं मूलैक्यवर्गः करणीयोगस्तथा मूलांतरवर्गः करण्यंतरं स्यात्। अत उक्तं "योगांतरे रूपवदेतयोः स्त" इति। अथ करण्योर्वधस्तन्मूलवधवर्गसमो भवतीत्यत उक्तं "वर्गेण वर्गं गुणयेदि"ति। तथा मिथःकरणीभागे यत्फलं तत्तन्मूलभागफलवर्गसमं भवतीत्यत उक्तं "भजेच्चे"ति। सर्वमुपपन्नम् ॥

अथैतदेव मुग्धच्छात्रा⌊प्र⌋तीत्यर्थं पुनरुदाहरणत्वेनोच्यते। तथात्र कल्पिते करण्यौ ⌊९।४⌋ । एतयो⌊रुक्तवद्योगे⌋ जातं ⌊२५ अंतरे च १ । अथ⌋ करण्योर्मूले

३।२ । अनयोरैक्यस्य वर्गो जातः करणीयोगोऽयं २५ । अ⌊थ⌋ मूलयोरंतरमिदं १ । ⌊अस्य वर्गो जातं⌋ करण्यंतर⌊मिदं १ । अथ क⌋रण्योर्वधो ३६ मूलवधवर्गेणानेन ३६ सम एव ॥

अथ करणीभागार्थमन्यौ कल्पितौ राशी १६।४ । अत्र भजनात्फलं ४ । अथ करण्योर्मूले ४।२ । अनयोर्मिथःभजने लब्धस्य वर्गः ४ । अनेन समं करणीभजनफलमित्यलम् ॥छ॥

अथ "लघ्व्या हृताया" इत्यत्रोपपत्तिः। तत्रोभयमूलयोः परस्परभजने यत्फलं तस्य सैकस्य वर्गो लघुमूलवर्गेण ह⌊तः सन्⌋ करणीयोगो भवति। तथा मूलभजनफलस्य निरेकस्य वर्गो लघुमूलवर्गेण हतः सन्नंतरं च भवतीति क्रिया दृष्टा। एवं प्रकृते मूलयोरज्ञानात् करणीत्वाकारेण ज्ञातयोर्मूलवर्गयोः परस्परभजने यल्लब्धं तन्मूलभजनफलवर्गसमं भवतीत्यत उक्तं "लघ्व्या हृतायास्तु पदं महत्या" इत्यादि स्पष्टमित्युपपन्नम् ॥

अत्राप्युदाहरणार्थं कल्पिते करण्यौ १६।४ । एतयोर्मूले ४।२ । अनयोः परस्परभजने फलमिदं २ । अस्य सैकस्य वर्गः ९ । अयं लघुमूलवर्गेणानेन ४ हतः सन् जातः करणीयोगः ३६ । तथा मूलभजनफलस्य निरेकस्य वर्गः १ । अयं लघुमूलवर्गेणानेन ४ हतः सन् जातमंतरमिदं ४ । एवं योगांतरे ३६।४ ॥

अथवा सूत्रक्रमेणापि न्यासः १६।४ । अत्र लघ्व्या हृताया महत्याः पदं २ । इदं द्विधा स्थाप्य सैकं निरेकं च जातं ३।१ । क्रमेण वर्गिते ९।१ । लघुहते जाते योगांतरे ३६।४ । एवं सर्वमनवद्यम् ॥

अथ करणीयोगवियोगार्थमुदाहरणमाह। द्विकाष्टमित्योरिति ॥

<द्विकाष्टमित्योस्त्रिभसंख्ययोश्च
योगान्तरे ब्रूहि पृथक् करण्योः ॥२५॥
त्रिसप्तमित्योश्च चिरं विचिन्त्य
चेत्षड्विधं वेत्सि सखे करण्याः ॥>

भोः सखे। चेत्करण्याः षड्विधं वेत्सि तर्हि योगांतरे ब्रूहीति संबंधः। अथ कयोरित्याह द्विकाष्टमित्योरित्यादि। स्पष्टम् ॥

तथा च न्यासः। क २ क ८ । अत्र "योगं करण्योर्महतीं प्रकल्प्ये"त्यादिना महती १० । अथ करण्योर्वधः १६ । अस्य मूलं ४ । द्विगुणं सत् जातो लघुः ८ । अनयोः रूपवद्योगांतरे १८।२ । एते एव योगांतरकरण्यौ क १८ । क २ ॥

अथवा न्यासः। क ८ क २ । लघ्व्या हृतायाः महत्याः पदं २ । एतद्द्विधा स्थाप्यैकत्र सैकमपरत्र निरेकं कृत्वा न्यासः ३।१ । स्वहतं ९।१ । लघुघ्नं सत् जाते योगांतरे क १८ । क २ । एवं सर्वत्र ॥

अत्र "द्विकाष्टमित्योस्त्रिभसंख्ययोश्चे"त्युदाहरणद्वये वधस्य मूलं संभवतीति [कार]णात्प्रकारद्वये[णापि] योगः संगच्छते। अथ "पृथक्स्थितिः स्याद्यदि नास्ति मूलमि"ति प्रदर्शनार्थमाह त्रिसप्तमित्योरिति। शेषं स्पष्टार्थम् ॥छ॥

अ[थ] करणीगुणनार्थमुदाहरणमाह। द्वित्र्यष्टसंख्या गुणक इति ॥

<द्वित्र्यष्टसंख्या गुणकः करण्यो
गुण्यस्त्रिसंख्या च सपञ्चरूपा ॥२६॥
वधं प्रचक्ष्वाशु विपञ्चरूपे
गुण्येऽथवा त्र्यर्कमिते करण्यौ ॥>

अत्र गुणकः क २ क ३ क ८ । तथा गुण्यः सपञ्चरूपा त्रिसंख्या करणीति न्यासः क ३ रू ५ । अथ गुणके द्विकाष्टमित्योः करण्योर्योगः संभवतीति योगं कृत्वा न्यासः क १८ क ३ । तथा गुण्येऽस्मिन् क ३ रू ५ रूपाणि विद्यंते। तेषां वर्गं कृत्वा करणीत्वं संपाद्यं यतो "वर्गेण वर्गं गुणयेद्भजेच्चे"ति पूर्वमेवोक्तम्। एवं तथाकृते जातं क २५ क ३ । अथ

अव्यक्तवर्गकरणीगुणनासु चिंत्यो।
व्यक्तोक्तखंडगुणनाविधिरेव ॥

इति सूत्रक्रमेण गुणनाज्जातं क ५४ क ४५० क ९ क ७५ ।

अथ द्वितीयोदाहरणे गुणये रूपाणामृणत्वं प्रकल्प्याह विपञ्चरूपेति। अथवा विपञ्चरूपे गुणये त्र्यर्कमिते करण्यौ गुणकः कल्पितः। गुणयस्तु प्रागुक्त एव। तत्र वधं प्रवक्ष्वेति। शेषं स्पष्टं ग्रंथतोऽप्यवबुध्यते ॥

अथर्णगतकरणीरूपाणां वर्गत्वमूलत्वव्यवस्थायां विशेषमाह। क्षयो भवेदिति ॥

<क्षयो भवेच्च क्षयरूपवर्ग-
श्चेत्साध्यतेऽसौ करणीत्वहेतोः ॥२७॥
ऋणात्मिकायाश्च तथा करण्या
मूलं क्षयो रूपविधानहेतोः ॥>

असौ क्षयरूपवर्गश्चेत्करणीत्वहेतोः साध्यते तर्हि क्षय एव स्यादिति संबंधः। क्षयरूपाण्यृणगतरूपाणि। तेषां वर्गश्चेत्करणीत्वसंपादनार्थं साध्यते तर्हि क्षय ऋणगतो भवेदित्यर्थः। अत्र "कृतिः स्वर्णयोः स्वमि"ति प्राङ्निरूपितसूत्रक्रमेणर्णवर्गस्य धनत्वप्राप्तौ क्षयो भवेदित्यनेन तद्वैलक्षण्यं द्योतितमिति ज्ञेयम् ॥

अथ "न मूलं क्षयस्यास्ति तस्याकृतित्वादि"त्येतस्य वैलक्षण्यं निरूपयन्नाह ऋणात्मिकाया इति। ऋणात्मिकायाः करण्याः मूलं चेद्रूपविधानहेतोः साध्यते तर्हि क्षय एव स्यादित्यृणगतकरणीनां रूपत्वहेतोर्मूले गृह्यमाणे तस्यर्णत्वमेव स्यादित्यर्थः क्षयरूपवर्गः क्षयो भवेदित्युक्तत्वात् ॥

अत्रोपपत्तिः। ऋणकरण्या मूले गृह्यमा'णेऽंत्यादिषमात्कृतिं त्यक्त्वे'त्यादिना करणीत्वाकारेण कृतो य ऋणगतरूपवर्गः स एव शुद्ध्यति यत ऋणगतरूपवर्ग ऋणकरणी भवति। तस्यां च वर्गराशेः शोध्यमानायां संशोध्यमानमृणं धनत्वमेति। तथा "धनर्णयोरंतरमेव योग" इत्यनेन च सूत्रक्रमेण वर्गराशिः शुद्ध्यति। तथा च यद्रूपवर्गः शोधित आसीत्तद्रूपमेव मूलं स्यात्। अत उक्तं।

क्षयो भवेच्च क्षयरूपवर्ग-
श्चेत्साध्यतेऽसौ करणीत्वहेतोरिति ॥

अत्रोदाहरणं तु। वर्गराशिः क २५ । अथास्य मूलापेक्षायां "त्यक्त्वांत्याद्विषमात्कृतिं द्विगुणयेदि"त्यादिना करणीत्वाकारेण साधित ऋणगतपंचरूपाणां वर्गः शुद्ध्यति। स चायं २५ । अथ पूर्ववत् "संशोध्यमानं स्वमृणत्वमेती"त्यादिनांतरे क्रियमाणे लब्धं मूलं रू ५ । नन्वत्रोभयकरण्योरंतरे क्रियमाणे "योगं करण्योर्महतीं प्रकल्प्ये"त्यादिसूत्रक्रमादंतरेण भाव्यमिति चेन्न। "योगं करण्योर्महतीं प्रकल्प्ये"त्यत्रापि महतीप्रकल्पनार्थं करण्योर्योगे क्रियमाणे एतत्सूत्रप्राप्तावतिप्रसंगः स्यात्। किं च। प्रकृते सूत्रप्राप्तावपि समकरण्योरंतरे शून्यावशेषत्वात्। उक्तं च।

करण्योः समयोर्योगे एका कार्या चतुर्गुणा।
तयोः स्यादंतरे शून्यमिति सर्वत्र निश्चयः ॥

इत्यादि। सर्वमुपपन्नम् ॥

अथैतत्सूत्रोदाहरणोपयोगि द्वितीयोदाहरणस्य न्यासः। तत्र गुण्योऽयं क २५ क ३ गुणकश्च क ३ क १२ रू ५ । अत्र गुणके रूपाणि संति। अतो "वर्गेण वर्गं गुणयेद्भजेच्चे"ति सूत्रक्रमेणर्णगतरूपाणां करणीत्वसंपादनार्थं वर्गे क्रियमाणे तत्रर्णत्वमेवेति सूत्रार्थः सिद्धः। तथाकृते जातो गुणकः क २५ क ३ क १२ । अत्रापि त्र्यर्कमितयोर्योगं कृत्वा जातं क २५ क २७ । अनेन पूर्वगुण्येऽस्मिन् क २५ क ३ गुणिते जातं गुणनफलं क ६२५ क ६७५ क ७५ क ८१ । अत्र वर्गराशी द्वौ स्यातां क ६२५ क ८१ । अनयोर्मूले गृह्यमाणे

ऋणात्मिकायाश्च तथा करण्या।
मूलं क्षयो रूपविधानहेतोरिति ॥

सूत्रार्थः सिद्धः। तथा च मूले रू २५ रू ९ । अनयोरंतरमेव योगोऽयं १६ । अस्य वर्गः २५६ । जातं करण्योरनयोः क ६२५ क ८१ अंतरम्। तथा पूर्वकेऽवशिष्टकरण्यौ क ६७५ क ७५ । अनयोरुक्तवदंतरं जातं क ३०० । एवमत्र क्रमेणांतरकरण्योर्न्यासः क २५६ क ३०० ॥

इति करणीगुणनम् ॥छ॥

अथ करणीभागहरणार्थं पू[र्व]गुणनफलं भाज्यं प्रकल्प्य तथा तद्गुणकमेव भाजकं प्रकल्प्य महतीं करणीमादीकृत्य च न्यासः। भाज्यः क ४५० क ७५ क ५४ क ९ । भाजकः क २ क ३ क ८ । अत्र भाजके द्विकाष्टमित्योः करण्योर्योगं कृत्वा न्यासः क १८ क ३ । अथ "भाज्याच्छेदः शुद्ध्यति प्रच्युतः सन्नि"ति प्रकारेण भागे ह्रियमाणे लब्धो गुण्यः रू ५ क ३ ॥

अथ द्वितीयोदाहरणे भाज्यः क ६७५ क ६२५ क ८१ क ७५ । भाजकः रू ५ क ३ क १२ । अत्र भाजके त्र्यर्कमितकरण्योर्लाघवार्थं योगं कृत्वा रूपाणां च करणीत्वं संपाद्य जातो भाजकः क २७ क २५ । अनेन पूर्ववद्भाज्याद्भागे ह्रियमाणे लब्धो गुण्यः रू ५ क ३ ॥छ॥

अथ "गुण्ये गुणे वा भाज्ये भाजके वा करणीनां करण्योर्वा यथासंभवं लाघवार्थं योगं कृत्वा गुणनभजने कार्ये" इत्युक्तत्वाद्द्वितीयोदाहरणे गुणनफलगतकरणीनां भजनार्थं न्यासः। भाज्यः क ८१ क ६२५ क ६७५ क ७५ । छेदः पूर्वगुणक एवायं रू ५ क ३ क १२ । अत्र भाज्ये करणीनां लाघवार्थं योगे क्रियमाणे "धनर्णयोरंतरमेव योग" इत्युक्तत्वादंतरमेव भवति। अतो "योगं करण्योर्महतीमि"ति प्रकारेण प्रथमद्वितीययोस्तृतीयचतुर्थयोश्चांतरे क्रियमाणे जाते करण्यौ क २५६ क ३०० । अथवानयोः करण्योः क ८१ क ६२५ वर्गराशित्वात्प्रकारेणाप्यंतरं तद्यथा। अनयोर्गृहीते पुनर्मूले रू २५ रू ९ अनयोर्योगे "धनर्णयोरंतरमेव योग" इति जातमंतरमेव योगोऽयं रू १६ । अस्य वर्गो जातमंतरं तयोः करण्योरेव क २५६ ॥

अथैतयोरंतरकरण्योर्भजनार्थं न्यासः। भाज्यः क २५६ क ३०० । भाजको गुणक एवायं क २५ क ३ क १२ । अत्रापि त्र्यर्कमितकरण्योर्यथोक्तयोगे कृते जातो भाजकः क २५ क २७ । अनेन भाज्याद्भागे ह्रियमाणे

भाज्याच्छेदः शुद्ध्यति प्रच्युतः सन्
स्वेषु स्वेषु स्थानकेषु क्रमेण।
यैर्यैर्वर्णैः संगुणो यैश्च रूपैरित्यादि ॥

सूत्रप्रकारैः क्रियमाणे पूर्वगुण्यस्थिताभ्यां करणीभ्यां क २५ क ३ एवं गुणितश्छेदो भाज्याच्छुद्ध्यतीति दृष्टं तद्यथा। पूर्वगुण्यस्थितकरणीभ्यामाभ्यां क २५ क ३ छेदेऽस्मिन् क २५ क २७ गुण्यमाने सति

अव्यक्तवर्गकरणीगुणनासु चिंत्यो।
व्यक्तोक्तखंडगुणनाविधिः ॥

इत्युक्तत्वाद् "गुण्यः पृथग्गुणकखंडसमो निवेश्य" इति खंडगुणनाविधौ क्रियमाणे तावदत्र गुण्यश्छेद एव। स चायं क २५ क २७ । पूर्वगुण्य एवास्य गुणकः। स चायं रू ५ क ३ । अत्र "वर्गेण वर्गं गुणयेद्भजेच्चे"त्युक्तत्वाद्रूपाणां करणीत्वं संपाद्य जातो गुणकः क २५ क ३ । अत्र गुणके खंडद्वयं वर्तत इति कृत्वा छेदरूपं गुण्यं द्विधा स्थाप्य न्यासः

क २५	क २७
क २५	क २७

अस्मिन् पूर्वगुण्ये रूपगुणकस्थितकरणीभ्यामाभ्यां क २५ क ३ गुणिते जातं

क ६२५	क ६७५
क ७५	क ८१

अथ "तैः खंडकैः क्रमहतः सहितो यथोक्त्ये"त्युक्तत्वादासां योगे क्रियमाणे "धनर्णयोरंतरमेव योग" इत्यंतरमेवोपपन्नम् ॥

अथैतासामंतरार्थं क्रमेण न्यासः क ६२५ क ८१ क ६७५ क ७५ । अत्र

प्रथमद्वितीययोस्तृतीयचतुर्थयोरुक्तवदंतरे कृते जातं भाजके करणीद्वयं क २५६ क ३०० । इदं भाज्यात्पूर्वगुणनफलादस्मात् क २५६ क ३०० "संशोध्यमानं स्वमृणत्वमेती"ति प्रकारेण प्रच्युतं सच्छुद्धतीति कृत्वा लब्धः पूर्वगुणय एवायं रू ५ क ३ । एवं सर्वत्र ज्ञातव्यं बुद्धिमता ॥

अथ प्रकारांतरेण करणीभागहारार्थं सूत्रमाह। धनर्णताव्यत्ययमीप्सिताया इति द्वाभ्याम् ॥

<धनर्णताव्यत्ययमीप्सितायाः-
श्छेदे करण्या असकृद्विधाय ॥२८॥
तादृक्छिदा भाज्यहरौ निहन्या-
देकैव यावत्करणी हरे स्यात् ॥>

<भाज्यास्तया भाज्यगताः करण्यो
लब्धाः करण्यो यदि योगजाः स्युः ॥२९॥
विश्लेषसूत्रेण पृथक् च कार्या-
स्तथा यथा प्रष्टुरभीप्सिताः स्युः ॥>

छेदे ईप्सितायाः करण्या धनर्णताव्यत्ययमसकृद्विधा[य ता]दृक्छिदा भाज्यहरौ तावन्निहन्यादिति संबंधः। तावत् कथम्। यावत् हरे छेदे एकैव करणी स्यात्। एवं तयैकया करण्या भाज्यगताः करण्यो भाज्याः। तत्र भागहरणे या लब्धाः करण्यस्ता यदि योगजा भवंति तदा व[क्ष्य]माणविश्लेषसूत्रेण तथा पृथक्कार्याः। तथा कथम्। यथा प्रष्टुः पृच्छकस्याभीप्सिताः स्युरिति [स्पष्टम्] ॥

[अ]त्रोपपत्तिः। छेदस्य धनर्णताव्यत्यये कृते स तु भाज्यहाराभ्यामन्यः कश्चित्तृतीयांको जायते। तेन भाज्यहारौ गुणितौ तथोक्तवद्गुतावंतरितौ वा लघू भवतः। यत ऋणगतछेदेन गुणितयोर्धनभाज्यहारयोरथवा धनछेदेन गुणितयोर्ऋणभाज्यहारयोर्मध्ये धनर्णत्वेन कियंत्योऽप्यंतरकरण्य उत्पन्नाः। एवं तासामंतरे कृते लघुत्वं जायत एवे[त्यत] उक्तं धनर्णताव्यत्ययमीप्सिताया इति। यद्वा येन केनाप्यंकेन भाज्यहारौ

संगुण्य भागे ह्रियमाणे फलं तु पूर्वफलसमं भवति। यथा कल्पितौ भाज्यहारौ १६।४ । अत्र भजनात्फलं ४ । अथ भाज्यहारौ द्वाभ्यां संगुण्य पुनर्भागे [ह्रि]यमाणे तदेव फलं लभ्यत इत्युपपन्नम् ॥छ॥

ननु विश्लेषसूत्रेण पृथक्कार्या इत्यत्र विश्लेषसूत्रस्याप्रसिद्धेस्तत्किं नामेत्याशंक्याह वर्गेणेति ॥

<वर्गेण योगकरणी विहृता विशुद्ध्येत्
खण्डानि तत्कृतिपदस्य यथेप्सितानि ॥३०॥
कृत्वा तदीयकृतयः खलु पूर्वलब्ध्या
क्षुण्णा भवन्ति पृथगेवमिमाः करण्यः ॥>

योगकरणी येन वर्गेण विहृता सती विशुद्ध्येत् तत्कृतिपदस्य यथेप्सितानि खण्डानि कृत्वा तदीयकृतयः पूर्वलब्ध्या क्षुण्णाः संत्य [इमाः] करण्यः पृथग्भवंतीति संबंधः। येन वर्गेण वर्गराशिना योगकरणी विहृता सती विशुध्यति निःशेषा भवति तत्कृतिपदस्य भागहारीभूतवर्गराशिमूलस्य यथेप्सितानि खण्डानि कार्याणि। [तत]स्तेषां खण्डानां वर्गाः पूर्वलब्ध्या क्षुण्णाः संतः पृथक्करण्यो भवंति। अत्र पूर्वलब्ध्येत्यनेन यत्र वर्गेण योगकरणी विहृता तत्र या लब्धिस्तयेत्यर्थः ॥

अत्रोपपत्तिः। [अत्र "लघ्व्या हृ]तायास्तु [पदं] महत्या" इत्येतत्सूत्रवैलोम्येन सूत्रमुपनिबद्धम्। तथाह्यत्रानेन योगसूत्रेण करण्योर्योगे कृते यदा स्वहतं लघु[घ्नं] कृतं तदा तत्र सैकनिरेकमूलयोर्मध्ये स्वहतमिति सैकमूलस्य वर्गो लघुगुणित आसीत्। अथ तेनैव वर्गेण चेदयं पुनर्भाज्यते तर्हि [येन लघुना] गुणितः स एव लब्धमागमिष्यति। एवमनेन योगकरण्याः सकाशाल्लघुकरणी ज्ञायते। अत उक्तं वर्गेण योगकरणी विहृतेति। अथ येन वर्गेण योगकरणी विहृता स वर्गस्तु "स्वहतमि"त्यादिना सैकनिरेकमूलस्य वर्गः कृत आसीत्। तन्मूलं गृहीत्वा तस्य चेत्खण्डानि [क्रियंते] तर्हि तदेव सैकं निरेकं वा पदं जायते। अत उक्तं खण्डानि तत्कृतिपदस्येति। अथैतस्य चेद्वर्गः क्रियते तर्हि ल[घ्व्या हृ]ताया महत्या लब्धं ज्ञायते। अथ

तच्चेत्पूर्ववर्गभजनलब्धं लघुना गुणयते तर्हि योगकरणी भवतीत्युपपन्नम् ॥छ॥

अथ "धनर्णताव्यत्ययमीप्सिताया" इत्यादिसूत्रक्रियां प्रतीति दर्शनार्थं पूर्वकथितभाज्यभाजकयोर्न्यासः। भाज्यः क ९ क ४५० क ७५ क ५४ । हारः क १८ क ३ । अथ "धनर्णताव्यत्ययमीप्सिताया" इति सूत्रक्रमेण भाजके त्रिमितकरण्या ऋणत्वं प्रकल्प्य छेदस्य न्यासः क १८ क ३ं । अत्र भाजके खण्डद्वयं वर्तते। अतस्ताभ्यां भाज्ये द्विधा गुणिते जातं क १६२ क ८१०० क १३५० क ९७२ । क २७ं क १३५०ं क २२५ं क १६२ं । अथैतासां करणीनां लाघवार्थं योगे क्रियमाणे तावत्संभवत्संप्रदायेऽस्मद्गणिते यथा।

करण्योः समयोर्योगे एका स्याच्चतुराहता।
तयोरेवान्तरे कार्ये शून्यं स्यादिति निश्चय इति ॥

तथा च समकरणीनामंतरे शून्यं जातम्। अतस्ता गताः ॥

अथोर्वरितानां करणीनां न्यासः क ८१०० क २२५ं क ९७२ क २७ं । अत्र प्रथमद्वितीययोस्तृतीयचतुर्थयोरंतरे [जा]तं करणीद्वयं क [५६२]५ क [६७५] । अथोक्तवद्विधा भाजकेऽपि गुणिते न्यासः क ३२४ क ५४ । क ५४ं क ९ं । अत्रापि समयोः करण्योर्नाशे तथोर्वरितयोरंतरे च "एकैव यावत्करणी हरे स्यादि"ति जातैका हारकरणी क २२५ । अनया भाज्यस्यास्य क ५६२५ क ६७५ भागे ह्रियमाणे लब्धं पूर्वगुणयोऽयं क २५ क ३ ॥

एवमनयैव रीत्या द्वितीयोदाहरणेऽपि जातम् ॥

तद्यथा गुणनफलमेवैतद्भाज्यः क ८१ क ६२५ं क ६७५ क ७५ं । तथा गुणकस्त्वयं क २५ं क २७ भाजकः। तथात्र भाज्ये लाघवार्थं प्रथमद्वितीययोस्तृतीयचतुर्थयोश्च करण्योरंतरमेव योगं यथोक्तं कृत्वा क्रमेण भाज्ये जातेऽन्तरकरण्यौ क २५६ं क ३०० । तथा च भाजकेऽस्मिन् क २५ं क २७ । पंचविंशतिकरण्या धनत्वं प्रकल्प्य छेदस्य न्यासः क २५ क २७ । अत्र भाजके

खण्डद्वयं वर्तते। अतस्ताभ्यां द्विधा भाज्ये गुणिते जातं क ६४००ं क ७५०० । क ६९१२ं क ८१०० । अथैतासां करणीनां लाघवार्थं योगे क्रियमाणे "धनर्णयोरंतरमेव योग" इत्यंतरमेवोपपन्नम् ॥

एवमत्र यथाक्रममंतरार्थं करणीनां न्यासः क ८१०० क ६४००ं क ७५०० क ६९१२ं । अत्र प्रथमद्वितीययोस्तृतीयचतुर्थयोरंतरे जातं करणीद्वयं क १०० क १२ । अथोक्तवद्विधा भाजकेऽपि न्यासः क ६२५ं क ६७५ । क ६७५ं क ७२९ । अत्रापि समयोः करणयोर्नाशे तथोर्वरितयोरंतरे "चैकैव यावत्करणी हरे स्यादि"ति जातैका हारकरणी क ४ । अनया भाज्यस्यास्य क १०० क १२ भागे ह्रियमाणे लब्धः पूर्वगुण्य एवायं क २५ क ३ ॥

अथ पूर्वोदाहरणे गुणनफले भाज्ये गुण्ये च भाजके कृते न्यासः। भाज्यः क ९ क ४५० क ७५ क ५४ । भाजकः क २५ क ३ । अत्रापि त्रिमितकरण्या ऋणत्वं प्रकल्प्य छेदस्य न्यासः क २५ क ३ं । एवमत्र भाजके खंडद्वयं वर्तते। अतस्ताभ्यां द्विधा भाज्ये गुणिते जातं क २२५ क ११२५० क १८७५ क १३५० । क २७ क १३५०ं क २२५ं क १६२ं । अथैतासां करणीनां लाघवार्थं योगे क्रियमाणे "धनर्णयोरंतरमेव योग" इत्यंतरमेवोपपन्नम्। तत्र समकरणीनां "योगं करण्योर्महतीमि"ति प्रकारेणांतरे क्रियमाणे शून्यमेव भवति। अतो याः समकरण्यस्तास्त्यक्ताः ॥

एवमुर्वरितकरणीनां न्यासः क ११२५० क १६२ं क १८७५ क २७ं । अत्रापि प्रथमद्वितीययोस्तृतीयचतुर्थयोश्च यथोक्तवदंतरे क्रियमाणे भाज्ये जातं करणीद्वयं क ८७१२ क १४५२ । अथोक्तवद्विधौ भाजकेऽपि गुणिते न्यासः क ६२५ क ७५ । क ७५ं क ९ं । अत्रापि समकरण्योर्नाशे तथोर्वरितयोरंतरे च कृते जातैका हारकरणी क ४८४ ॥

एवं जातभाज्यभाजकयोर्न्यासः क ८७१२ क १४५२ । क ४८४ । अत्र भागहरणे लब्धो गुणकः क १८ क ३ । इह खलु पूर्वगुणके खण्डत्रयमासीत्। अत इयं योगकरणीति कल्प्यते। अतोऽस्या विश्लेषसूत्रविधिना विश्लेषः साध्यते। तथा च योगकरणी तावदियं क १८ । इयं वर्गेण नवमितेन ९ हृता शुद्ध्यति। तत्र

लब्धं २ । अथ छेदीभूतानां नवानां मूलं ३ । अस्य खंडे १।२ । अनयोर्वर्गौ १।४ । पूर्वलब्धेनानेन २ गुणितौ जाते पृथक् करण्यौ क २ क ८ । एवं क्रमेण जातो गुणकः क २ क ३ क ८ ॥छ॥

इति करणीभजनम् ॥छ॥

अथ करणीवर्गोदाहरणमाह। द्विकत्रिपंचप्रमिता इति ॥

<द्विकत्रिपञ्चप्रमिताः करण्य-
स्तासां कृतिं त्रिद्विकसंख्ययोश्च ॥३१॥>

<षट्पञ्चकत्रिद्विकसंमितानां
पृथक् पृथङ्मे कथयाशु विद्वन्।
अष्टादशाष्टद्विकसंमितानां
कृती कृतीनां च सखे पदानि ॥३२॥>

अत्र पद्यार्थः सुलभ एव ॥

अथ वर्गार्थं प्रथमोदाहरणस्य न्यासः क २ क ३ क ५ । अत्र करणीवर्गविधौ

स्थाप्योऽंत्यवर्गो द्विगुणांत्यनिघ्ना।

इति पाटीगणितोक्तवर्गप्रकारमनुस्मरन् वर्गः कार्य इत्याचार्येणोक्तम्। तत्रायं विशेषः। तथाकृते वर्गो न भवतीति कृत्वा द्विगुणांत्यनिघ्ना इत्यत्र

वर्गेण वर्गं गुणयेद्भजेच्च।

इति विशेषस्मरणाद्रूपात्मकयोर्द्वयोः करणीत्वसंपादनार्थं वर्गं कृत्वा अपरानकान् गुणये[त्]। तथा च "द्विगुणांत्यनिघ्ना" इत्यत्र "चतुर्गुणांत्यनिघ्ना" इति सूत्रार्थः संपन्नः। एवं मूलकरणीनां वर्गे [कृ]ते ये वर्गराशयो भवे[युस्ते]षां मूलैक्यं रूपाणि

प्रकल्पयेदिति ज्ञेयम् ॥

एवमत्र करणीवर्गे सहजाः निमित्तजाश्च वर्गराशयो भवंतीति दृष्टम्। तत्र विषमस्थानस्थितत्वे सति समद्विघात इति जाताः सहजाः। समस्थानत्वे सति विषमघातजा निमित्तजाः। ते यथा चतुर्षु चतुर्भिर्गुणितेषु जाता १६ इत्यादि सहजाः। द्वाभ्यामष्टसु द्वात्रिंशत्सु वा गुणितेषु जाता १६।६४ इत्यादि निमित्तजाः। एवमत्र करणीवर्गे क्रियमाणे ये सहजा वर्गराशयस्तावतामेव मूलैक्यं रूपाणि प्रकल्पयेदित्यर्थः। निमित्तजास्तु करण्य एव कल्प्याः। तासां यथासंभवं योगे क्रियमाणे समद्विघात इति स्वलक्षणजाः। सहजाः सर्वेऽपि वर्गराशयो भवेयुः। अतस्तावतां सर्वेषां मूलैक्यं रूपाण्येव करणीवर्गे स्युः। नैकादिसंकलितमितकरण्य इति सत्यम्। तत्रापि क्रियाविच्छित्तिर्न भवति। अतो"ऽत्रापि यथासंभवं करण्योः करणीनां लाघवार्थं योगं कृत्वा वर्गवर्गमूले कर्तव्ये" इति ग्रंथकर्त्रैवोक्तत्वात् तथा च यत्र क्वापीष्टमूलकरणीवर्गे क्रियमाणे सहजा निमित्तजा वा वर्गराशय एव सर्वे स्युस्तथा तेषां यन्मूलैक्यं तदेव मूलगतसर्वकरणीनां योगो भवति। एवं मूलकरणीयोगे जाते विश्लेषसूत्रेण यथाभीष्टं विश्लेषयित्वा मूलकरण्यो भवंति। तद्यथा मूलकरण्यः क १८ क ८ क २ ।

अव्यक्तवर्गकरणीगुणनासु चिंत्यो।
व्यक्तोक्तखण्डगुणनाविधिः ॥

इति प्रकारेणैताः स्वाभिरेव गुणिता जाताः। क ३२४ क १४४ क ३६ । क १४४ क ६४ क १६ । क ३६ क १६ क ४ । एते सर्व एव सहजा निमित्तजाश्च वर्गराशय एव जाताः। अतस्तेषां सर्वेषां मूलानामैक्यमेव रू ७२ । जातः करणीवर्गः रू ७२ । अयमेव मूलकरणीनां योगः क ७२ । एवं मूलकरणीनां खंडत्रयमासीदिति कृत्वा

वर्गेण योगकरणी विहृता विशुद्ध्येत् ॥

इति षट्त्रिंशता शुद्धा लब्धं २ । षट्त्रिंशन्मूलस्यास्य ६ खण्डानि ३।२।१ । एषां

वर्गाः १।४।१ । पूर्वलब्ध्यानया २ क्षुण्णा जाता मूलकरण्यः क १८ क ८ क २ ॥

अथवा एतादृक्स्थलेऽपि मूलकरणीस्थानप्रमितानामेव सहजवर्गाणां मूलैक्यं रूपाणि प्रकल्प्येतरवर्गास्तु करण्य एव। ता यथासंभवं संयोज्य वर्गे एकादिसंकलितमित्युक्तकरण्यः स्युः। ततोऽपि "रूपकृतेः प्रोज्झ्ये"ति प्रकारेण मूलं संभवत्येव। एवं सर्वत्र। तथाकृते जाताः क्रमेण वर्गाः। रू १० क २४ क ४० क ६० । रू ५ क २४ । रू १६ क १२० क ७२ क ६० क ४८ क ४० क २४ ॥छ॥

एवं करणीवर्गमु⌊क्त्वेदानीं⌋ करणीमूले क⌊रण⌋सूत्रमाह। वर्गे करण्या यदि वा करण्योरिति द्वाभ्याम् ॥

<वर्गे करण्या यदि वा करण्यो-
स्तुल्यानि रूपाण्यथवा बहूनाम्।
विशोधयेद्रूपकृतेः पदेन
शेषस्य रूपाणि युतोनितानि ॥३३॥>

<पृथक् तदर्धे करणीद्वयं स्या-
न्मूलेऽथ बह्वी करणी तयोर्या।
रूपाणि तान्येव कृतानि भूयः
शेषाः करण्यो यदि सन्ति वर्गे ॥३४॥>

वर्गे वर्गराशौ करण्याः करण्योः करणीनामथ बहूनां चतसृणां पंचानां वा तुल्यानि रूपाणि रूपकृतेः सकाशाद्विशोधयेत्। तत्र यच्छेषं तस्य पदेन रूपाणि पृथग् युतोनितानि कार्याणि। तदर्धे करणीद्वयं स्यात्। अथैवं कृतेऽपि यदि वर्गे शेषाः करण्यः संति तर्हि मूले या बह्वी करणी स्यात्तान्येव रूपाणि कृतानीति प्रकल्प्य भूयः कार्यं यावद्वर्गो निःशेषः स्यादिति। ननु रूपकृतेः सकाशात्करणीं वा करण्यौ

करण्यो वा शोधयेदित्यनेनैव चारितार्थ्यात्तुल्यानि रूपाणीति पदमनर्थकमिति चेन्न। रूपकृतेः सकाशात्करणीशोधने

लघ्व्या हृतायास्तु पदं महत्या ॥

इति सूत्रप्राप्तावतिप्रसंगः स्यात्। तन्निरासार्थं तुल्यरूपग्रहणम् ॥

अत्रोपपत्तिः। इह तावदुक्तवत्करणीवर्गे क्रियमाणे मूलराशौ यावंत्यः करण्यः संति तावतीनां वर्गा एव भवन्ति। ततश्चतुर्गुणितांत्यकरणीगुणिता अन्या अपि करण्यो भवंति। एवं तस्मिन् यावतो वर्गराशयो जातास्तावतां मूलैक्यं रूपाणि कल्पितानि। एवं यानि रूपाणि जातानि स एव मूलगतकरणीनां योगो ज्ञातः। अथ संक्रमणविधिना योगगतकरणीनां पृथक्करणार्थमंतरापेक्षायां रूपकृतेः सकाशाच्छेषकरणीतुल्यानि रूपाणि शोधितानि। तेषु शोधितेषु करण्यंतरस्य वर्गोऽवशिष्यते। तस्य मूलं करणीनामंतरं ज्ञायते। यतः राश्योर्योगवर्गाच्चतुर्गुणे राशिघाते शोधिते सति राश्यंतरवर्गोऽवशिष्यते। प्रकृते तु यानि करणीतुल्यरूपाणि स एवोत्पत्स्यमान मूलकरणीद्वयघातश्चतुर्गुणो भवति। यतश्चतुर्गुणोऽंत्यगुणिताग्रिमोऽस्ति स एव तयोर्घातश्चतुर्गुणः। तदूने योगवर्गेऽतरवर्गोऽवशिष्यत एव। एवमंतरे ज्ञाते ततो

योगोऽंतरेणोनयुतोऽर्द्धितस्ताविति ॥

संक्रमणसूत्रक्रमेण करणीयोगत्वेन ज्ञातानि रूपाणि द्विधास्थाप्यांतरेणोनयुतान्यर्द्धितानि च कृत्वा करणीद्वयं साधितम्। तत्र याल्पीयसी करणी स्यात्सैका मूलकरणी जाता। तथा या च बहुतरा स्यात्सैवोर्वरितकरणीनां योगो जातः। अथ तान्येव रूपाणि कृत्वा पुनरंतरज्ञानार्थं रूपकृतेः सकाशाच्छेषकरणीं विशोध्य [शेषस्य] मूलमुर्वरितकरणीनामंतरं स्यात्। पुनस्तेनापि द्विधा रूपाणि युतोनितान्यर्द्धितानि च कृत्वा पुनः करणीद्वयं [सा]धि[तम्]। एवं याव[द्वर्गो] निःशेषः स्यादत उक्तं।

वर्गे करण्या यदि वा करण्योस्तुल्यानि रूपाणि ॥

इत्याद्युपपन्नम् ॥छ॥

अथ [प्रथम]वर्गस्य मूलार्थं न्यासः। रू १० क २४ क ४० क ६० । अथ सूत्रावतारो यथास्मिन्वर्गे चतुर्विंशतिचत्वारिंशन्मितकरण्योस्तुल्यानि रूपाण्येतानि ६४ रूपकृतेरस्याः १०० सकाशाद्विशोध्य शेषं ३६ । अस्य पदं ६ । अनेन रूपाणि द्विधास्थाप्य युतोनितानि कृत्वा जातं १६।४ । अनयोरर्धे करणीद्वयं जातं क ८ क २ । अथ वर्गे एका करण्यवशिष्टा। तन्मूलार्थं मूलगतकरणीद्वयमध्ये बह्वी करणीयं क ८ । एतान्येव रूपाणि प्रकल्प्यैतत्कृतेरस्याः ६४ षष्टि करणीतुल्यानि रूपाण्यपास्य शेषं ४ । अस्य मूलं २ । अनेन रूपाणि युतोनितान्यर्द्धितानि च कृत्वा पुनर्जातं करणीद्वयं क ३ क ५ । एवं क्रमेण करणी[वर्गमू]लं क २ क ३ क ५ । एवमन्यत्रापि ज्ञेयमिति। शेषं स्पष्टम् ॥छ॥

अथ करणीवर्गे धनर्णत्वव्यवस्थाविशेषमाह। ऋणात्मि[का चे]दिति ॥

<ऋणात्मिका चेत्करणी कृतौ स्या-
द्धनात्मिकां तां परिकल्प्य साध्ये।
मूले करण्यावनयोरभीष्टा
क्षयात्मिकैका सुधियावगम्या ॥३५॥>

कृतावृणात्मिका चेत्करणी स्यात् तदा तां मूलसिद्धौ धनात्मिकां परिक[ल्प्य] मूले करण्यौ भवेताम्। अनयोर्मध्ये सुधियैका क्षयात्मिकावगम्या ॥

अत्रोपपत्तिः। यदि करणीवर्गे ऋणगता करणी स्यात्तदा तत्तुल्यरूपाणां रूपकृतेः सकाशाच्छोधने प्राप्ते 'संशोध्यमानमृणं धनं स्यादि'ति उभयोर्योग एव स्यात्। कृते च योगे क्रियाविच्छित्तिः। अतः

कृतिः स्वर्णयोः स्वं स्वमूले धनर्णे ॥

इत्यादिवन्मूल एव धनर्णत्वं कल्प्यम्। वर्गे तु धनात्मकत्वमेव। अत एव साध्ये इति विशेषणमित्युपपन्नम् ॥

अत्रोदाहरणं वृत्तार्धेनाह। त्रिसप्तमित्योर्वदेति ॥

<त्रिसप्तमित्योर्वद मे करण्यो-
र्विश्लेषवर्गं कृतितः पदं च ॥>

भोः सखे। त्रिसप्तमित्योः करण्योर्विश्लेषवर्गं वद तथा कृतितश्च पदं वदेति संबंधः। विश्लेषवर्गः क्षयगतवर्गः। तथा च न्यासः। ⌊क ३̇ क ७ ।⌋ क ३ क ७̇ । अनयोः सूत्रक्रमेण पृथग् वर्गे क्रियमाणे सम एव वर्गः स्यात्। स यथा रू १० क ⌊८४̇ । अत्र⌋ वर्गे ⌊ऋणकरणीधनत्वं⌋ प्रकल्प्य ये लब्धे मूलकरण्यौ तयोर्मध्ये 'एकाभीष्टा क्षयगा स्यादि'ति जातं क ३̇ क ७ ॥

⌊अ⌋थाधिककल्पनया पुनरुदाहरणांतरमाह। द्विकत्रिपंचप्रमिताः करण्यः स्वस्वर्णगा इति ॥

<द्विकत्रिपञ्चप्रमिताः करण्यः
स्वस्वर्णगा व्यस्तधनर्णगा वा ॥३६॥
तासां कृतिं ब्रूहि कृतेः पदं च
चेत्षड्विधं वेत्सि सखे करण्याः ॥>

भोः सखे। त्वं चेत्करण्याः षड्विधं वेत्सि तर्हि प्रागुक्ता एव द्विकत्रिपंचप्रमिताः करण्यः स्वस्वर्णगाः प्रकल्प्याथवा व्यस्तधनर्णगाः प्रकल्प्य तासां कृतिं कृतेश्च पदं ब्रूहीति संबंधः। स्वं च स्वं चर्णगा च स्वस्वर्णगाः। द्वे धनकरण्यावेकर्णगा चेत्यर्थः। तथा व्यस्तधनर्णगा इत्यनेन द्वे ऋणकरण्यावेका धनमिति स्पष्टम् ॥

एवमत्र न्यासः। क २ क ३ क ५̇ । क २̇ क ३̇ क ५ । अनयोर्वर्गः सम एव

ऋणात्मिका चेत्करणी कृतौ स्यात् ॥

इत्युक्तत्वात्। तथा च वर्गोऽयं रू १० क २४ क ४०̇ क ६०̇ । अत्र मूलार्थमृणकरण्योरेव तुल्यानि रूपाणि धनानीमानि १०० । एतानि रूपकृतेरपास्य शेषस्य मूलेन रूपाणि युतोनितानि कृत्वा जातं तदर्धं ५ ।

अथ धनकरण्योस्तुल्यानि रूपाणि ६४ रूपकृतेरपास्य शेषमूलेनानेन ६ युतोनितानां रूपाणामर्धे क २ क ८ । अनयोर्मध्ये महत्या ऋणत्वं प्रकल्प्य तान्येव रूपाणि कृत्वोक्तवत्करण्यौ क ३ क ५ । एवमत्रापि महत्या ऋणत्वं प्रकल्प्यम्। शेषं स्पष्टम् ॥

अथ करणीवर्गे संभवदभिप्रायेण कंचिन्नियममाह। एकादिसंकलितमितेति ॥

<एकादिसंकलितमित-
करणीखण्डानि वर्गराशौ स्युः ॥३७॥
वर्गे करणीत्रितये
करणीद्वितयस्य तुल्यरूपाणि ॥>

<करणीषट्के तिसृणां
दशसु चतसृणां तिथिषु च पञ्चानाम् ॥३८॥
रूपकृतेः प्रोज्झ्य पदं
ग्राह्यं चेदन्यथा न सत्क्वापि ॥>

वर्गराशावेकादिसंकलितमितकरणीखंडानि स्युः। एकादिर्यस्य तदेकादि। एकादि च तत्संकलितं च। तन्मितानि करणीखण्डानीति। अयमर्थः। अभीष्टकरणीवर्गे रूपाणामवश्यं भावनियमः। तथा चैककरणीवर्गे रूपाण्येव भवंति। तथा करणीद्वयस्य चेद्वर्गः क्रियते तर्हि रूपाण्येका करणी स्यात्। तथा तिसृणां वर्गे क्रियमाणे रूपाणि करणीत्रयं स्यादिति क्रमेणैकद्वित्रिचतुष्पंचादिसंकलितमितकरणी-खण्डानि भवंतीत्यर्थः ॥छ॥

अथ करणीवर्गमूलानयने मुग्धच्छात्रसंदेहनिरासार्थं तुल्यरूपनियममाह वर्गे करणीत्रितये इत्यादि। स्पष्टार्थम् ॥

अथ रूपकृतेरिति। उक्तकरणीखण्डतुल्यरूपाणि रूपकृतेः सकाशात् प्रोज्झ्य शोधयित्वा पदं ग्राह्यम्। अथ क्वापि चेदन्यथेति। चेदन्यथोक्तप्रकारवैलक्षण्येन मूलं लभ्यते तदप्यसत्। तद्यथोक्तप्रकारस्तु। करणीषट्के प्रथमं तिसृणां करणीनां तुल्यानि

रूपाणि रूपकृतेरपास्य ततो द्वयोस्तत एकस्यास्तुल्यानि मूलं ग्राह्यम्। एवमुक्तक्रमादन्यथा क्रमं विहाय क्वचि"च्चतुर्गुणाः सूर्यतिथी"त्यादिषूदाहरणेष्वन्यथा मूलं गृह्यते। तद्यथा प्रथममाद्यकरण्यास्तुल्यानि रूपाणि ततो द्वयोस्ततः शेषाणामेवमपि कृते मूलं लभ्यते। परं तु तदसत्यतस्तथा मूलस्य सा कृतिर्न भवतीत्यर्थः ॥

अथोत्पत्स्यमानयेति ॥

<उत्पत्स्यमानयैवं मूलकरण्याल्पया चतुर्गुणया ॥३९॥
यासामपवर्त्तः स्याद्रूपकृतेस्ता विशोध्याः स्युः ॥>

उत्पत्स्यमानयाल्पया करण्या चतुर्गुणया च यासामपवर्त्तः स्यात्ता एव रूपकृतेर्विशोध्याः स्युः ॥

अथ प्रकारांतरेण मूलकरणीज्ञानं चाह ॥

अपवर्त्ते या लब्धा मूलकरण्यो भवंति ताश्चापि ॥४०॥
शेषविधिना न यदि ता भवंति मूलं तदा तदसत् ॥

अस्यार्थः। अपवर्त्ते या लब्धा लब्धसंख्याकास्ताश्चापि सर्वा मूलकरण्यो भवंति। एतावतैव कर्मणा सर्वा मूलकरण्यो ज्ञाता भवंतीत्यर्थः। यदि शेषविधिना

मूलेऽथ बह्वी करणी तयोर्या रूपाणि ॥

इत्युक्तेन ता ज्ञाता मूलकरण्यो न भवंति न संवादयंति संवादं न प्राप्नुवंति तदा तदपि मूलमसदित्यर्थः। सर्वमुदाहरणावसरे व्याख्यास्यामः ॥

अत्रोपपत्तिः। तत्रैकादिसंकलनं नामैकद्वित्र्यादीनामुत्तरोत्तरं योगः। स यथा

१	२	३	४	५	६	७	८	९
१	३	६	१०	१५	२१	२८	३६	४५

अत्रैकस्य संकलनं एकः। तथा द्वयोस्त्रयः। त्रयाणां षट्। चतुर्णां दश। पंचानां पंचदशेत्यत उक्तं "वर्गे करणीत्रितये" इत्यादि ॥

अथ उत्पत्स्यमानयेति। करणीवर्गस्य मूले ग्राह्ये योत्पत्स्यमाना मूलकरणी सा तावदंत्या करणी। तस्यां ज्ञातायामन्यापि ज्ञातव्येति प्राप्ते "स्थाप्योऽंत्यवर्गो द्विगुणांत्यनिघ्ना" इत्येतस्य सूत्रस्य वैलोम्येन सूत्रमुपनिबद्धम्। तद्यथा "स्थाप्योऽंत्यवर्गो द्विगुणांत्यनिघ्ना" इत्यत्र चतुर्गुणांत्यनिघ्ना इत्यनेन करणीवर्गे अं⌊त्याति⌋रिक्तान्याः करण्यश्चतुर्गुणितांत्यगुणिता आसन्। इदानीं ता एव चतुर्गुणितांत्यभक्ताः संत्यः स्वस्ववर्गाविशेषा भवंति। ⌊त⌋ता⌊स्ता⌋श्च रूपकृतेः शोध्य चेत्पूर्ववन्मूलं गृह्यते तर्हि मूलकरण्यो भवंतीत्यत उक्तं यासामपवर्तः स्यादित्याद्युपपन्नम् ॥छ॥

अत्रोदाहरणमाह। वर्गे यत्र करण्यो दन्तैः सिद्धैरिति ॥

<वर्गे यत्र करण्यो दन्तैः सिद्धैर्गजैर्मिता विद्वन् ॥४१॥
रूपैर्दशभिरुपेताः किं मूलं ब्रूहि तस्य स्यात् ॥>

एवमत्र न्यासः रू ⌊१०⌋ क ३२ क २४ ⌊क ८⌋ । अत्र वर्गे करणीत्रितयं वर्तते। अतः करणीद्वितय⌊स्य तुल्य⌋रूपाणि रूपकृतेरपास्य यावदुक्तवन्मूलं गृह्यते तावन्न लभ्यत इति कृत्वा सर्वासां तुल्यानि रूपाणि ६४ अपास्यैभ्यः शेषं ३६ । अस्य पदेन ६ रूपाणां युतोनितानामर्धे करण्यौ क ८ क २ । अस्य मूलस्यायं वर्गो न भवतीति ⌊दुष्टो⌋द्दिष्टमेतदित्यर्थः ॥

अथ उत्पत्स्यमानयैवमित्येतद्विषयीभूतमुदाहरणमाह। वर्गे यत्र करण्य⌊स्ति⌋थिविश्वहुता⌊श⌋नैरिति ॥

<वर्गे यत्र करण्यस्तिथिविश्वहुताशनैश्चतुर्गुणितैः ॥४२॥
तुल्या दशरूपाढ्याः किं मूलं ब्रूहि तस्य स्यात् ॥>

तथा च न्यासः रू १० क ६० क ५२ क १२ । अत्र वर्गे करणीत्रितयं वर्तते। ⌊अतः क⌋रणीद्वितयस्य द्विपंचाशद्द्वादशकमितस्य तुल्यरूपाण्येतानि ६४ रूपकृतेरस्या १०० अपास्य शेषस्य पदं ६ । अनेन रूपाणि युतोनितानि। तेषामर्धे क २ क ८ । अत्रोत्पत्स्यमानाल्पा करणीयं क २ । अनया चतुर्गुणया क ८ तुल्यरूपोपयोगि द्विपंचाशद्द्वादशकमितकरण्योरपवर्त्तो न गच्छतीति कृत्वा ते करण्यौ न शोध्ये। "यासामपवर्त्तः स्याद्रूपकृतेस्ता विशोध्याः स्युरि"त्युक्तत्वादत एतदसदित्यर्थः ॥

अथ "यासामपवर्त्तः स्यादि"त्येतत्संभवोदाहरणमाह। अष्टौ षट्पंचाशदिति ॥

<अष्टौ षट्पञ्चाशत् षष्टिः करणीत्रयं कृतौ यत्र ॥४३॥
रूपैर्दशभिरुपेतं किं मूलं ब्रूहि तस्य स्यात् ॥>

अत्र न्यासः रू १० क ८ क ५६ क ६० । अत्राद्यं खंडद्वयं क ८ क ५६ । एतत्तुल्यरूपाण्येतानि ६४ रूपकृतेः शोध्य शेषमूलं ६ । अनेन पूर्ववल्लब्धं करणीद्वयं क २ क ८ । अत्राल्पया चतुर्गुणयानया ८ तस्य खंडद्वयस्यापवर्त्तनेन लब्धे खण्डे १।७ । अत्र ⌊सू⌋त्रावतारो यथापवर्त्तादपि लब्धा इति। पूर्वकथितापवर्त्तादपि या लब्धाः करण्यस्ता यदि मूलकरण्यो न भव⌊ति⌋ तर्हि शेषविधिना 'शेषस्य पदेने'त्यादिना कार्याः। तेनापि ता यदि न भवंति तदा मूलं सन्न भवति। समीचीनं ⌊न⌋स्यात्। आसन्नमिति वा पाठः। एवं प्रकृते खंडद्वयेऽस्मिन् १।७ शेषविधिना करण्यौ नोत्पद्येते। अतस्ते ⌊न श⌋ोध्ये इत्यर्थः ॥

अथोदाहरणांतरमाह। चतुर्गुणाः सूर्यतिथीषु ⌊रुद्र⌋नागर्तव इति ॥

<चतुर्गुणाः सूर्यतिथीषु रुद्रनागर्तवो यत्र कृतौ करण्यः ॥४४॥
सविश्वरूपा वद तत्पदं ते यद्यस्ति बीजे पटुताभिमानः ॥>

अत्र न्यासः रू १३ क ४८ क ६० क २० ⌊क ४४ क ३२⌋ क २४ । अत्र करणीषट्के तिसृणां करणीनां तुल्यानि रूपाणि प्रथमं रूपकृतेरपास्य मूलं ग्राह्यम्। पश्चाद्द्वयोस्तत एकस्यैवंकृते मूलाभावः। अथान्यथा संभवदभिप्रायेण

प्रथमकरण्या⌊स्तु⌋ल्यानि ⌊रूपाण्येतानि ४८⌋ । रूपकृतेरस्याः १६९ अपास्य शेषं १२१ । अस्य मूलं ११ । अनेन रूपाणि युतोनितानि। तेषामर्धे १।१२ । अत्र बह्वी करण्येव रूपाणि कृत्वोक्तवदग्रिमकरणीद्वयतुल्यरूपाणि शोध्य पुनः करण्यौ क २ क १० । अत्रापि बहुकरणीं रूपाणि कृत्वा तदग्रिमकरणीत्रयतुल्यरूपाण्येतानि १०० । रूपकृतेरस्या १०० शोध्य शेषपदं ० । अनेन रूपाणि ⌊युतो⌋नितानि। तेषामर्धे ५।५ । एवं क्रमेण मूलं क १ क २ क ५ क ५ । एवमिदमसदिति प्रतिभाति यतोऽस्य वर्गो न भवतीत्यर्थः। एव⌊मे⌋वंविधकरणीवर्गस्थले आसन्नमूलमेवानेयम् ॥

तथा चास्मत्तातचरणैः स्वकृतसिद्धांतसुंदरे बीजगणिताध्याये आसन्नमूलानयनप्रकार उक्तः। स यथा ॥

आसन्नमूलेन हृतात्स्ववर्गाल्लब्धेन मूलं सहितं द्विभक्तम्।
भवेत्तदासन्नपदं ततोऽपि मुहुर्मुहुः स्यात् स्फुटमूलमेवमिति ॥

अस्यार्थः। यस्य कस्यापि वर्गराशेरवर्गराशेर्वासन्नमूलं ग्राह्यम्। तेन स्वीय एव वर्गो भाज्यः। तत्र यल्लब्धं तेनासन्नमूलं सहितं कार्यम्। तच्च द्विभक्तं सदासन्नमूलं भवति। एवं मुहुर्मुहुः कर्त्तव्यं यावन्निःशेषो वर्गः स्यादित्यर्थः ॥

अत्रोदाहरणं ⌊तु⌋। इष्टराशिः ५ । अस्य कल्पि⌊तं⌋ मूलं २ । अनेन स्ववर्गो हृतः $\frac{५}{२}$ । लब्धेनानेन मूलमिदं २ सहितं $\frac{९}{⌊२⌋}$ । द्विभक्तं $\frac{९}{४}$ । इदमासन्नमूलम्। अथ पुन"रासन्नमूलेने"ति कृते लब्धं मूलं पंचानां $\frac{२}{१४}$ । इत्यलमतिविस्तरेण ॥छ॥

अथान्यदुदाहरणमाह। चत्वारिंशदिति ॥

<चत्वारिंशदशीतिद्विशतीतुल्याः करण्यश्चेत् ॥४५॥
सप्तदशरूपयुक्तास्तत्र कृतौ किं पदं ब्रूहि ॥>

अत्र न्यासः रू १७ क ४० क ८० क २०० । ⌊अ⌋त्रोक्तवन्मूलं क १० क ५ क २ ॥ छ ॥

दैवज्ञज्ञानराजात्मजकविगणकाचार्यसूर्या⌊भि⌋धान-
प्रोक्ते सद्बीजभाष्ये ⌊सुज⌋नबुधजनाशेषभूषाविशेषे ।
जातः सूर्यप्रकाशेऽपटु वटु हृ⌊दयध्वांतविध्वंसदक्षे
स्वर्णैकानेकवर्णप्रकरण⌋करणीषड्विधानां समूहः ॥ छ ॥ छ ॥

इति श्रीमद्दैवज्ञपंडितसूर्यविरचिते सूर्यप्रकाश⌊नाम्नि⌋ भास्क⌊रीय⌋बीजभाष्ये ष⌊ड्विधं प्रकर⌋णं समाप्तिमगमत् ॥ छ ॥

< 3. तृतीयोऽध्यायः >

< कुट्टकाधिकारः >

श्रीगजाननाय नमः ॥ छ ॥

प्रचंडकर⌊म⌋डलोज्ज्वल⌊क⌋पोलली लालस⌊त्⌋-
प्रसन्नमधुपांग⌊ना⌋र⌊णि⌋तजातहासोत्सवम् ॥
सरत्सरलशुंडिकाकलितचंडगंडस्थलं
स्थलांबुजकला⌊य⌋माल⌊म⌋भिनौमि लंबोदरम् ॥१॥

< A. सामान्यकुट्टकः >

एव⌊म⌋व्यक्तादिकरणयंतानां षड्विधान्युक्त्वाधुना वक्ष्यमाणव⌊र्ग⌋प्रकृत्यनेकवर्णाद्युपयो⌊गि⌋त्वेन⌊ कुट्टकमारभमाणस्तावत्त⌊त्स्व⌋रूपं निरूपयन्नाह। भाज्यो हारः क्षेपकश्चेति ॥

<भाज्यो हारः क्षेपकश्चापवर्त्यः
केनाप्यादौ संभवे कुट्टकार्थम् ॥४६॥
येन छिन्नौ भाज्यहारौ न तेन
क्षेपश्चैतद्दुष्टमुद्दिष्टमेव ॥>

आदौ संभवे सति केनाप्यंकेन कुट्टकार्थं भाज्यो हारः क्षेपक⌊श्चा⌋पव⌊र्त्य⌋ इति संबंधः। भज्यतेऽसौ भाज्यस्तथा ह्रियते तेनेति हारस्तथा क्षिप्यते स क्षेपः। एवमेते त्रयो यत्र भवंति तत्रैव कुट्टकः ⌊सं⌋भव⌊ति⌋। अत्र संभवे सतीत्यनेनापवर्त्तसंभवे ह्यपवर्त्तः कार्यः। तदसंभवे यथावस्थितैरेव क्षेपहारभाज्यैः कुट्टकवि⌊धि⌋ः संपादनीयः। परं त्वपवर्त्त्य एवेति नियमेनेति सूचितम्। अत्र कुट्टक इति रूढः शब्दः। अथापवर्त्तेऽपि कुट्टकसंभ⌊वासंभवे⌋ विशेषमाह येनेति। येनांकेन भाज्यहारौ छिन्नौ तेनैव क्षेपश्चेच्छिन्नो

न शुद्ध्यति तर्हि तद्दुष्टमु[द्दिष्टम्]। अय[मर्थः]। येनांकेन भाज्यहारयोरपवर्त्तः कृतस्तेनैव चेत्क्षेपस्यापवर्त्तो न गच्छति तर्हि तद्दुष्टमुद्दिष्टम्। धूर्त्त[त्वेन पृष्ट]मित्यर्थः ॥

अ[था]पवर्त्तानियमे [स]ति के केन वांकेन भाज्यहारक्षेपा अपवर्त्त्या इति व्याकुलचित्तानां मुग्धच्छात्रा[णां] संशयो माभूदित्यप[वर्त्ता]किज्ञानार्थं सूत्रमाह। परस्परं भाजितयोरिति ॥

<परस्परं भाजितयोर्ययोर्यः
शेषस्तयोः स्यादपवर्त्तनं सः ॥४७॥
तेनापवर्त्तेन विभाजितौ यौ
तौ भाज्यहारौ दृढसंज्ञकौ स्तः ॥>

द्वयोः परस्परं भाजितयोः सतो[र्यच्छेषं तत्तयोरपवर्त्तनं स्यात्]। द्वयोर्भाज्यभाजकयोरित्यर्थः। अत्रापवर्त्तनं नाम निःशेषभागहरणे निमित्त<१>भूतोऽकः कश्चिदित्यर्थः। एवमपवर्त्तने [ज]ाते यौ भाज्यहारौ स्वेनापवर्त्तेन विभाजितौ तौ दृढसंज्ञकौ स्तः। अत्र भाज्यहाराविति द्विवचनमुप[लक्ष]णमे[व]। तेन भाज्यहारक्षेपा भाजिताः संतो दृढसंज्ञकाः स्युरित्यर्थः। अत्र दृढसंज्ञसंज्ञकत्वं नामाविकृतत्वं विवक्षितम् ॥

अथ कुट्टकप्रतिपाद्यार्थसिद्ध्यर्थं सूत्रमाह। मिथो भजेत्तौ दृढभाज्यहारावित्यादितस्त्रि[भिर्वृ]त्तैः ॥

<मिथो भजेत्तौ दृढभाज्यहारौ
यावद्विभाज्ये भवतीह रूपम् ॥४८॥
फलान्यधोऽधस्तदधो निवेश्यः
क्षेपस्तथांते समुपान्तिमेन ॥>

<स्वोर्ध्वे हतेऽन्त्येन युते तदन्त्यं
त्यजेन्मुहुः स्यादिति राशियुग्मम् ॥४९॥
ऊर्ध्वो विभाज्येन दृढेन तष्टः
फलं गुणः स्यादपरो हरेण ॥>

तौ दृढभाज्यहारौ तावन्मिथो भजेदिति संबंधः ॥ तावित्यनेन यौ पूर्वसूत्रनिष्पन्नावित्यर्थः। तावत्कथम् ? यावदिह भाज्ये रूपं भवति। परस्परभजने यो हि भाज्यत्वेनोपस्थितो राशिस्तत्र यावद्रूपमेकोऽवशेषः स्यात्तावद्भजेदित्यर्थः। एवं तत्र मिथो भागहरणे फलानि लब्धान्यधोऽधःस्थाप्यानि। ततस्तदधः क्षेपो निवेश्यस्तथांते सर्वेषामधः शून्यं स्थाप्यम्। एवं यथा फलवल्ली भवति तथा लेख्यम्। एवं कृते सत्युपांतिमेन स्वोर्ध्वे हतेंऽत्येन युते च सति अंत्यं त्यजेत्। एवं मुहुर्वारं वारं कर्त्तव्यं यावद्राशियुग्मं स्यात्। अत्रोपांतिमेत्यनेनांत्यस्योपसमीपे तिष्ठतीत्युपांतिमः। तथात्रांत्यः शून्यं तदुपरिष्ठ उपांतिमः। परिशेषात् क्षेप इति यावत्। तत उत्तरोत्तरं य उपांतिमः स्यात्तेन स्वोर्ध्वस्थितं हन्यात्। तत उक्तवद्राशियुग्मे कृते सति तत्र य ऊर्ध्वो राशिः स दृढेन भाज्येन तष्टः सन् शेषं फलं स्यात्। तथा अपरोऽधःस्थितो राशिर्दृढहारेण तष्टः सन् शेषं गुणः स्यात्। अथ यत्र भागहरणे फलानुपयोगे सति शेषेणैव प्रयोजनं तत्र तष्ट इति सांकेतिकः शब्दः प्रयुज्यते ॥

अथ गुणलब्ध्योः सिद्धौ तदिति कर्त्तव्यतायां तत्संभवे विशेषमाह। एवमिति ॥

<एवं तदैवात्र यदा समास्ताः
स्युर्लब्धयश्चेद्विषमास्तदानीम् ॥५०॥
यथागतौ लब्धिगुणौ विशोध्यौ
स्वतक्षणाच्छेषमितौ तु तौ स्तः ॥>

एवं तदैव स्यात् य[दात्र] कुट्टके ता लब्धयः समाः स्युः। उक्तकर्त्तव्यताप्रकारस्तदैवेत्ये[व]कारेण नियमः कृतः। अयमर्थः। उक्तवन् "मिथो

भजेत्तौ दृढभाज्यहाराविʺत्यादिना सूत्रेण भाज्यहारयोः परस्पर[भज]ने या लब्ध[यस्ता] यदि समाः समसंख्याका भवंति तर्ह्युक्तक्रिययैव लब्धिगुणावानेयौ। अथ ता एव लब्धयो [य]दि विषमा विषमसंख्याका भवंति तर्हि यथागतौ लब्धिगुणौ स्वतक्षणाच्छोध्यौ त[च्छेषं]मितौ तु पुनस्तौ लब्धिगुणौ स्तः। अत्र तष्टविधौ यो हारः स तक्षण इत्युच्यते। एवं प्रकृते तु पूर्वोक्तराशियुग्मे "ऊर्ध्वो विभाज्येन दृढेन तष्ट" इत्यत्र दृढभाज्यहारावेव गुणलब्ध्योस्तक्षणौ भव[तस्त]था च विषमलब्धिपक्षे प्रोक्तवद्ये गुणलब्धी ते स्वतक्षणाभ्यां शुद्धे सत्यौ गुणलब्धी भवेतामिति भावः ॥

अत्रोपपत्तिः। इह तावत्कुट्टकेन लब्धिगुणावानीयेते। तत्रोद्देशक्रमो यथा अयं भाज्यः केनापि गुणितः क्षेपयुतः स्वहारेण हृतश्च सन् निरवशेषो भवति। एवं येन गुणितः स चाज्ञातोऽस्ति। अथ प्रस्तुत यथावस्थितस्यैव भाज्यस्य हारेण भागे ह्रियमाणे यद्यपि भाज्यो निरवशेषो न भवति तथापि भजनानंतरं कियदवशेषं स्यात्तज्ज्ञानार्थं परस्परभजनं कृतम्। तद्यथा। हारहृतो भाज्यो निरवशेषो न [वृ]त्तः। तत्र यच्छेषमुर्वरितं तस्मात्कतिगुणात्पुनः ह<ा>रः शुद्ध्यतीति ज्ञानार्थं भाज्यावशेषेण पुनः ह<ा>रो भक्तः। तत्र या फलवल्ली तस्याः क्रमेण लब्धिगुणरूपत्वं दृष्टम्। एवं फलवल्ल्याः क्षेपासंस्कृतभाज्यादागतत्वादस्फुटत्वम्। ततोऽस्याः क्षेपस[व]र्णि[ता]याः सकाशाद्य[ज्जा]तं राशियुग्मम्। तावेव स्फुटौ लब्धिगुणौ। परं तु तयोर्बहुत्वाज्जडांकोत्पत्तौ तल्लघवीकरणार्थं [दृ]ढभाज्यहाराभ्यां तक्षणं कृतम्। एतदेव मनसि निधायाचार्येण "इष्टाहतस्वस्वहरेण युक्ते"त्येतत्प्रकारांतरं रचितम्। अयमर्थः। दृढभाज्यहाराभ्यां राशियुग्मे भक्ते यदवशेषं तौ लब्धिगुणौ। अथ यल्लब्धं तदेवेष्टं प्रकल्प्य "इष्टाहतस्वस्वहरेण युक्ते" इत्यादिकरणेन पुनस्तद्राशियुग्ममेव लब्धिगुणौ भवेतामिति भावः। अथ द्वयोर्गुणलब्ध्योरपेक्षायां राशिद्वयेनैव भाव्यम् ॥ अत उक्तं "मुहुः [स्या]दिति राशियुग्ममि"ति।

अथ ह<ा>रभक्ते भाज्ये फलं लभ्यत इति लब्धेर्भाज्यांतःपातित्वं दृष्टम्। तथा लब्धिगुणो ह<ा>रो भाज्याच्छुद्ध्यतीति। गुणस्य च ह<ा>रांतःपातित्वं दृष्टम्। अत उक्तं "ऊर्ध्वो विभाज्येन दृढेन तष्टः फलं गुणः स्यादि"ति। तथात्र "मिथो भजेत्तौ दृढभाज्यहारौ" इत्यादिना फलवल्ल्यां गृह्यमाणायां प्रथमं फलं

ह<ा>रभक्तभाज्याल्लभ्यते द्वितीयं तु भाज्यावशेषभक्ताद्धरत इत्यत "ऊर्ध्वो विभाज्येने"ति नियमः कृत इत्युपपन्नम् ॥ छ ॥

अथोपपत्त्युपसंहारकथनव्याजेनास्माभिरपि कारिकाभिः किंचित्प्रपंच्यते ॥

भाज्यस्यावयवष्कश्चित्क्षेप इत्यभिधीयते ॥
भाज्यहारांतरगतावेतौ लब्धिगुणौ मतौ ॥१॥

रूपावशेषभाज्यस्य फलवल्ली गुणो यदि ॥
तदा क्षेपावशेषस्य कः स्यादित्यनुपाततः ॥२॥

यद्राशियुग्मं तन्माने गुणलब्धी स्मृते ततः ॥
तल्लघ्वीकरणार्थं च मुनिभिस्तक्षणं कृतम् ॥३॥

एतदेवाभिसंधायेत्याचार्येणाग्रतः स्मृतम् ॥
इष्टघ्नस्वस्वहाराढ्ये गुणाप्ती स्तोऽथवेतरे ॥४॥

द्वयोरेकतराज्ञाने द्वितीयागमनं त्विति ॥
मुग्धच्छात्रचमत्कारकारणं किंचि[दुच्]यते ॥५॥

गुणेन गुणितो भाज्यः क्षेपयुक् छेदभाजितः ॥
गुणाज्ञाने फलाज्ञाने फलं तल्लभ्यते स्फुटम् ॥६॥

फलज्ञाने गुणाज्ञाने फलेन गुणितो हरः ॥
क्षेपोनो भाज्यभक्तश्च तत्र स्याद्गुणकः स्फुटः ॥७॥

यत्र क्वापि हरादूनो दृढभाज्यः प्रजायते ॥
व्यत्यासो लब्धिगुणयोस्तत्र कार्यो मनीषिभिः ॥८॥

लब्धयो विषमा यत्र क्षेपशुद्धिर्भवेद्यदि ॥
यौ तत्र लब्धिगुणकौ तावेव हि परिस्फुटौ ॥९॥

इदानीं पूर्वोक्तकुट्टककारणीभूतक्षेपहारभाज्यानां मध्ये चैकतमस्यापवर्त्तासंभवे सति विशेषमाह। भवति कुट्टविधेरिति ॥

<भवति कुट्टविधेर्युतिभाज्ययोः
समपवर्त्तितयोरथवा गुणः ॥५१॥
भवति यो युतिभाजकयोः पुनः
स च भवेदपवर्त्तनसंगुणः ॥५२b॥>

अथवा युतिभाज्ययोः समपवर्त्तितयोः सतोः कुट्टविधेः सकाशाद्गुणो भवति। युतिः क्षेपः। तस्य भाज्यस्य चापवर्त्तने कृते सति यद्यपि हारो नापवर्त्तितस्तथापि गुणो लभ्यत एवेत्यर्थः। अथ पुनर्युतिभाजकयोः समपवर्त्तितयोः सतोर्यश्च गुणो भवति सोऽपवर्त्तनसंगुणः सन् गुणः स्यात्। युतिभाजकयोः क्षेपहारयोरपवर्त्तने कृते सति भाज्यापवर्त्तव्यतिरेकेण यो गुणः साध्यते सोऽपवर्त्ताकेन गुणितः सन् गुणो भवतीत्यर्थः ॥

अत्रोपपत्तिः। तत्र लब्धेः प्रयोजनाभावाद् यद्यपि युतिभाज्यावपवर्त्तितौ तथापि गुणो लभ्यत एव यतो गुणस्य ह<ा>रांतःपातित्वं दृष्टम्। एवं ह<ा>रे त्वविकृते गुणोऽप्यविकृत एवेत्यत उक्तं भवति कुट्टविधेरित्यादि। अथ भवति यो युतिभाजकयोरिति तत्र क्षेपभाजकयोरपवर्त्तने कृते सति ह<ा>रापेक्षया भाज्यो ह्यपवर्त्ताकगुणितोऽधिको वृत्तः। एवं ह<ा>रस्य न्यूनत्वात्तदंतःपाती गुणोऽप्यपवर्त्ताकगुणितो न्यूनो वृत्तः। स च यद्यपवर्त्ताकेन गुण्यते तर्हि गुणकः स्यादित्युपपन्नम् ॥छ॥

एवं कुट्टकसिद्ध्यर्थं सूत्रसंदर्भमुक्त्वाधुना गुणलब्ध्योरानयने संभवदभिप्रायेण कंचिन्नियममाह। गुणलब्ध्योः समं ग्राह्यमिति ॥

<गुणलब्ध्योः समं ग्राह्यं धीमता तक्षणे फलम् ॥५२c॥>

धीमता गुणलब्ध्योस्तक्षणे फलं समं ग्राह्यम्। गुणलब्ध्योस्तक्षण इत्यत्र पूर्वोक्तकुट्टककर्मणा फलवल्लीगुणानानंतरं राशियुग्मे सिद्धे

ऊर्ध्वो विभाज्येन दृढेन तष्टः।
फलं गुणः स्यादपरो हरेण ॥

इत्यादिना यदा दृढभाज्यहाराभ्यां क्रमेण राशियुग्मस्य भागो ह्रियते तदोभयोः समं फलं ग्राह्यमिति नियमः कृत इत्यर्थः यतो यद्गुण एव दृढभाज्यः शोधितस्तद्गुणेनैव दृढहारेण शुद्धेन भाव्यमित्युपपन्नम् ॥

अथोत्तरार्धेन विशेषांतरमाह। योगज इति ॥

<योगजे तक्षणाच्छुद्धे गुणाप्ती स्तो वियोगजे ॥५३b॥>

योगजे गुणाप्ती तक्षणा⌋च्छु⌊द्धे सत्यौ वियोगजे भवतः। योगजे धनक्षेपाभिप्रायेण साधिते ये गुणलब्धी ते स्वतक्षणाद् दृढभाज्यहारसंज्ञकाद्यदि शोध्येते तर्हि वियोगजे भवत ऋणक्षेपाभिप्रायेण भवत इत्यर्थः ॥

अत्रोपपत्तिः। दृढभाज्यहारभजनावशेषीभूतौ लब्धिगुणौ यदि स्वभागहारा⌊भ्यां⌋ शोध्येते तर्ह्यंतरजे स्तो यतः क्षेपो भाज्यान्न्यूनीक्रियत इति स्पष्टम् ॥छ॥

अथ हारस्य धनर्णत्वे सूत्रमाह। धन⌊भाज्योद्भव इति⌋ ॥

<धनभाज्योद्भवे तद्वद्भवेतामृणभाज्यजे ॥५४a॥>

धनभाज्यो<द्भवे> तद्वत्पूर्ववद्भवे<...

<हरतष्टे धनक्षेपे गुणलब्धी तु पूर्ववत् ॥५३a॥>

...>दिति। अथ ऋणभाजके गुणलब्धी पूर्ववद्भवेताम्। कस्मिन्सति? धनक्षेपे हरतष्टे ⌊सति। यदि भाजक ऋणः स्यात्तर्हि ते⌋न भाजितस्य धनक्षेपस्य शेषं क्षेपं प्रकल्प्य कुट्टकविधिसंपादनेन लब्धिगुणावानेयावित्यर्थः ॥

अथ पुनर्विशेषांतरमाह। क्षेपतक्षणलाभाढ्या इति ॥

<क्षेपतक्षणलाभाढ्या लब्धिः शुद्धौ तु वर्जिता ॥५४b॥>

अथवा लब्धिः क्षेपतक्षणलाभाढ्या कार्या। क्षेपस्य तक्षणं क्षेपतक्षणम्। तस्मिन् यो लाभो लब्धस्तेनाढ्या संयुता लब्धिर्लब्धिर्भवतीत्यर्थः। इदं तु धनक्षेपविषयकम्। अथ शुद्धौ तु वर्जिते इत्यृणक्षेपे सति लब्धिः क्षेपतक्षणलाभवर्जिता कार्या। अयमर्थः। हरतष्टे धनक्षेपे इत्येनेन हारेण क्षेपस्य तक्षणे कृते यल्लब्धं तेन ल⌊ब्धिर्ध⌋नक्षेपे सति युक्ता। तथर्णक्षेपे सति वर्जिता लब्धिर्भवतीति भावः ॥छ॥

अथ पुनर्विशेषांतरमाह। अथवा भागहारेणेति ॥

<अथवा भागहारेण तष्टयोः क्षेपभाज्ययोः।
गुणः प्राग्वत्ततो लब्धिर्भाज्याद्धतयुतोद्धृतात् ॥५५॥>

अथवा प्रकारांतरेण भागहारेण त⌊ष्ट⌋योः क्षेप⌊भ⌋ाज्ययोः सतोः प्राग्वद्गुणो ज्ञेयः। पूर्ववत्कुट्टकविधिना गुणो ज्ञेयः "भवति कुट्टविधेर्युतिभाज्ययोरि"त्युक्तत्वा⌊त्⌋। ए⌊वं⌋ कृते गुण एव लभ्यते न तु लब्धिः। तथा ⌊च गु⌋णे ज्ञाते ततो ⌊लब्धिर्ज्ञेया⌋। भाज्ये गुणगुणिते क्षेपयु<ते> ह<ा>⌊रभ⌋क्ते च फलं लभ्यत इत्यर्थः। अथवा ह<ा>रेण युतो⌊द्धृ⌋ताद्भाज्याद्वा लब्धिगुणौ साध्यौ। एवमेतत्सर्वमप्युदाह⌊रणा⌋वसरे ⌊स्प⌋ष्टं ⌊सवासनं⌋ निरूपयिष्यते ॥

⌊अ⌋थ गुणकाभावसंभवमाह। क्षेपाभाव इति ॥

<क्षेपाभावोऽथवा यत्र क्षेपः शुद्ध्येद्धरोद्धृतः।
ज्ञेयः शून्यं गुणस्तत्र क्षेपो हारहृतः फलम् ॥५६॥>

यत्र क्षेपाभावे सति तथा यत्र च ⌊हरोद्धृ⌋तः सन् क्षेपः शुद्ध्येत्तत्रोभयत्रापि शून्यं गुणो ज्ञेयः। अथात्रापि विशेषमाह क्षेपो हारहृतः फलमिति। यत्र हार⌊हृतः⌋ क्षेपः शुद्ध्येत् ⌊तत्रैव⌋ क्षेपो हार⌊हृतः⌋ सन् फलं ⌊भ⌋वति। तथा यत्र च स्वरूपेणैव क्षेपाभावस्तत्र हारेण किं ⌊भाज्यम्⌋। अतस्तत्र गुणलब्धी शून्यमेव भवत इ⌋त्यर्थः ॥

⌊अत्रोप⌋पत्तिः। तत्र प्रथमं क्षे⌊पाभ⌋ाव इति ⌊यत्र⌋ क्षेपाभावस्तत्र "⌊मिथो भ⌋जेत्तौ दृढभाज्यहारा"वि⌊त्यादिना परस्परं भजनात्फलवल्ल्यां गृह्यमा⌋णा⌊यां "उ⌋पांतिमेन ⌊स्वोर्ध्वे⌋ हते" इत्यादिना उपांतिमः क्षेप एव। ⌊स⌋ च प्रकृते शून्यप्रमितः। ⌊तेनोर्ध्वगुणने⌋ शून्यगुणितः शून्यमेव भवतीति ⌊सर्व⌋त्र शून्यमेव स्यात्। अत⌊स्तत्र⌋ शून्यं गुण इत्युपपन्नम्। अथ यत्र क्षे⌊पः⌋ शु⌊द्ध्ये⌋दिति तत्र "हरतष्टे धनक्षेप" इति सू⌊त्रक्रमेण हारेण क्षेपस्तष्टः⌋ स⌊न् नि⌋रवशेषो भवति। तथा च ⌊क्षे⌋पाभा⌊वे⌋ गुणः शून्यमित्युचितम्। अथ क्षेपो हारहृतः फलमित्य⌊ने⌋न "क्षे⌊प⌋तक्षणलाभा⌊ढ्या" इति सूत्रार्थ ए⌋व सिद्धः। अथाकृतेऽपि क्षेपाभावे कथं वा गुणः शून्यं स्यादित्यत्रोच्यते। यदेकगुणः क्षेपो हारहृतः सन् शुद्ध्यति तर्हि ⌊द्वित्र्यादि⌋गुणोऽपि ह<ा>रहृतः सन् शुद्धो भविष्यत्येव। एवं फलवल्ल्याः क्षेपे गुणिते सति निष्प⌊न्न⌋राशियुग्ममध्येऽधःस्थितो राशिर्यदा हारेण भाज्यते तदा निरवशेषो भवत्येवे<ति>। अतस्तत्रापि शून्यं गुणो जायत इत्युचितम्। अथ क्षेपो हारहृतः इति तत्र फलं नाम गुणकगुणिते भाज्ये क्षेप⌊युते ह<ा>रभक्ते⌋ च ⌊लब्धम्⌋। तथा च प्रकृते गुणः शून्यम्। तेन भाज्ये गुणिते शून्यमेव भवति। तस्मिंश्च क्षेपयुते ह<ा>रेण भाज्य इति प्राप्ते परिशेषा⌊द्राग⌋हारेणैव क्षेपो भाज्यः। तत्र ⌊फ⌋लं लभ्यत इत्युपपन्नम् ॥छ॥

अथ प्रागुक्तकुट्टककर्मालसानां विनेयानामनेकगुणल⌊ब्ध्यागम⌋ने च⌊मत्का⌋रार्थं सूत्रमाह । इष्टाहतस्वस्वहरेणेति ॥

<इष्टाहतस्वस्वहरेण युक्ते
ते वा भवेतां बहुधा गुणाप्ती ॥>

ते गुणाप्ती इष्टाहतस्वस्वहरेण युक्ते सत्यौ बहुधा भवेता[मिति। उक्तवत्कुट्टकविधिना] ये गुणलब्धी साधिते ते इष्टाहतस्वस्वहरेण युक्ते लब्धिगुणौ स्तः। अयमर्थः। एकद्वित्र्यादिना येन केनापीष्टे[न स्वस्वहरौ दृढभा]ज्यहारसंज्ञकौ संगुणय ताभ्यां क्रमेण पूर्वागते गुणलब्धी युक्ते सत्यावन्यौ लब्धिगुणौ भवेताम्। एवं बहुधानेकशो [लब्धिगुणाः स्युरि]ति भावः ॥

अत्रोपपत्तिः। तत्र फलवल्लीक्षेपयोर्गुणनाद्यज्जातं राशियुग्मं तस्मात्क्रमेण दृ[ढभा]ज्यहाराभ्यां भा[गे] ह्रि[यमा]णे [ह्यव]शेषमितौ लब्धिगुणौ जातौ। अथ लब्धिगुणात्मिकाभ्यामवशेषाभ्यां स्वहारीभूतावेकगुणौ दृढभाज्यहारौ चेद्योज्येते तर्हि पुन[र्ल]ब्धि[गुणौ] भवेतां यतः स्वस्वाव<शेषा>धिकौ स्यातामित्युपपन्नम् ॥छ॥

इदानीं कुट्टकसूत्रविषयीभूतमुदाहरणमाह। एकविंशतियुतं शतद्वयमिति ॥

<एकविंशतियुतं शतद्वयं
यद्गुणं गणक पञ्चषष्टियुक् ॥५७॥
पञ्चवर्जितशतद्वयोद्धृतं
शुद्धिमेति गुणकं वदाशु तम् ॥>

भो गणक। तं गुणमाशु वदेति संबंधः। यत्त[द्यो]र्नित्यसंबंधात्त[च्छ]ब्दो यच्छब्दमपेक्षत इति न्यायात्तं कमित्याह य[द्गु]णमिति। यद्गुणमेकविंशतियुतं शतद्वयं पंचषष्टियुक् सत् पंचवर्जितशतद्वयोद्धृतं शुद्धिमेति। तमिति। एवमत्र "भाज्यो हारः क्षेपकश्चापवर्त्यः" इत्यादिना सूत्रेण "भज्यतेऽसौ भाज्यः" इति व्युत्पत्त्या एकविंशतियुतं शतद्वयमेव भाज्यो जातः पंचषष्टिः क्षेपस्तथा पंचवर्जितशतद्वयमेव हारः। एवमेतेषां क्रमेण न्यासः।

भा २२१ क्षे ६५
हा १९५

अथै[ते]षां लघ्वीकरणार्थमपवर्त्तः कार्य इति प्रथमपद्योक्तम्। तथा च 'परस्परं

भाजितयोर्द्वयोर्यच्छेषं तयोः स्यादपवर्त्तनं तदि'ति द्वितीयसूत्रक्रमेण। अत्र परस्परभाजितयोर्भाज्यहारयोः शेषमपवर्त्तांको लब्धः १३ । अनेन भाज्यहारक्षेपा अपवर्त्तिताः संतो जाता दृढसंज्ञकाः।

भा १७ क्षे ५

हा १५

अथ "मिथो भजेत्तौ दृढभाज्यहारौ" इत्यादिना दृढभाज्यहारयोर्यावद्रूपं शेषं तावन्मिथो भजने यानि फलानि तान्यधोऽधःस्थाप्य तदधश्च क्षेपमंते शून्यं च स्थाप्य जाता फलवल्ली।

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अथ "उपांतिमेन स्वोर्ध्वे हते" इत्यादिनोक्तवत्कर्मणि कृते जातं राशियुग्मम्।

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एतत् दृढभाज्यहाराभ्यामाभ्यां १७।१५ तष्टं सत् क्रमाज्जातौ लब्धिगुणौ ६।५ । अत्र राशियुग्मतक्षणे उभयत्रापि जातं समं लब्धं २ । एवमागताभ्यां लब्धिगुणाभ्यामनेकशो लब्धिगुणानानेतुं पूर्वोक्तसूत्रमुपन्यसति "इष्टाहत" इति। एवमत्र मौलिकौ गुणलब्धी ५।६ । एतयोर्हारौ १५।१७ । एतावेकेनेष्टेन संगुण्य मौलिकलब्धिगुणयुतौ जातावन्यौ लब्धिगुणौ २३।२० । एवं द्विकेनेष्टेन ४०।३५ । त्रिकेण ५७।५० । एवमनेकशः ॥छ॥

एवं क्षेपस्य धनर्णत्वप्रकल्पनयोरुदाहरणमुक्त्वाधुना तस्य धनर्णत्वं प्रकल्प्य "भवति कुट्टविधेरि"त्येतत्सूत्रविषयीभूतमुदाहरणमाह। शतं हतं येन युतं नवत्येति ॥

<शतं हतं येन युतं नवत्या
    विवर्जितं वा विहृतं त्रिषष्ट्या ॥५८॥
निरग्रकं स्याद्वद मे गुणं तं
    स्पष्टं पटीयान् यदि कुट्टकेऽसि ॥>

भो गणक। त्वं चेत्कुट्टके पटीयानसि तर्हि तं गुणं मे स्पष्टं वदेति संबंधः ॥ अतिशयेन पटुरिति पटीयान् कुशल इत्यर्थः। अथ तं कमित्याह येनेति। येन हतं शतं नवत्या युतं विवर्जितं वा सत् त्रिषष्ट्या विहृतं निरग्रं स्यादिति।

एवमत्र न्यासः।

भा १०० क्षे ९०
हा ६३

अत्रापवर्तासंभवादेत एव दृढभाज्यहाक्षेपाः। अथात्रापि पूर्ववत्कुट्टकविधौ क्रियमाणे जाता फलवल्ली।

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उक्तवद्गुणाप्ती १८।३० । अथ नवतिक्षेपस्य ऋणत्वं प्रकल्प्य तथा "योगजे तक्षणाच्छुद्धे गुणाप्ती स्तो वियोगजे" इति सूत्रक्रमेणागते गुणलब्धी स्वतक्षणाभ्यां शुद्धे सत्यौ पुनर्जाते गुणलब्धी ४५।७० ॥

अथ पुनर्भाज्यहाक्षेपाणां न्यासः।

भा १०० क्षे ९०
हा ६३

अत्र "भवति कुट्टविधेरि"ति सूत्रक्रियाप्रदर्शनार्थं भाज्यक्षेपौ दशभिरपवर्त्त्य न्यासः।

भा १०   क्षे ९

हा ६३

अत्र परस्परभजनात्फलवल्ली।

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उक्तवल्लब्धिगुणौ ७।४५ । अथ फलवल्ल्यां लब्धयो विषमाः संति। अतः "यथागतौ लब्धिगुणौ विशोध्यौ स्वतक्षणादि"ति कृते लब्धो गुणः १८ । लब्ध्या प्रयोजनं ना[स्]ति "सम[ा]पवर्त्तितयोरथवा गुणः" इति सूत्रे गुणस्यैव निर्देशात् ॥

अथ "भवति यो युतिभाजकयोः पुनरि"त्येतत्क्रियादर्शनार्थं हक्षेपौ नवभिरपवर्त्त्य न्यासः।

भा १००   क्षे १०

हा ७

अत्रोक्तवत्फलवल्ली।

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तथा च लब्धिगुणौ ३०।२ । अत्र गुणोऽयं २ । अपवर्त्तकेनानेन ९ गुणितो जातो गुणः। [स] एव १८ यतः "स च भवेदपवर्त्तनसंगुणः" इत्युक्तत्वात्। अथर्णगतनवतिक्षेपजे गुणलब्धी ४५।७० । अत्र "इष्टाहत[ा]स्वस्व[ा]हरेण युक्ते" इत्यादिना पुनर्गुणलब्धी १०८।१७० ॥ १७१।२७० । एवमनेकधा ॥

अथोर्वरितसूत्रक्रियासंदर्शनार्थं भाज्यस्यर्णत्वं प्रकल्प्य पुनरुदाहरणांतरमाह। यद्गुणा क्षयगषष्टिरन्वितेति ॥

<यद्गुणा क्षयगषष्टिरन्विता
वर्जिता च यदि वा त्रिभिस्ततः ॥५९॥
स्यात्त्रयोदशहृता निरग्रका
तं गुणं गणक मे पृथग्वद ॥>

तं गुणं पृथक् मे वदेति। पूर्ववदेव श्लोकप्रयोजना। तथा च न्यासः।

भा ६०̇ क्षे ३

हा १३

अत्र फलवल्ली।

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उक्तवद्गुणाप्ती २।९ । अत्र लब्धयो विषमा इति कृत्वा स्वतक्षणाच्छोधितौ जातौ लब्धिगुणौ ५१।११ । अत्र क्षेपस्य धनत्वं प्रकल्प्य स क्षेपो यदर्णगते भाज्ये योज्यते तदा "धनर्णयोरंतरमेव योगः" इति कृत्वा जाते गुणलब्धी २।९̇ । अथ क्षेपस्यर्णत्वं प्रकल्प्य स क्षेपो यदर्णगते भाज्ये योज्यते तदा 'क्षययोर्योगे युतिः स्यात्' इति कृते गुणलब्धी ११।५१̇ । एवं "योगजे तक्षणाच्छुद्धे गुणाप्ती स्तो वियोगजे" इत्यनेनैव सर्वं सिद्धम्। परं तु मंदावबोधार्थमाचार्येण धनभाज्ये भवे"त्तद्भवेतामि"त्युक्तम्। शेषं स्पष्टं ग्रंथतोऽप्यवबुध्यते ॥छ॥

अथ भाजकस्यर्णत्वं प्रकल्प्योदाहरणांतरमाह। अष्टादश हताः केनेति ॥

<अष्टादश हताः केन दशाढ्या वा दशोनिताः ॥६०॥
शुद्धभागं प्रयच्छन्ति क्षयगैकादशोद्धृताः ॥>

अत्र पदार्थः सुगमः। तथा च न्यासः।

भा १८ क्षे १०

हा ११̇

अत्र भाजकस्य धनत्वं प्रकल्प्य परस्परभजनया फलवल्ली।

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एवमुक्तवज्जातौ लब्धिगुणौ १४।८ योगजौ। अथ "योगजे तक्षणाच्छुद्धे गुणाप्ती स्तो वियोगजे" इति कृते जाते वियोगजे गुणलब्धी ३।४ । अत्रोदाहरणे भाजकस्यर्णत्वाल्लब्धिर्ऋणगता ज्ञेया ३।४̇ । यतः "स्वयोरस्वयोः स्वं वधे स्वर्णघाते क्षयो भागहारेऽपि चैवं निरुक्तमि"ति पूर्वमेवोक्तम् ॥छ॥

अथ "हरतष्टे धनक्षेपे" इत्येतद्विषयीभूतमुदाहरणांतरमाह। येन संगुणिताः पंचेति ॥

<येन संगुणिताः पञ्च त्रयोविंशतिसंयुताः ॥६१॥
वर्जिता वा त्रिभिर्भक्ता निरग्राः स्युः स को गुणः ॥>

अत्रापि पदार्थः सुगमः। एवमत्र न्यासः।

भा ५ क्षे २३

हा ३

अत्र फलवल्ली।

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तथा जातं राशियुग्मम्।

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अत्रोर्ध्वराशौ पंचसंख्याकेन स्वतक्षणेन तष्टे सति नव लभ्यंते। तथाधस्तने राशौ त्रिभिस्तष्टे सप्त लभ्यंते। एवं लब्ध्योरसमानत्वादेतदसंगतं "गुणलब्ध्योः समं ग्राह्यं" इत्युक्तत्वात्। अतोऽत्र सूत्रांतरेणार्थसिद्धिः। तथा हि "हरतष्टे धनक्षेपे गुणलब्धी तु पूर्ववदि"ति सूत्रक्रमादत्र ह<ा>रतष्टक्षेपस्य शेषं क्षेपं प्रकल्प्य न्यासः।

भा ५ क्षे २

हा ३

अत्र परस्परभजनात्फलवल्ली।

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अतो जाते गुणलब्धी २।४ । एते स्वहाराभ्यां शुद्धे जाते वियोगजे गुणलब्धी १।१ । अथास्मिन्नेवोदाहरणे "क्षेपतक्षणलाभाढ्या लब्धिः शुद्धौ तु वर्जिते"त्येतस्य सूत्रस्य विषयो दृश्यते। तथात्र क्षेपस्यास्य २३ हारेणानेन ३ तक्षणे कृते लब्धं ७ । अनेनाढ्या योगलब्धिरियं ४ जाता लब्धिः ११ गुणश्च प्राक्तन एवायं २ । अथ 'शुद्धौ विवर्जिते'ति तत्र क्षेपतक्षणलब्ध्यानया ७ वियोगलब्धिरियं १ वर्जिता सती जाता पुनर्लब्धिः ६ गुणश्च प्राक्तन एव १ । अथवा क्षेपतक्षणलब्धेनानेन ७ तक्षणीभूतौ भाज्यहारावेतौ ५।३ संगुण्य पूर्वागतराशियुग्मादस्मात् ४६।२३ संशोध्य जातौ पुनर्गुणलब्धी २।११ ॥

अत्रोपपत्तिः। हारेण क्षेपतष्टे सति क्षेपस्तु लघुर्जातः। ततस्तदुत्पन्ना लब्धिरपि लघ्वी जाता। सा तु लघुक्षेपोपयोगिनी न तु वृहत्क्षेपे। अथ यद्गुणो ह<ा>ः क्षेपाच्छुद्धः। तेन चेत्प्राक्तनलब्धिर्योज्यते तर्हि महती लब्धिर्भवति क्षेपस्य योज्यत्वादित्युपपन्नम् ॥छ॥

अथ 'क्षेपाभावे तथा यत्र क्षेपः शुद्धेद्धरोद्धृतः' इत्येतत्सूत्रविषयीभूतमुदाहरणमाह। येन पंच गुणिता इति ॥

<येन पञ्च गुणिताः ससंयुताः
पञ्चषष्टिसहिताश्च तेऽथवा ॥६२॥
स्युस्त्रयोदशहृता निरग्रका-
स्तं गुणं गणक कीर्तयाशु मे ॥>

अत्र पदार्थः सुलभ एव। तथा च न्यासः।

भा	५	क्षे	०
हा	१३		

अत्र क्षेपाभावे गुणाप्ती ०।० । अथवा "इष्टाहतस्वस्वहरेणे"त्यादिनैकगुणह<ा>क्षेपे गुणाप्ती १३।५ ॥

द्वितीयोदाहरणे न्यासः।

भा	५	क्षे	६५
हा	१३		

अत्र 'हरोद्धृतः क्षेपः शुद्ध्यती'ति कृत्वा जातो गुणः ० । तथा 'क्षेपो हारहृतः सन् फलं भवती'त्यादिना फलं च ५ ॥छ॥

< B. स्थिरकुट्टकः >

एवं सामान्यतः कुट्टकं निरूप्येदानीं ग्रहगणितोपयोगिस्थिरकुट्टसिद्ध्यर्थं सूत्रमाह। क्षेपं विशुद्धिं परिकल्प्य रूपमिति ॥

<क्षेपं विशुद्धिं परिकल्प्य रूपं
पृथक् तयोर्ये गुणकारलब्धी ॥६३॥
अभीप्सितक्षेपविशुद्धिनिघ्ने
स्वहारतष्टे भवतस्तयोस्ते ॥>

क्षेपं रूपं विशुद्धिं परिकल्प्य तय<ोः> पृथग् ये गुणकारलब्धी भवतस्ते अभीप्सितक्षेपविशुद्धिनिघ्ने स्वहारतष्टे च सत्यौ गुणकारलब्धी भवेतामिति दंडान्वयः। विशुद्धिर्नाम ऋणगतक्षेपस्तं रूपमेकसंख्याकं प्रकल्प्येत्यर्थः। शेषं स्पष्टम् ॥

एवमत्र प्रथमोदाहरणे रूपं क्षेपं प्रकल्प्य। न्यासः।

भा	१७	क्षे	१
हा	१५		

तथा फलश्रेणी।

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गुणलब्धी ७।८ । एते ऋणक्षेपत्वात् स्वतक्षणाभ्यां शुद्धे सत्यौ गुणलब्धी ८।९ । एते "अभीप्सितक्षेपविशुद्धिनिघ्ने"त्यादिनात्राभीप्सितः प्राक्तन एव क्षेपोऽयं ५ । अनेन गुणलब्धी निघ्ने ४०।४५ । स्वहारतष्टे च जातौ लब्धिगुणौ ११।१० ॥

अत्रोपपत्तिः। तत्र रूपक्षेपे विशुद्धौ कल्पिते सति "उपांतिमेन स्वोर्ध्वे हते" इत्यादिना वल्लीगुणने कृते एकेन गुणितं तदेव स्यादित्युक्तत्वात्सा फलवल्ली यथास्थितैवासीत्। तत्रोक्तवद्यौ लब्धिगुणौ तौ प्राचीनलब्धिगुणापेक्षया क्षेपगुणौ न्यूनौ

वृत्तौ। तौ चेत् क्षेपेण संगुण्य स्वस्वहाराभ्यां भाज्येते तर्हि प्राक्तनौ लब्धिगुणौ भवतः। अत्राभीप्सितक्षेपनिघ्ने ये जाते त एव लब्धिगुणौ भवतः। परं तु तल्लघवीकरणार्थमुक्तं स्वहारतष्टे इति। एवमेतत्प्रकारांतरमेवोक्तमन्यथा "योगजे तक्षणाच्छुद्धे" इत्यनेनैवार्थसिद्धिः। तथा चात्र कलितोऽनुपातः यद्येकमितर्णक्षेपेणैते गुणलब्धी तर्ह्यभीष्टेन किमिति। एवमत्र रूपक्षेपस्य ऋणताप्रकल्पनं तु प्रकारवैचित्र्यद्योतनार्थम्। तेन तदकल्पनेऽपि लब्धिगुणौ स्यातामेवेत्युपपन्नम् ॥छ॥

इदानीं कुट्टककथनप्रयासप्रयोजनं प्रदर्शयन् तेनैव ग्रहसाधनार्थं सूत्रमाह। कल्प्याथ शुद्धिर्विकलावशेषमिति सार्धवृत्तेन ॥

<कल्प्याथ शुद्धिर्विकलावशेषं
षष्टिश्च भाज्यः कुदिनानि हारः ॥६४॥>

<तज्जं फलं स्युर्विकला गुणस्तु
लिप्ताग्रमस्माच्च कलालवाग्रम्।
एवं तदूर्ध्वं च तथाधिमासा-
वमाग्रकाभ्यां दिवसा रवीन्द्वोः ॥६५॥>

अथ विकलावशेषं शुद्धिः कल्प्या तथा षष्टिश्च भाज्यः कल्प्यस्तथा कुदिनानि च हारः। एवं भाज्यहारक्षेपैः कुट्टकः कार्यः। अत्र शुद्धिर्नाम ऋणगतक्षेप इति पूर्वमेवोक्तम्। ततस्तज्जं फलं विकलाः स्युः। गुणस्तु लिप्ताग्रं स्यात्। कुट्टकादागतौ यौ लब्धिगुणौ तयोर्मध्ये लब्धिर्विकला भवति। गुणस्तु कलावशेषं स्यादित्यर्थः। अथ कलावशेषं शुद्धिं प्रकल्प्य षष्टिश्च भाज्यः कुदिनानि च हारः। तत्रापि कुट्टकविधिना ये गुणाप्ती तयोर्मध्ये लब्धिः कला भवति। गुणस्तु लवाग्रं स्यात्। अथ लवाग्रं शुद्धिं प्रकल्प्य त्रिंशद्भाज्यः कुदिनानि च हारः। पुनः कुट्टकविधिना ये गुणलब्धी तयोर्मध्ये लब्धिर्हि भागाः गुणस्तु राश्यग्रं स्यात्। अथ राश्यपेक्षायां द्वादश भाज्यो राश्यग्रं क्षेपशुद्धिः कुदिनानि च हारः। तत्राप्युक्तवद्ये गुणाप्ती तयोर्मध्ये लब्धी राशयः गुणस्तु भगणाग्रं स्यात्। एवं भगणाधिमासावमदिवसरवींदुदिवसाद्यानेयम् ॥

एवमत्रोपपत्तिः। तत्र "द्युचरचक्रहतो दिनसंचयः क्वहहृतो भगणादि फलं ग्रहः" इति सिद्धांतोक्तमध्यग्रहानयनसूत्रे तावदनुपातः। यदि कल्पकुदिनैः कल्पभगणा लभ्यंते तदाभीष्टाहर्गणादिनैः किमित्यत्र कुदिनानां प्रमाणत्वेन हारत्वं तथाहर्गणस्येच्छारूपत्वेन गुणत्वं च दृष्टम्। अथैतत्सूत्रक्रमेण यावद् ग्रहः साध्यते तावद्भगणानां फलत्वेन प्रथमं ग्रहस्य भगणा एव लभ्यंते। अथ राश्यपेक्षायां भगणशेषं द्वादशभिः संगुण्य यदि कुदिनैर्भाज्यते तर्हि राशयो लभ्यंते। अथ राश्यवशेषं त्रिंशता संगुण्य यावत्कुदिनैर्भाज्यते तावद्भागा लभ्यंते। ततो भागशेषं षष्ट्या संगुण्य यावत्कुदिनैर्भाज्यते तावत्कला लभ्यंते। अथ कलावशेषमपि षष्ट्या संगुण्य यावत्कुदिनैर्भाज्यते तावद्विकला [ल]भ्यंते। एवमुर्वरितं विकलावशेषम्। तथा च भगणशेषात्फलं राशयः। राश्यवशेषात्फलं भागाः। भागावशेषात्फलं कलाः। का[ल]ावशेषात्फलं विकलाः। एवं भगणमादीकृत्य पूर्वपूर्वापेक्षया उत्तरोत्तरस्य फलत्वं दृष्टम्। अथामुनैव प्रकारेण भगणावशेषमादीकृत्य यावंत्यवशेषाणि जातानि तैः स्वस्वभाज्या एकद्वादशकत्रिंशत्षष्टिषष्टिप्रमिताः फलानयनार्थं संगुणिताः। अतोऽवशेषाणामुत्तरोत्तरापेक्षया पूर्वपूर्वस्य गुणरूपत्वं च दृष्टमित्यत आचार्येणेदं सूत्रमुपनिबद्धमिति सामान्यतो विचारः ॥छ॥

अथ परमार्थतस्तु मध्यग्रहानयनसूत्रक्रियावैलोम्येन विकलावशेषाद् ग्रहानयनमुक्तम्। तत्र कलावशेषं षष्ट्या संगुणितं कुदिनैर्भक्तं लब्धं विकलाः। तत्र शेषं विकलावशेषं जातमासीत्। अथास्मादेव विकलावशे[ष]मवशेषत्वेनाधिकमुर्वरितमासीत्। इदानीमस्मिन् भाज्याच्छोधिते भाज्यो भागहरेणे निःशेषो भविष्यतीत्यत उक्तं <u>कल्प्याथ</u> <u>शुद्धिर्विकलावशेषमिति</u> ॥

अथ येन कलाशेषेण षष्टिः पूर्वं गुणितासीत्तत्प्रस्तुतमज्ञातम्। तस्य ज्ञानार्थं पूर्वं या षष्टिर्गुण्याभूत्सैवेदानीं वैलोम्येन भाज्यः कल्पिता। तत्र भाजकत्वं तु कुदिनानामेवेत्यत उक्तं षष्टिश्च भाज्यः कुदिनानि हार इति। एवं भाज्यहारक्षेपेषु सिद्धेषु कुट्टकविधिना यो गुण उत्पद्यते तदेव कलावशेषं स्याद् यतः पूर्वं कलाशेषेण षष्टिर्गुणितासीत्। अथात्र या लब्धिरुत्पन्ना <सा> विकला भवंति यतः पूर्वं

कलाशेषगुणितायाः षष्टेः कुदिनैर्भागे दत्ते लब्धं विकला अभूवन्नित्यत उक्तं तज्जं फलं स्युर्विकला गुणास्तु लिप्ताग्रमिति। अथैवमेवाग्रे नियोजनीयमित्युपपन्नम् ॥छ॥

अथैतदेव छात्रावबोधार्थमुदाहरणत्वेन स्पष्टं निरूप्यते। तत्र प्रथमं विकलावशेषज्ञानार्थं "द्युचरचक्रहतो दिनसंचयः" इत्यादिना ग्रहः साध्यते। एवमत्र ग्रहभगणाः कल्पिताः ३ कुदिनानि ११ अहर्गणः ३ । अथ सूत्रक्रमेण जातो भगणाद्यो ग्रहः।

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अत्र विकलावशेषं ७ । एतच्छुद्धिं प्रकल्प्य कुट्टकार्थं न्यासः।

भा ६० क्षे ७̇
हा ११

अत्र जाता फलवल्ली।

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लब्धिगुणौ च ⌊१७⌋।३ । एते योगजे स्वतक्षणाच्छुद्धे वियोगजे ४३।८ । अत्र लब्धिरियं ४३ जाता विकलाः।

अथ कलानयनार्थं हा ११ गुणास्तु कलावशेषं जातं ८ । इमं शुद्धिं प्रकल्प्य "षष्टिर्भाज्यः कुदिनानि हारः" इत्यादिना पुनः कुट्टकार्थं न्यासः।

भा ६० क्षे ८̇
हा ११

अत्र प्रागवज्जातौ लब्धिगुणौ ३२।६ । अत्रापि लब्धिः कला जाताः। गुणो हि

भागशेषम्। इमं शुद्धिं प्रकल्प्य त्रिंशन्मितं भाज्यं प्रकल्प्य पुनर्न्यासः।

भा ३० क्षे ६

हा ११

पूर्ववज्जाते गुणलब्धी ९।२४ । अत्र लब्धिर्भागा जाता गुणस्तु राश्यवशेषं ९ । इमं शुद्धिं द्वादश भाज्यं प्रकल्प्य पुनर्न्यासः।

भा १२ क्षे ९

हा ११

अत्र लब्धिगुणौ ९।९ । अत्र लब्धयो विषमास्तथा ऋणक्षेपः। [अतो] लब्धिगुणौ यथावस्थितावेव ९।९ । उक्तं च।

लब्धयो विषमा यत्र क्षेपः शुद्धिर्भवेद्यदि।
यौ तत्र लब्धिगुणकौ तावेव हि परिस्फुटौ ॥

इति। एवमत्र लब्धिरियं ९ जाता राशयो गुणस्तु भगणावशेषं ९ । इमं शुद्धिं प्रकल्प्य तथा कल्पितभगणान् भाज्यमेकं कुदिनानि हारं प्रकल्प्य च न्यासः।

भा ३ क्षे ९

हा ११

फलवल्ली।

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गुणलब्धी ३।० । अत्र लब्धिरियं ० जाता भगणाः। गुणोऽयम् जातोऽहर्गणः ३ । अत्रोदाहरणे यथा तथा कल्पनया ग्रहः समानीत इति कृत्वा भगणानयनार्थं कल्पितभगणा एव भाज्यः कल्पितः। अन्यथा कल्पभगणा एव भाज्यः कल्प्यः। तथा चात्र गुणस्त्वधिमासशेषम्। एवमुत्तरोत्तरमधिमासावमाद्यमुक्तवदानेयमित्यलमतिविस्तरेण ॥छ॥

< C. संश्लिष्टकुट्टकः >

अथ संश्लिष्टकुट्टकसिद्ध्यर्थं सूत्रमाह। एको हरश्चेद्गुणकौ विभिन्नाविति ॥

<एको हरश्चेद् गुणकौ विभिन्नौ
तदा गुणैक्यं परिकल्प्य भाज्यम्।
अग्रैक्यमग्रं कृतमुक्तवद्यः
संश्लिष्टसंज्ञः स्फुटकुट्टकोऽसौ ॥६६॥>

यदा एको हा[रः] गुणकौ च विभिन्नौ स्यातां तदा विभिन्नयोर्गुणयोरैक्यं भाज्यः [क]ल्प्यः। तथा अग्रैक्यमवशेषैक्यमग्रं क्षेपं कल्पयेत्। स क्षेपोऽनुक्तोऽपि ऋणगतो ज्ञेयः। एवं भाज्[य]हारक्षेपेषु सिद्धेषूक्तवदसावाचार्यैः संश्लिष्टसंज्ञः स्फुटकुट्टकः कृतः। संश्लेषः संयोगोऽविश्लेषः। तत्पूर्वकः संश्लिष्टकुट्टकः। गुणावशेषयोगेन साधित इत्यर्थः। गुणकावशेषाभ्यां भाज्यक्षेपौ संपाद्य कुट्टकविधिना गुणकमानयेदिति भावः। यतः क्षेपस्तु भाज्यावशेषमिति पूर्वमेवोक्तम् ॥

<अत्रोदाहरणमाह। कः पञ्चनिघ्नः इति ॥>

<कः पञ्चनिघ्नो विहृतस्त्रिषष्ट्या
सप्तावशेषोऽथ स एव राशिः।
दशाहतः स्याद्विहृतस्त्रिषष्ट्या
चतुर्दशाग्रो वद राशिमेनम् ॥६७॥>

<स्पष्टम् ॥>

तथात्र न्यासः।

गु	५	शे	७	गु	१०	शे	१४
हा	६३			हा	६३		

अत्र "एको हरश्चेदि"ति सूत्रक्रमेण गुणैक्यं भाज्यं शेषैक्यं क्षेपं च प्रक⌊ल⌋्प्य न्यासः।

भा	१५	क्षे	२१
हा	६३		

अथ भाज्यादयस्त्रिभिरपवर्त्त्य पुनर्न्यासः।

भा	५	क्षे	७
हा	⌊२१⌋		

अत्रोक्तवत्कुट्टकविधिना जाते गुणाप्ती १४।३ । एवमनेकधा ॥छ॥

दैवज्ञज्ञानराजा⌊त्म⌋जकविगणकाचार्यसूर्याभिधान-
प्रोक्ते सद्बीजभाष्ये सुजनबुधजनाशेषभूषाविशेषे।
सम्यक् सूर्यप्रकाशेऽपटु⌊बटु ह⌋दयध्वांतविध्वंसदक्षे
जातः सर्वोपपत्तिर्बहुगुणफलभाक्कुट्टकः कोऽपि धन्यः ॥छ॥

इति श्रीमद्दैवज्ञपंडितसूर्यविरचि⌊ते⌋ सूर्यप्रकाशनाम्नि भास्करीयबीजभाष्ये कुट्टकाधिकारः समाप्तिमगात् ॥ छ ॥

CHAPTER III

APPARATUS CRITICUS

FOR THE

SANSKRIT *TEXT ALPHA*

< 1. प्रथमोऽध्यायः >
< उपोद्घातः >

Page 2.

P. 2, 3 – p. 5, 5. श्रीगणेशाय ... °वसरे] A missing folios 1 – 2, text in ε || 3. श्रीगणेशाय नमः] श्रीवरदमूर्तिर्जयति N । श्रीसरस्वत्यै नमः om. Rβ (except W, ॐ श्रीभास्कराय नमः T, श्रीलक्ष्मीनृसिंहाय नमः I) । श्रीगुरुभ्यो नमः om. εβ (except W) || 4. °युगुलो° LTζ || 5. श्रीकण्ठपीठे स्फुरतिफणि° LD || 7. °नंतभावं IM । यज्जाति° D, मज्ज्योति° I, मज्योति° M || 8 – 11. याव° ... भजे om. LD || 9. °धैरवि° T, °धैरधि° ζ || 10. °गतव्यक्ता° α || 13. °जलधिप्रोत्तुंग°] °गहनाकूपार° β (°गहनंकूपार° TH) || 15. °तात°] °तत्ता° I, °तता° M || 17 – 18. विश्लेषे ... पूरयन् om. B || 17. समुल्हासयन् NL || 18. °नुरक्ता° R, °नुसक्त° DT, °नसक्त° ζ ||

Page 3.

2. रचयेत्सूर्य° LS || 4. का β (except L) || 7. श्रीमार्त्तंड° LD || 8. °विकसित° β (except L) || 9. °सादुण्य° R, °षाड्गुण्य° β (except LH) || 10 – p. 4, 24. °ल्मिम् ... °दव्यक्तशा°] L missing folio 2 || 11. °निहंतुं B, °निहितं εζ || 13. °रादावारब्ध°] °रारब्ध° β (°शरब्ध° T) । °त्साधारणा° IM । °कारणां εH, °कारणा° ιS || 15. °णाकार्थैः IM || 17 – 20. Verse 1a - d. θ || 23. नमस्कारणे ε, मस्कारे B || 24. °पि om. D, वि° RB ||

Page 4.

1. हि] ह RBD, हं N || 3. °तासत्सां° IM || 5. °न्नमस्कार° DT || 8. भू° om. BTζ || 8 – 9. °धरादेरु° ... कवयः। य° om. N || 11. चैतदपि + कार्यं β । घटं° R, धष्ट° IM || 13. सकर्तृत्वे B, सकर्तृके ε (सकर्तके R) || 14. °द्भटव° om. ε || 15. °भूतं + तं εDT || 18. महत्ववे द्यो° ε, मरुत्व द्यो°

D । °क्षमा° ε, °क्षत्वा° B ॥ 20 – 22. एका° ... तथा om. B ॥ 20. °प्रायेण तदुक्त° ζ ॥ 24. °दव्यक्त°] °द्गणित° ε (°गणिन° N) । °जनेता° εβ (except H; °जनयिता° S) । °रमप्यनेनैव] °रमपि B, °रमथनेनैव εT, °रमनेनैव ι, °रं अनेनैव θ ॥ 24 – 26. °नेनैव ... वंदे om. B ॥ 25. °ति^{2} om. IM ॥ 26. तदुत्पा° IM । नत्वत्र D, मन्वत्र IM ॥

Page 5.

1. जनेतु° ζ (except H), विनेतु° εγ । तस्यैवा IM ॥ 1 – 2. तुं पितु° L, तत्पितु° ζ (except H) ॥ 2. °रुत्पादस्य IM । प्रणानि° N, प्रणाति° R, प्रणीत° IM, प्रागति° L ॥ 5. A starts from पितुरेव (folio 3) ॥ 6 – 9. प्रथितः ... °रणेति om. RB ॥ 8. लब्धवाव° ζ (except MS) ॥ 11. °व्याजेन तस्यति° R, °व्याजेति° ζ (°व्याजे° IM, °न तस्या° in margin W^1) । संख्या परिसंनं IM, संख्यानं BL ॥ 12. ज्योतिषका: R, ज्योतिषिका: BDTW, ज्यातिषिका: S, ज्योतिविका: IM । यद°] दय° IM ॥ 14. °गणित°] °गणन° β (°गणत° B, °गणित° L) । °द्योतनेनैतदा°] °द्योतनैतदा° δ (°द्योतनैतद° L) ॥ 14 – 15. °तनेनैतदा° ... कौशलमासी° om. B ॥ 14. °प्येकतर°] °प्येकत° ε (र in margin A) ॥ 16. °पनोदनार्थ° ζ (except W) ॥ 16 – 17. कृत्स्नस्य ... व्यक्तस्य व्यक्त° om. D ॥ 17. °स्यैक°^{1}] °स्यै° IM । पाटी°] पात्य° ε ॥ 19. स्व + गुरु° ζ । °विशेषं β ॥ 20. °मभि°] °मसिभि° N, °मति° IM । संश्लिष्य (संश्लिल्य B) + प्रकृतिं β ॥ 21. स्वाभीष्ट° RDT, स्वाधिष्टा° B । पद्येन प्रणमति β । तद°] यद° ζ ॥ 22. °साम्य + प्रधान° β । प्रयोजना D^1 (प्र in margin), योजनं IM ॥ 23. संख्या: IM ॥ 24. चातुर्विंश° IM । तद्विदति N, तद्विदंति RDTWθ, तद्विदात B, तद्वदंति LIM । इति om. β । तदधीते om. H, दधाते B । तद्वेदेत्यण् om. H, तद्वेदेत्यणः A (तदेत्यणः R) T, तद्वेत्यण B, वेदेत्यण् L, तद्वेदेत्यण्: W, तद्वदेत्यणः IM ॥

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2. °त्पत्ति + प्रति° ζ (except H, + प्रतिपादन° W) । सर्गेति ι, सर्गमिति S, सर्गमति H । तथा ... स्वकृत°] तदुक्तमाचार्यैः β । °शिरोमणावुक्ता R, °शिरोमणौ β ॥ 3. °पुरूषेति R, °पुरुषाभ्यामिति B ॥ 3 – 4. महानस्य ... °त्यादि om. RB ॥ 4. अहंकारोऽभू° om. ζ । °दिति δ, इति WH, °ति IMS ॥ 5. तथा om. B, + च ζ । °स्मत्पितृचर° εγ, °स्मत्तातचर° ζ (°स्मात्तातचर° S) ॥ 6. °ना चेति om. β ॥ 7 – 8. व्यक्तस्य ... कारणमिति om. N ॥ 8. व्यक्ति] व्यक्त D, व्यक्ति Rζ (except H) । कारणं LD ॥ 9. समस्तस्य om. H, समग्रस्य (समग्र B) β । °व्याक्तं IM । ज्ञानयत A (except ε) ॥ 10. °संपादना°] °संपादनार्था° R, °संपादनार्थ° N, °समर्थना° H ॥ 11. अथ + गणितपक्षे तु β ॥ 11 – 12. गणितस्य ... °त्युत्पादकमिति om. LD ॥ 11. एतैरेव] तैरेव β ॥ 13. अथ om. A । यदिति] दिति B, यदि IM । °धिष्ठितमाश्रितं ζ (see line 14) ॥ 17. कारणमिति] कारणं β ॥ 20. पूर्वं प्रोक्तमिति om. L । पूर्वं om. A ॥ 21 – p. 7, 2. Verse 2a - d. Lθ ॥ 21. व्यक्ताम° S ॥ 22. प्रश्नान्नो L ॥

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1. शक्त्या मीदधी° L ॥ 3. पूर्वं व्यक्तं प्रोक्तम् om. B ॥ 4. °रल्पबुद्धिभि° om. A । °व्युत्तया A (except ε) । नितांतु IM ॥ 5. शक्यः D, शक्त्याः IM । दुखगमा N, दुर्गम Bδζ । इत्यर्थः] °त्वात् δζ ॥ 5 – 6. किंभूतं ... °मित्यर्थः om. B ॥

< 2. द्वितीयोऽध्यायः >
< षड्विधं प्रकरणम् >

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4. निरूपणे प्रसंगेन T, निरूपणेन BD । सर्व° om. β । गुणन° om. D, गुणना° L । °भजना° om. L, °भजनम° B । °द्य° om. BTζ ॥ 4 – 5. °यास्य ... °नाह om. N ॥ 4. °स्य om. β ॥ 5. °व्यवकलनं (°व्यवकलने ζ) β । °मुपेंद्रवज्रा° om. β । योगे युतिः स्यादिति om. LS ॥ 6 – 7. Verse 3a - b. Lθ ॥ 6. स्या° om. H ॥ 7. योग + इति L ॥ 8. स्यात् ... °मेव om. LD ॥ 9 – 10. संबंधः ... युति° om. D ॥ 9. संबंधः ... तयोः om. B । 9. क्षयौ] तौ Tζ । °तथा om. L, + स्वं च स्वं च ते (om. T) तयोः BTζ ॥ 10 – 12. तथात्र ... °त्वात् om. NB ॥ 12. °त्वात् + 19 – 22. च ... भवेदिति β (तच्च बहुंकसदृशमेव तदुक्तं । स्वयोर्योगे (°र्यो° om. B) स्वमेव (स्व° om. B) स्यादस्वयोरस्वमेव च (च om. S) । धनर्णयोस्तु संयोगे (संयोगो H) बहुंकसदृशं (°सदृशो H) भवेदिति) ॥ 13. यथा om. Rβ (except L) । °करणार्थे ζ (except S) ॥ 14 – 16. यद्युभे ... स्पष्टम् om. A ॥ 14. °श्वरं च] °श्व चरं L ॥ 15. ऋणयोर्योगे LW ॥ 16. धनर्णयोर्योगे L, धनयोर्योगु D, धनयोर्योगे Tι । युति B । तत्र] अथ यदि β ॥ 17. च] तदा δζ ॥ 17 – 18. °तरे ... °त्वादं° om. L ॥ 18. °जाती°] °जातीति IM ॥ 18 – 19. °यत्वादं° ... °मृणत्वं om. IM ॥ 18 – 19. कर्पूरा° ... °मृणत्वं] अतः धनर्णयोर्योगेऽंतरं (°र्योगो° BH) स्यात् । अथ स्वयोर्योगे स्वमेवेति β ॥ 19 – 22. च ... भवेदिति after 12. °त्वात् β ॥

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1. अत्रो° ... स्पष्टम् om. β । अथ] यथा β ॥ 2. सकाशा°] सकलौशा° ε । °स्त्रींश्च] °स्त्रांश्च N, °त्रयाश्चं R, °स्त्रयश्च ζ (except H) ॥ 4. यदा + दशभ्यः BTζ ॥ 5. °गत° om. ζ, °त° om. γ (except L) । ऋणत्वमेव] ऋणमेव B, ऋणत्वमेवेति D ॥ 5 – 8. भवति ... °शेषस्य om. A ॥ 5. भवति om. D ॥

6. ज्ञातव्यम्] ज्ञाते LD ॥ 7. °ऽवशेषं] च शेषं BTζ ॥ 8. °शेषस्यैव T, °शेषस्यै LD । भवतीत्युपपन्नम् om. LD । °पन्नम् + 14 – 15. °तान्येव ... कुर्यात् L ॥ 9 – 14. अथात्र ... °स्थि° om. D ॥ 9. अथात्र ... °वाह] अथोक्तेऽर्थे शिष्यबोधार्थमुदाहरणचतुष्टयमुपजातिकयाह (from Kṛṣṇa's *BP* p. 11, line 1) L । °बोधोदा° T, °बोधायोदा° Bζ । पूर्ववृत्तेनैवाह om. D, तेनैव वृत्तेनाह β । रूपत्रयमिति om. LS ॥ 10 – 13. Verse 3c - 4b. Lθ ॥ 12. क्षयः स्वं θ । पृथक् पृथक् चेत् θ ॥ 13. °वैषि + रूपत्रयं रूपचतुष्टयं चेति द्वयमप्यृणमित्येकं द्वयमपि धनमिति द्वितीयं आद्यधनमपरमृणमिति तृतीयं प्रथममृणमितिरद्धनमिति चतुर्थमेवं चत्वार्युदाहरणानि धनर्णयोरिति (from Kṛṣṇa's *BP* p. 11, lines 6 – 8) । शेषं स्पष्टं L ॥ 14. अत्र ... °स्थि° om. L (see ad. line above) । यानि[1] om. β । धनगतानि + रूपाणि BTζ । तान्यथाव° A (except R, तानि व्यथाव° N) । यथास्थितान्येव BTζ (°न्येव om. IM) ॥ 14 – 15. °तान्येव ... कुर्यात् after 8 °पन्नम् L ॥ 14. तथा + च β (except LH) ॥ 15. धनर्णत्व° om. A, धनत्वं ऋणत्व° B । योगोंऽतरं + वा A । प्रकृते om. ζ ॥ 16. ३ । ४ RBDS ॥ 17. ७̇ om. N, ७ RBD । जातः RL, यातं BT । ४̇ om. D, ४ RBS ॥ 18. १̇] १ RBD । ३̇] ३ RBDMS । योगे] अंतरे A ॥ 19. संशोध्यमानमिति om. LS ॥ 20 – 21. Verse 4c - d. Lθ ॥ 21. °वच्चेति L ॥ 22 – 23. स्वत्वं धनत्वमेति] स्वत्वमेति धनत्वं ζ, सत्वं धनं एति B ॥ 23 – p. 10, 1. एति ... संशोध्यमानत्वं om. T ॥ 23. प्राप्नोतीत्यर्थः A ॥ 23 – 24. तत° ... °त्यर्थः om. A ॥

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1. तत्र शोध्य° NLDζ । °मृणगत्वं BTι, °मृणत्वं D । पर्यायाः ALDIM ॥ 2. सुकारमेव N, युक्तमेव β (युक्तमेवा B, युक्तमेवे D) । क्रियमाणे°] कृतिश्च ε । अभावाभावे A (except ε) β (अभावे D) । भावविनियय T, भावविनिमय Dζ ॥ 3. अन्य° ... स्यात् om. T ॥ 3 – 4. युतिर्न ... °पन्नम् om. B । अतः ... °पन्नम् om. LD ॥ 3. शोध्यमानः ζ ॥ 4. °पन्नम् + Appendix #1. β ॥ 5. त्रयाद्वयमिति om. LS ॥ 6 – 7. Verse 5a - b. Lθ ॥ 7. शोध्य L । शेषमिति L ॥ 8. सर्वं om.

β । स्पष्टं β ॥ 9. धनर्ण°[1]] धनर्णयोः BTζ । करण° om. β ॥ 10. स्वयोरस्वयोः स्वमिति om. LS ॥ 11 – 12. Verse 5c - d. Lθ ॥ 11. वधः Lθ ॥ 12. निरुक्तमिति L ॥ 14. °हारे°] °हरे° IM ॥ 14 – 15. °गुणने धनम् तथास्वयोः om. ζ ॥ 15. तथा°[1] ... धनम् om. N । °गतयोरपि वधे β ॥ 15 – 16. तथा[3] ... स्यात् om. A ॥ 16. °हरे[1] IM । °हरे[2] IM ॥ 18. भाज्ये] ह्रियमाणे δH, ह्रियमाणेन BιS । ऋणमेव β । स्यात् + भागहारेऽपि चैवं निरुक्तमित्युक्तत्वात् (°मित्युक्त° om. W, corr. W[1] in the right margin, °क्त° om. IM) β ॥ 18 – 19. ततः पुनश्च°] एवं तस्य च° (च om. BD) β ॥ 19. °गत°[1] om. β ॥ 20. अन्य Tι (corr. W[1] in the left margin) S । ऋणयोः] शोध्यमानमृणं धनं स्यादिति धनयोः β ॥ 20 – 22. °त् ... °मित्या° om. N ॥ 21. अत्रोदाहरणी तु R, यथा β । ३ं] ३ RBDM । °हरे° IM ॥ 22. ३ं] ३ RBD ॥ 23. ६ om. BD । जात इत्युपपन्नम्] अतः अस्वयोरपि वधे स्वं भवति β ॥ 24. भागहरे° IM । निरुक्तमिति तत्र° β ॥ 24 – p. 11, 1. °हारेण भागे β ॥

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1. हरो[1] εβ (हारेणेदो B) । °च्छु° om. R, सु° B, बु° IM । हरो[2] Lζ ॥ 2. सन् भाज्याच्छोधितः om. T । च om. LT ॥ 4. धनं ... °मिति om. LS । धनं om. N, धने H । धनेर्णमिति B, धनेनर्णमृणेननिघ्नमिति D ॥ 5 – 6. Verse 6a - b. Lθ ॥ 5. धनं] धने H ॥ 7. तथा om. L । रूपाष्टकं ... चेति om. LS, रूपाष्टकमिति H ॥ 8 – 11. Verse 6c - 7b. Lθ ॥ 10 – 11. स्यादृतं L, स्याद्धृतं S ॥ 11. बोबुधीषीति L ॥ 12. उपपत्तावुदाहृतमपि om. ε ॥ 13 – 14. कृतिः स्वर्णयोरिति om. LS ॥ 15 – 16. Verse 7c - d. Lθ ॥ 16. °कृतित्वादिति L ॥ 17. °श्र्णगतयो° L, तयो गुणन° R, तपोणयो° N ॥ 18 – 19. °मूले इति A (except ε) H, °मूलमिति RBTιS ॥ 19 – 21. धनांक° ... मूलमिति om. L ॥ 19. °मृणांकस्य R, °मृणस्य मूलं ζ, °मृणांकवर्गस्य BT ॥ 20. अत्रो° ... ज्ञेया om. β ॥ 21 – 22. मूलाभावः ... °वर्गस्य om. N ॥ हेतो° ... °त्वादिति om. B ॥ 22 – 23. तस्य°[2] ... °त्वात्] यत्र त्रयाणां वर्गे (वर्गो L, वर्वेण T) नव (°व

om. T) धनगता भवंति । अथ (अ IM) तेषां नवानामृणत्वं तदेव (यत्र ... °त्वं त° om. B) भवति (ति B, स्यात् L) । यदा एकत्रये (एकंत्रयेण L, एकत्त्रये ζ (except H)) ऋणत्वं कल्पितं स्यात् । तथाहि β ॥ 23. अयमर्थः om. β ॥

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1. तथा त्रयः] त्रयश्च β । ३ं] ३ RD, ३० B ॥ 2. °दिति भावः] °दित्युपपन्नम् β ॥ 3. अत्रोदाहरणमाह om. ALD । धनस्य रूपेति om. ALDS ॥ 4 – 7. Verse 8a - d. θ ॥ 8. स्पष्टम् om. ALD ॥ 9 – 14. °षड्विधम् ... धनर्णं त° om. R ॥ 11. एवं ... शून्यषड्विधं om. IM । एवं ... °मुक्ते° om. β । °दानीं] इदानीं β । स्वयोग इति om. RLS ॥ 12 – 13. Verse 9a - b. Lθ ॥ 13. °समेति + इति L ॥ 14. स्वयोगे ... स्यात् om. θ । च^1 om. Rζ । थव स्यात् R, तथैवेति ι ॥ 15. यथावत्स्थित° B, यथास्थित° RTζ ॥ 15 – 16. यतो ... विकरोति om. β ॥ 16. स्वरूपं ε । °श्रुतं A (°श्वतमं R) Bι । सकृद्धिप° D, साद्धिसप° B, सद्धिप° Tζ, स्तद्धिप° R, दात्तद्धिष° N ॥ 17. वैपरीत्यमाप्नो° ζ ॥ 17 – 18. संशोध्यमानं ... °त्वात् om. β, which has instead Appendix #2. ॥ 19. रूपत्रयं स्वमिति om. LS ॥ 20 – 21. Verse 9c - d. Lθ ॥ 21. चेति L ॥

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2. वधादाविति om. LS, वधविति B ॥ 3 – 4. Verse 10a - b. Lθ ॥ 4. राशिरिति L ॥ 6. स्यादतः β । संख्या°] सं° ι (corr. W^1 in the top margin), स्व° S ॥ 6 – 7. °भावादि भावः LW (corr. W^1 in the left margin), °भावाभावः B ॥ 7 – 12. एवमेत° ... °निवेति after p. 14, 16 इति β ॥ 8. °बीजे] °बीजगणिते ζ (°बीगणिते M) । यथा om. β ॥ 9 – 11. °वशात्स्वता° ... आत्माभ्यास° om. T ॥ 9. °वशास्वता° R, °वशारवता° N, °वत्सात्स्वता° L, °वशोच्चता° ζ (°वशोत्स्वता° W^1 because W^1 erases च्च and writes त्स्व in the right margin, °वेशोच्चता° M) ॥ 12. संसृतिपदं β (except L) ॥ 13. स्वहारो D ॥ 14. द्विघ्नमिति om. LS, विघ्नमिति B ॥ 15 – 16. Verse 10c - d. Lθ ॥ 16. वर्गे S । चेति L ॥

18. स्वहारस्यांकस्य β, स्वहारांकस्य N । प्रकट इति RIM । अनंत इति] अनंतो राशिः स्वहर उच्यत इति (from Kṛṣṇa's *BP* p. 28, 11-12) S; अनंत इति + अनंतो राशिः स्वहर इत्युच्यते (from Bhāskara's *BG*, p. 5, 11) H; अनंतर N ॥ 19. अथ तस्या° β । अस्मिन्निति om. εLS, ग्मस्मिन्निति B ॥

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1 – 4. Verse 11a - d. Lθ ॥ 1. स्वहरेर्गा L ॥ 4. °नंते°] °नत° S । यद्वदिति L ॥ 5. बहुष्कंकेषु N, बहुषकेषु R, बहूंकेषु B, बहुष्वनेकेषु ζ ॥ 6 – 7. भिन्नांके ... °त्र्यादि° om. B ॥ 7. राशौवेक° IM ॥ 8. °रत्वाद्विकार ζ, °रत्वान्त्रिकाविकार R ॥ 10. °त्व°[1] om. H । °मनंतत्व° om. εβ । °चमत्कारे IM ॥ 11. यद्वल्लय° B, यद्वा ल्लय° ζ (यथा ल्लय° H) ॥ 12. तद्वदिति om. β ॥ 13. नास्तीति] नास्ति तद्वदिति β । यदुक्तं Bζ ॥ 14. °संवादो RBT ॥ 16. इति + एवमेत° ... °निवेति β ॥ 19. एव IM । °धमुक्ते°] °धं निरूप्ये° β (°धं निरूप्यो° L) ॥ 19 – 20. °द्विधं ... °त्तावदि° om. N ॥ 20. विक्षु° IM, विविक्षु° T । यावत्तावदिति om. LS ॥ 21 – p. 15, 2. Verse 12a - d. Lθ ॥

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2. °वर्यैरिति L । °वर्यैः + आचार्यवर्यैरव्यक्तानामेतदाद्या मान° (नाम° D) °संज्ञाः कल्पिताः। अथैतदाद्याः का इत्याह (इत्यमाह ι, इत्थमाह S) यावदिति (यावदिति om. D, यावत्तावदिति S)। β ॥ 6. °वधारणे] च धारण B, °वसाधारण L, च साधारण D ॥ 7. कर्त्तुम्] कर्त्तुं (कंतु IM) + तच्च ζ (except H) ॥ 9. योगोऽंतरमिति om. LS ॥ 10 – 11. Verse 13a - b. Lθ ॥ 11. °स्थितिश्चेति L ॥ 12. समजातीय° A ॥ 13. °तरं + वा β(+ वार्वा B), + च सस्° R ॥ 15. °वर्गाणां om. D, °वर्णानां εL, °वर्गा× T, °स्य Bζ । केवलव्य° IM । °स्थितिरे° om. ζ (पृथग्वेति ι, पृथगेवेति θ) । सुगमम् om. β ॥ 16. अत्रोदाह°] ज्ञापयितुमुदाह° β । स्वमव्यक्तमिति om. β ॥ 17 – 20. Verse

13c - 14b. S; but L transposed to after p. 16, 5. °कुर्वन्नाह || 19. पक्षयरेतयोः L || 21. स्पष्टार्थम् om. β, स्पष्टार्थं R ||

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1 – 2. अथा° ... °रणमाह om. β, (see p. 15, 14 – 16) || 1. °वर्णानां N || 2. धनाव्यक्तवर्गत्रयमिति om. L || 3 – 4. Verse 14c - d. LH || 5. तथा om. β | पुनश्छात्रशिक्षायै दृढी° β | °कुर्वन्नाह + p. 15, 17 – 20. Verse 13c - 14b. L | धनाव्यक्तयुग्मादिति om. L || 6 – 7. Verse 15a - b. LH || 6. धनव्यक्त° L || 7. वदाशु + इति L || 8. सर्वं स्पष्टार्थम्] न्यासः या २ या ६ रू ८ शोधिते जातं या ८ रू ८ (from Bhāskara's *BG*, p. 7, 7 – 8) H || 9. स्याद्रूपवर्णेति om. LS | °ति + व्यक्तां IM || 10 – 13. Verse 15c - 16b. Lθ || 10. वर्णो] वर्णौ LS || 13. °घाते + p. 17, 16 – 17. Verse 16c - d. S || 14. रूप° ... स्यात् om. B | अत्र om. β || 15. ह्यव्यक्त¹] त्वव्यक्त γ (except L), °ऽव्यक्त ζ || 15 – 18. नन्वत्र ... भवतीति om. β || 17. न ... °क्रियते om. R | व्यक्तस्य] अव्यक्तस्य A || 19 – p. 17, 3. अत्रो° ... एव भवतीत्युपपन्नम् transposed to after p. 17, 20. सिद्ध्यतीत्यर्थः β || 19. °पतिः | β | °व्यक्ते च रूपेषु ε | °ष्वव्यक्ता भवंति | β || 21. °व्यक्त + रूप° β || 22. शुध्यत्यतो β (शुद्ध्यति ततो D, शुद्धतो T) | कृत्वा om. β ||

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3. भवति β || 3 – 4. °त्युपपन्नम् ... संपन्नः om. β || 5 – 7. अथ ... प्रसिद्धमेव repeated with omissions in Appendix #3. (after line 20. सिद्ध्यतीत्यर्थः and after तर्हि पुनरव्यक्त एव भवति) β || 5. वधे + तु ζ | द्वौ + च β || 6. येषां] एषां IM | °पंचाद्याः β || 6 – 7. यतः ... प्रसिद्धमेव om. β, but see Appendix #3. || 7. °भयत्रापि NDTζ | °शब्दे Dζ || 9 – 10. तद्भवति ... गुणने om. ε || 9. तद्भवति भावितं] तद्भावितं भावितकं β (तद्भवति भावितकं D, तद्भावितकं L) || 11. तथा¹ गुण° N, तथान्य° β | तथा² om. β | भावितकं δζ || 12. °गुणनानोप° γ (except D), °गुणनेनोप° WH, °गुणनंनोप° IM | संभावितः]

भावितः ζ ॥ 13. यावत्तावत् RBζ । माकाभेति N (corr. N¹ in the left margin), यात्काभेति R, कालेति T, याकाविति ζ (या का इति H) । गुणिते + सति BTζ ॥ 14. °मादीकृत्य लेख्यं β (°मादौकृत्य लेख्यं D) । °मिति भावः om. β ॥ 15. भागादिकमिति om. LW (add. W¹ in the top margin) ॥ 16 – 17. Verse 16c - d. LH; but S after p. 16, 13. °घाते ॥ 17. तदत्रेति L ॥ 20. सिद्ध्यतीत्यर्थः + p. 16, 19 – p. 17, 3. अत्रो° ... पुनरव्यक्त एव भवति + Appendix #3. β ॥ 21. गुणनादौ α (गुणनादी L, गुणादौ ζ) ॥ 21 – 22. गुण्यः पृथगिति om. LS ॥

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1 – 2. Verse 17a - b. Lθ ॥ 2. यथोक्त्या + 17 – 18. Verse 17c - d. LS ॥ 4. यत्] यः ε ॥ 5. यथोक्त्येनेन R, यथोक्तेत्यनेन D, यथोक्तचोपपन्ने B, यथोक्तोत्पन्नेन T, यथोक्त्योत्पन्नेन Wθ, योक्त्यासन्नेन IM । °स्समयो° om. Bζ (°र्वा corr. W¹ to र in the text of W, and समयोर्वा° add. W¹ in the top margin) । व्यक्तव्यक्तयो° R, व्यक्ताव्यक्तेयो° B, व्यक्तयो° ζ (°व्यक्ता° add. W¹ in the top margin) ॥ 6 – 7. इति ... स्पष्टैव om. N ॥ 7. स्पष्टैव ... °मुच्यते om. β ॥ 8. भाज्ये A ॥ 9. °जकांकस्य यत्प्रमितावृत्तयः शुध्यंति तत्पूर्व° β ॥ 10. °वर्णाः + रूपाणि च β । गुणाके°] भाजके° β । °त्र om. β ॥ 11. गुणखण्डानि LTIM । गुण्यरूपाणि स्थाप्येत्येकं β (स्थाप्येत्येवं WS, संस्थाप्येत्येवं H, स्थाप्य प्रत्येकं γW¹, प्र° add. W¹ in the right margin) ॥ 11 – 12. कृते सति गुणनफलं A ॥ 12. °दशसु] °दश εL, °दना B ॥ 13. अथ + द्वादशानां β ॥ 14. लभ्यत] भवति BD ॥ 15. °वर्गे (°वर्गो N) + च β ॥ 17 – 18. Verse 17c - d. H; but LS transposed to after Verse 17a - b. ॥ 18. °त्वमत्रेति L ॥ 19. अव्यक्त°¹ ... चिंत्यः om. H । अत्र अव्यक्त°¹ β, अथ व्यक्त° N । °सु अत्र A (°त्र om. N), °सु β (+ चिंत्यौ ζ, चिंत्यौ erases W¹) । चिंत्यः] °मत्र ζ (°त्यत्र W¹ in the right margin) ॥ 20. °त्ताव° om. AL, add. L¹ above the line ॥ 23. °तोक्तस्तत्रक्रमेण D, °तोक्तेस्तत्क्रमेण ζ ॥

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1. यावत्तावत्पंचकमिति om. LS ‖ 2 – 5. Verse 18a - d. Lθ ‖ 5. कल्पित्वा L । तु] च S ‖ 6. ११ om. RT, १ BD, °पं N ‖ 8. २ om. R, २ BD ‖ 9 – 14. करणा° ... एवात्र om. B ‖ 9. भाज्याच्छेद इति om. LS, भाज्यादिति DTιH ‖ 10 – 13. Verse 19a - d. Lθ ‖ 10 – 11. सन्तस्त्वेषु H, सन् S ‖ 12. संगुणैर्यैश्च S ‖ 13. स्युस्त्रेति L ‖ 14 – 16. ता ... शुद्ध्यति om. ζ (add. W[1] in the top margin) ‖ 14. सन्न ε, सम् IM । त ε ‖ 15 – 22. भाजयितुं ... °पपन्नम् om. A, which has instead Appendix #4. ‖ 15 – 18. योग्यो ... °णितानां om. B ‖ 17. पूर्वं° W, पूर्वं IMS । स्थाप्यानि ζ (W erases and corrects) ‖ 19 – 20. गुणक .. संगुणित: om. D ‖ 20. अतो यै (यैं L) वर्णै: BLT ‖ 21. पूर्वं δH ‖

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2. २ RBDM । च om. εLT ‖ 3. हरोऽयं L । २] ३ A (except R, रा N) ‖ 4 – 5. छेदस्थि° ... तथा च om. IM ‖ 5. °च्छुद्धंति + ता एव लब्धयो भवंति BTζ । च] चात्र Bζ (+ (4 – 5. छेदस्थि° ... °च्छुद्धंति + ता एव लब्ध) B) । २] २ RBDM । या ३ रू २ om. β ‖ 6. शेषं om. ε । ७ om. εT । रू रं ε, रू २ D, रू २ रु B ‖ 7. यद्गुणैरेव] य रू २ गुणैरेव N, या रू २ गुणैरेव R, षड्गुणैरेव रू २ D ‖ 8. हत° R, दत्त° Nβ । °त्पंचगुणकमेव ζ ‖ 9. भवंति] भवति ε ‖ 10. सूत्र° ... अथ om. N ‖ 10. अथ] अत: ζ ‖ 11. °भाजकयो° A ‖ 12. ३ BDζ, ३ं R । २ om. R, रं N, २ BD ‖ 13. या ३ रू २ om. D, after 15. भाज्या IM, after भाज्यात् H (२ H) ‖ 15. जातं β (+ रू IMθ), ऋणं A (ऋणां R) । १ om. T, १ RBM, एव D ‖ 16. ऋणत्वे सति after 17. २ ζ ‖ 17. ३ DMH । २ M, ३ R, ३ T, रं N, रं २ D ‖ 18. या ३ रू २ om. N, after 16. स्व° T, after 19. कृ° IM, after 19. कृते H । ३] ३ BDM । २] रं R, ३ T, २ BDM ‖ 19. एवमेवमत्र IMS, एवमेव HW (changed to एवमत्र W[1]) । शोध्य° Lζ (सं add. W[1] in the right margin) । धनत्वमेति तथा (धनत्वमेतिथा B) β ‖ 19 – p. 21, 17. °णय: ... °रूपा° om. N ‖

20. एवमेतदुपपन्नं A || 21. अ (त D) थाव्यक्तोदा° β | रूपैः ... °मिति om. LS, रूपैः षड्गिरिति β || 22 – 23. Verse 20a - b. Lθ || 23. मे + इति L ||

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3. °रिति] °रेवमत्रेति ζ || 4. सूत्रक्रमेणाव्यक्त° β (°क्रमेण व्यक्त° BTS) || 5. °करण° om. β | सूत्र° (°माह om.) ζ | कृतिभ्य आदायेति om. LS || 6 – 9. Verse 20c - 21b. Lθ || 9. तथैव शेषम्] पदानि चैवमिति L || 10. °रपि हर्ति LT, °रपि निहर्त D | द्विनिघ्नी om. α, add. ιS | संबंधः om. β || 11. वर्गराशौ om. ζ (add. W in the bottom margin) | ये om. ιS (add. W in the bottom margin) | पदानि + मूलानि (+ लानि B, पदानि T) β || 12. °काबोधये° IM | वेत्कृतीति R, ये संतीति BTIMH || 13. तद्रूपपदानि A (रुपपदानि R, om. N) | °पदं + च β || 14. यथा om. β | विद्यते] वर्तते β || 15. स्थानद्वये खण्डयेन R, स्थानद्वयेन γ (except D) H | गुणनप्राप्ते BD | गणितस्तत्र IM || 16. जातः] भवति β | वर्णो] वर्गो εL || 17. °खण्डेन β | तत्र om. β || 18. यावत्तावद्भवति om. D, वातावद्भवति R, वर्णश्च (वर्गश्च L) भवति β | वर्गरूपः IM || 20. स om. BIM | युक्त° ζ || 21. अतो द्वयोश्चा° ιS || 22. एवमनेक° A (एवमनेन° N) | निरूपयिषु°] विवक्षु° β (विविक्षु° T) | °स्तावत्त (त D) त्सं° δ | व्यवकलनं om. LT || यावत्तावत्कालकेति om. LS ||

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1 – 4. Verse 21c - 22b. Lθ || 1. °त्कालक°] °त्काल° S || 4. स्युस्तैः S, स्युस्ते इति L || 7. पृथ स्थ्छिति° IM || 9. यावत्तावत्त्रयमिति om. LS || 10 – 13. Verse 22c - 23b. Lθ || 10. कालको H || 11. द्विगुणमितैस्ते L, द्विगुणितमितेस्ते S || 13. कृतेश्चेति L || 14. अत्र] अत्रांकस्थापनमेव व्याख्या β | ३ BDM, इ R | २ं om. R, २ां N, २˙ B || 15. ६ M, द RBT | ४ εBM, ध D | गुणयः[2]] गुणयं DTζ || 16. स्थाप्यं DWS, सं (+ से N) स्थाप्य εH | °खण्डैर्गु° γθ || 17. समानजा° IM, मजा° B | तत्तद्वर्गो R, तद्वर्गा° β (तद्वगा° B,

स्तदवर्गा° IM) ॥ 18 – 19. तद्भावितं ... °वद्गुणने om. T ॥ 19. पूर्वं कथि° ζ, पूर्कथि° B । गुणनफलं LDζ, गुणकफलं BT ॥ 20. १२ं [1] om. R, १२ BDM । ꣿ[1]] ० DM, टं RB । या १२ं om. T । १२ं[2] om. L, १२ εDM, १२० B । का[3] om. L । ꣿ[2]] ० DM, ट RB ॥ 21. ग्रंथश्वा° IM ॥

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2. विवक्षुरादौ] निरूपयितुमादौ LD, निरूपदषुरादौ B corrected to निरूपतुमारादौ B[1], निरूपयिषुरादौ Tζ । व्यवकलनं om. BL ॥ 3. योगं करणयोरिति om. LS, योगं करणयोर्महर्ती (°र्महंती BD) प्रकल्प्येति (प्रकल्येति B, °मकल्प्यति D) β ॥ 4 – 7. Verse 23c - 24b. Lθ, + Verse 24c - 25b. S ॥ 5. वधस्य] घातस्य LH, पातस्य S ॥ 6. स्ते S ॥ 7. °द्भजेच्चेति L ॥ 10. °र्योगं α, °र्योगस्य H ॥ 10 – 11. अथ ... प्रकल्पयेत् om. NT ॥ 11. लघुरिति α (लघु श्रणां B), लघुमिति LD । °लघ्वो εβ (°लघ्यो B) ॥ 11 – 13. °वद्[1] ... निरूप° om. L ॥ 12. योगोंतरं α (योगांतरं N), योगमंतरं H । कुर्यादित्यर्थः] कुर्यात् β । स्वरूपं om. T, स्वरूपयन् IM ॥ 13. भजेत् न] भवेन्न T, भाजयेन्न D, भजेत् ζ (न add. W[1] in the right margin) ॥ 14 – 17. °त्वं नाम ... स्यादि° om. D ॥ 4. वर्गात्वमि° R, वर्गत्वेनाभि° β (वर्गत्वनाभि° S, वर्गकित्वेनाभि° W) ॥ 15 – p. 24, 13. °यणेन ... °पपन्नम् om. R ॥ 18. °योगवियोगमाह ζ (°योगमाह H), °योगाचाह B । लघ्व्या हताया इति om. LT ॥ 19 – 22. Verse 24c - 25b. LH and S after p. 23, 7 ॥ 22. मूलमिति L ॥

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2. निरेके उभयत्र IM ॥ 4. योगों° α । °तरं ζ । °माणं ζ । मूलं] पद β ॥ 5. °मित्यत आह ζ । पृथक्°] पृथ° IM ॥ 7. °वद्गुणसाम्ये° (°सौम्ये° N) A । कथं + न ζ । °त्रोपतिरुच्येते N, °त्रोच्यते β ॥ 9. किल्पितौ A । यावन्भवति B, यावद्भवति Nζ । चात्रोक्त° β (except T; च चोक्त° B) ॥ 10. °वर्ग° om. β ॥ 11. २ om. N, १ IM । ५१ om. N, ५० BW, २५ HIM, १५ S ॥

11 – 12. अस्य वर्गः १८।१२ om. A ॥ 12. १२] २ β ॥ 13. २] १ ζ। ३ om. N, २ θ, १२ IM, ५२ W ॥ 14. करणी अत्रो° ε, करणीसू° IM। तत्र] अत्र ζ। °योगो om. ε, °योयोगो D ॥ 16. तथैक्य° R, नथैक्य° N। ज्ञायेते A (except R) T, जायते B ॥ 17. °वर्गो ε। °र्योगे ζ, °र्थो R ॥ 18. मूलं om. Nγ (except L) ॥ 19. मूलवधसमं A (मूलर्वर्गसमं N) ॥ 20. चेत्यादि] च इति β ॥ 22. °त्यतर्त्क IM ॥ 24. सर्वमुपपन्नम् om. β ॥ 26. करणै IM ॥

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2. वर्गो εL ॥ 4. °त्पलं IM ॥ 5. °भाजने ε ॥ 6. °फलमित्यलम्] °फलं स्यात् β ॥ 7. इत्यत्रोपपत्तिः] इति β। तयोभय° IM ॥ 7 – 8. °लं तस्य सैकस्य om. B ॥ 8 – 9. करणी° ... सन्न° om. S ॥ 8 – 9. भवति ... च om. A ॥ 9 – 10. क्रिया ... °वर्गयोः om. B ॥ 10. प्रकृति A (प्रकृते N, कृते R), प्र T। °भजनो IM, °भजना S। यल्लब्ध + स्य मूलं A ॥ 11. हताया° γ (except L)। महत्य इत्यादि IM, महत्यादि A ॥ 12. स्पष्टमित्युपपन्नम् om. β ॥ 13. कल्पित° A ॥ 15 – 16. जातः ... सन् om. εT ॥ 15. जातः करणी° om. ζ। योगो + यं ζ। तथा om. ζ। मूलभजनस्य फलस्य B, अस्यैव २ ζ। अयं om. ζ ॥ 16. °वर्गेणा° ... सन्] °वर्गेण हतः सन्नंतरमिदं ४ ζ। जातमंतरमिदं ४ om. S ॥ 16 – 19. एवं ... ४ om. R ॥ 17. अथ AH, अवा IM। सूत्रक्रमेणापि om. β ॥ 17 – 19. अत्र ... ४ om. D ॥ 18. संस्थाप्य β (except L)। ३१ NB ॥ 18 – 20. वर्गिते ... °हरण° om. B ॥ 18 – 19. वर्गितं लघुहतं AL ॥ 19. °हते (°हतं L) + च Lζ। सर्वत्रमनवद्यं R, सर्वमुपपन्नं δζ ॥ 20. द्विकाष्टमित्योरिति om. LS ॥ 21 – 24. Verse 25c - 26b. Lθ ॥ 22. पृथक्] सखे S ॥ 23. °मित्याश्च S ॥ 24. °ङ्घ्रिं S। करणया इति L ॥

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1. संबंधः om. β ॥ 2. °रिति स्पष्टम् β ॥ 3. तथा च om. β ॥ 4. अथ ... जातो om. A ॥ 5 – 9. १८ ... योगांतरे om. A ॥ 7 – 9. लब्ध्या ... क २ om.

D || 8. एतद्विधा BTι || 9. २] २ IMS || 11. कारणान् N, कारकात् R,
२

कृत्वा β | संभवति β || 13 – p. 28, 17. °थ करणी° ... सूत्रा° A om. folio 10, text in ε || 13. द्वित्र्यष्टसंख्या गुणक इति om. LS || 13. द्विश्रष्ट° N, द्विश्रष्ट° R || 14 – 17. Verse 26c - 27b. Lθ || 17. गुणो° θ | करणयौ + इति L || 18. °रूप β (except W) | त्रिसंख्याक β (except L, त्रिसंख्यक D, त्रिसंख्यांक T, त्रिसंख्याकः + ३ | ५ H) || 19 – 20. करणीति ... तथा om. β (except L; °रणीति ... तथा in the bottom margin W[1]) || 19. °ति न्यासः om. L | अत्र LW[1] || 21 – p. 27, 1. वर्ग[2] ... °वर्ग° om. N ||

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3. सूत्र°] तत्र° β (except LW) | क ४५० om. R, क ४५ NH || 5. गुणये om. β | गुणयस्तु ... एव om. ε || 6. प्रचक्ष्वेति + संबंधः ε || 7. °व्यवस्थयां N, °वस्थायां R, °व्यस्था + space B, °व्यवस्थार्थं ζ | क्षयो भवेदिति om. LS, क्षयो भयेदिति I || 8 – 11. Verse 27c - 28b. Lθ || 11. °हेतोरिति L || 13. यों ऋण° α (except W) || 14 – 15. °तिः ... °प्राप्तौ क्ष° om. N || 15. °प्राप्तिर्भवे° ζ (corr. to °प्राप्तौ भवे° W[1] in the text, and add. क्षयो W[1] in the right margin) || 15 – 16. द्योतित° ... °क्षणयं om. S || 16. °क्षणयं + ६ IM | निरूपय R, द्योतयन्नाह β || 17. ऋणात्मिकाया इति om. β (except L; in the bottom margin W[1]) || 21. °कारणे LIM | एवं ε | °वर्गो[2] εγ, °वर्गेण ζ || 22. धनत्वमेव γ (त्वमेव T), धनमेव ζ || 22 – 24. तथा ... मूलं om. β || 23. च[1] om. N ||

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3. अत्रोदाहरणं तु] यथा β | २५ RBDMH, रं ५ N | अस्य (°था° om.) β || 4. °द्विषमादित्या° (°त्कृतिं द्विगुणये° om.) β | °दिनास्मात् β (°दिनास्मरणात् D, °स्मात् M) | साधितो° εL || 5. २५ RBDH, क २५ं N || 6. ५ BDIM, २५ ε, १५ं T || 7. करणयो° ... °त्यादि°] करणयोरिति β ||

8. °प्र°² om. β (except L) ।। 9. किं + तु BT ।। 9 – 13. सम° ... इत्यादि om. β ।। 15. क १२ om. S, रू १२ IM । रू ५ BDM, रू पं ε, क ५ L । सत्यंतो IM ।। 16. वर्ग² ε ।। 17. तत्र° om. β । क २५ ADH, २५ B ।। 18. अत्रापि ... क २७ om. A । २५ BDH ।। 19. ६२५ Rγ (except T) MH, ं २५ N । ६७५ L, ६५ N, ६७५ + क ७५६७५ H । ७५ LDM, ं७ R ।। 20 – p. 29, 1. अनयो° ... रू १ om. H ।। 21 – p. 29, 1. ऋणात्मि° ... मूले om. L ।।

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1. व + कृते ζ । रू २५ RDM, रूपं २५ T, नू २५ S, रू ५ L ।। 1. °मेव ... अ° om. B ।। 1. °यं + रू β (except T, नू S) । १६ εDTM ।। 1 – 2. अस्य ... अंतरम् om. A ।। 2. २५६ BDM । ६२५ DM ।। 3. क¹ om. DM । क ७५ om. R, क ७५ DMH । °दंतरे β । क³ om. β ।। 4. एवमत्र ... ३०० om. A । २५६ DTM ।। 6. °भागहारार्थं β ।। 7 – 9. भाज्यः ... अथ om. D ।। 9. करणयो° om. β (except L) । ३ A (except N, ३ R) ।। 10. प्रकारेण] क्रमेण A ।। 11. °हरणे + न्यासः BL । भाज्याः A (except ε) । क ६२५ A (om. R) DTM । ७५ ADTM, २५ B ।। 11 – 13. भाजकः ... जातो om. T ।। 12. ५ BDM । क १२ क १२ B, क १२ क ३ ζ (except W), क २ R ।। 13. २५ ADM ।। 14. ३] २ IMS, ३ R ।। 15 – p. 31, 4. अथ ... बुद्धिमता om. A ।। 15. अथात्र ζ । गुणे] गुणके ζ ।। 17. °ल° om. I (at the end of folio 10r.) M । भाजनार्थं Dζ (except W), भाजनार्थे T । क ६२५ om. B, क ६२५ DM ।। 18. ७५ BDMH । ५ BDM ।। 19. संभवति LD ।। 21. २५६ DM, ६२५ H । ६२५ DM ।। 23. १६ DM ।। 23 – 24. जातमंतरं तयोः] जातमंतरयोः γ (जातमेतरयोः L, + १ B), जातमंतरयोः S ।। 24 – p. 30, 1. क २५६ ... भाज्यः om. B ।। 24. क २५६ D, रू २५६ IH, रु २५६ M ।।

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1. क २५६ D, कयो २५६ B, रू २५६ं I, रु २५६ M ॥ 2. २५ DMH । °थोक्तवद्योगे ζ ॥ 3. २५ BDMH ॥ 9. २५ं] २५ BDMH ॥ 10 – 12. चिंत्यो ... °गुणना° om. ζ (in the bottom margin W[1]) ॥ 13. क २५ं क २७] space in B । २५ DMS ॥ 13 – 15. स^2 ... गुणकः om. D ॥ 15. गुणके करणीद्वयं ζ ॥ 16. स्थाप्य om. B, स्माप्य D, स्थाप्यं IM, संस्थाप्य H ॥ 17. २५ DTMH ॥ 18. क २५ं क २७ after 19. गुणिते B; after 19. जातं IM । २५ DTMθ ॥ 19. °गुणयरूप° γS ॥ 20. ६२५ γ (except L) MS ॥ 21. क ७५ं क ८१ om. B; after 22. क्रियमाणे D; after 23. धनर्णं° IM ॥ 21. ७५ DTMH ॥ 22. यथोक्ते° γ, यथोत्यो° IM ॥ 23. °वोत्पन्नं δH ॥ 24. ६२५ γM । क ८१ after क ७५ं ζ । ७५ BDMH । अथ LT ॥

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1. २५६ BDM ॥ 2. २५६ BDMH ॥ 3. प्रच्युतः सन् Lζ । लब्धं δ ॥ 5. °हार्थं R, °हारे β (°हार L, °हारो T, °हरे IM) ॥ 5 – 6. धनर्णता° ... द्वाभ्याम् om. LS, धनर्णतेति द्वाभ्यां H ॥ 7 – 14. Verses 28c - 30b. Lθ ॥ 9. °हरो S ॥ 11. °स्तथा H ॥ 14. पृष्टु° S । स्युरिति L ॥ 15 – 16. भाज्य° ... °दिति] भाज्यः तत्र माहरणो या लब्धाः करणयस्ता यदिति R ॥ 15. °हारौ AB ॥ 16. हरे om. β (except L) ॥ 17. तयैकया रणया A (except ε, तयैक रणय R), तयैक रणया B ॥ 19. पृष्टः om. H, प्रष्टः R, पृष्टु BT, पृष्टः N ॥ 20. सु त R, सु न N, सति ζ ॥ 21. कश्चित् om. β । जातस्तेन A । °हरौ IM । यथोक्त° LB ॥ 23. धनर्णत्वे ζ । कियंतो° ζ । उत्पद्यंते β (उत्पद्यते IM) ॥ 24. एवत्यत S, एवेत्य IM, एवेस्यत R । °हरौ RBD ॥

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1. °हरौ LH ॥ 2. ४^1 om. εD । °हरौ DH ॥ 4. पृथक् + च ζ । कार्यो A ॥ 5. वर्गेणेति om. LS, वर्गेण योगकरणीति BWH, वर्गेण योगे करणीति

IM ॥ 6 – 9. Verse 30c - 31b. Lθ ॥ 6. विशुद्धेत् S ॥ 7. खंडा तत्कृ° L ॥ 9. करणय इति L ॥ 10. वि°² om. γ ॥ 11. क्षणाः N, क्षुस्माः R, क्षुणाः γ (क्षुन्ना D), गुणयाः ζ (गुणिताः H) । इतिमा R, इति भा N ॥ 12. योकरणी IM । वि°² om. BLT ॥ 13. °हारीभूतवर्ग°] space in B ॥ 14. क्षुणाः A (कुस्माः R) γ (क्षुण D), क्षुणणाः I, क्षुंणः M, गुणिताः H ॥ 16. अत्रोनेन योगसूत्रेण करणयार्योर्गपपति N, अत्रोने योगसूत्रे कारणयोभपपत्तिः R । अत्र] तत्र A । इत्येतस्य β (except H) । °लोम्येनैतत् β (°लोक्येनैतत् M), °लोम्ये R ॥ 17. क्रियमाणे β ॥ 18. सैकमूलस्य D, सैकमूल L, सैकस्य A (सैकस्त N, सैक सा R) ॥ 19. लब्धभागनिष्पत्ति N, लाघुमागमिष्यति R, लब्धत्वेन (लघ्यत्वेन D, लघुत्वेन ζ) ज्ञायते β ॥ 21. विहृतेति ... °करणी om. N । विहृतेति] विहृतेति च दे R, विहतेति B, विहृतेनेति D, हृतेति IM ॥ 23. ज्ञायते A ॥ 24. जायते ζ ॥

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1. तत्पूर्व° (°च्चे° om.) β । °वर्ग° om. R । °लब्धं om. β, °स्तु R । तर्हि] तदा A ॥ 3. °व्यताय° R, °व्यत्तय° T, °त्प्रत्यय° IM । °सूत्रे ζ ॥ 4. °कल्पित° BL । भाज्यः om. RBD ॥ 4 – 5. हारः ... ३ after 5. °क्रमेण A, after 5. °करणया D, after 6. ऋणात्वं IM ॥ 6. ३ BDM, ३० R । भाजके] भा क १८ क ३ । जक A (भाजक ε) ॥ 7. अतस्ताभ्यां] तथा चानेन A । द्विधा भाज्ये β (except H), भूतात्येद्विधा R । गुणिते + सति BTζ (except M) ॥ 8. क³ ... १६२ after 10. °च्चतुरा° A (except N, after 11. तयोरे° R), after Appendix #5, line 1. धº B, after Appendix #5, line 1. धनर्णयोरंतरं (and before °रमेव योगः) IM ॥ 8. २७ εD । १३५० RD, ३५० N । २२५ εD । १६२ ABDM ॥ 9 – 12. ताव° ... गताः] Appendix #5. β ॥ 9. °त्संभवत्संभव° A (except ε) । °दाया° A (°दायो° ε) ॥ 13. अथोर्वि° R, एवमुर्व° β (एवं सर्वमुर्व° B) । २२५ εDM । २७ NDM, ५४ R ॥ 14 – 15. अत्र ... क १ om. R ॥ 14. ६७५ ζ (except M) ॥ 15. क³ ... १ after 16. याव° B, after 16. स्या° IM । ५४ om. R,

५४ A (पं ४ N) BDM । ९ं om. R, ९ ABDM ॥ 17. हर° Bζ, हा° R । क^{1} om. ζ । भाज्यत्थास्य R, भाज्यस्य B, भाज्यस्याकस्य IM । ५६२५ + क २२५ (under ५६२५) NT, ५६२५ + क २७२ R, ५६२५ + २२५ (under ५६२५) ζ (२५ S) । ६७५ं WI, ७५ N ॥ 17 – 20. भागे ... ७५ं om. L ॥ 19 – p. 34, 25. जातम् ... कल्प्यते om. R ॥ 20 – p. 34, 23. तद्यथा ... °न्न्यासः om. A ॥ 19. जातम् om. L, जातः D ॥ 20. ६२५ DM । ७५ BDMH ॥ 21. कं २५ D, क २५ M । लघ° IM ॥ 22. °तीयोस्तृ° IM ॥ 23. २५६ BDM । २५ BDM ॥

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1. ६४०० γ (except T) Mθ ॥ 1 – 2. क^{3} ... ८१०० after 2. °तरमे° B, after 3. इ° IM ॥ 2. ६९१२ DMθ, ६९ B ॥ 3. °मेवोत्पन्नं LD ॥ 4. ६४०० γMθ ॥ 5. ६९१०२ B, ६९१२ DMH । °स्तृतीयचतुर्थयो° om. T । °र्थयोरंतरमेव D, °र्थयोरेवांतरे ζ, °रंतरे T ॥ 6. क^{2} ... ६७५ after 8. जा° T । ६२५ BDMH । ६७५] ६७५ं TIS ॥ 6 – 7. क^{4} ... ७२९ after 7. हरे IM, under 6. क^{2} ... ६७५ T ॥ 6. ६७५ं] ६७५ BTζ (except W), ७५ D ॥ 8 – 10. °करणी ... पूर्वोदाहर° om. B ॥ 8. क^{1} om. WH ॥ 11. ४६० H, ४ क ५० BT ॥ 12. त्रि° om. B । °मित° om. γ । °करेणया B, °करणयं D । ३ DM, ३४ B ॥ 13 – 14. क^{1} ... १३५० after 15. यो°^{3} T ॥ 14. क^{3} ... १६२ं under 13 – 14. क^{1} ... १३५० T, after 15. क्रि° B, after 15. °योरं° IM । २७ क १३५० DM । २२५ DM, २१५ B । १६२ γ (except L) MH ॥ 15 – 16. योग ... शून्यमेव om. B ॥ 18. १६२ DM, १६ B । २७ DM ॥ 20. १४५ BT ॥ 21. क^{3} ... ९ं after 22. हा° B, after 22. ४८४ IM । ७५ं] ७५ γ (except L) M । ९ BDM ॥ 22. क om. Lζ ॥ 23. जातयोर्भाज्य° ζ, भात्य° D । क ४८४ after 24. खण्ड° B । क^{3}] भाजकः L, छेदः T, छद D ॥ 24. लब्धं A, लब्धेर् D । गुणकः om. A, गुणकं B । क^{1} + २२५ A ॥ 25. इयं + करणी ζ ॥ 26. इयं^{2} om. β । नवमितेन (नवमितेर्न M) वर्गेण β । ९ om. β (except L) ॥

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2. जातौ LD, ज्ञाते N । क २ क ८] २।८ BTζ ॥ 3. क ३ om. Nγ, ३ IM ॥ 5. द्विकत्रिपंचप्रमेता इति om. LS ॥ 6 – 11. Verses 31c - 32d. Lθ ॥ 7. द्वित्रिक° θ ॥ 8. °त्रिकद्विक° L, °द्वित्रिक° S ॥ 11. कृतानां Lθ । पदानि + न्यासः क २ क ३ क ५ क ३ क २ क ६ क ५ क ३ । क २ क १८ क ८ क २ स्छाप्योंत्यवर्गो द्विगुणांत्यनिघ्ना इति कृते जाता यथाक्रमं वर्गाः रू १० क २४ क ४० क ६० रू ५ क २४ रू १६ क १२० क ७२ क ४८ क ६० क ४० क २४ अत्रापि यथासंभवं करणीनां योगं कृत्वा वर्गवर्गमूले कर्तव्ये क १८ क ८ क २ योगे जातं करणी ७२ अस्य वर्गः रू ७२ इति मूलं अथ टीका H (from Bhāskara's *BG*, pp. 17-18, with slight modifications) ॥ 12 – 13. °र्थः ... वर्गा° om. H ॥ 13. °हरणन्यासः A (°हरणे न्यासः R) ॥ 15. °त्यावर्गो A (except ε) ॥ 16. °वर्गप्रकारमनुस्मरन्] °सूत्रेण B, °सूत्रकारणे L, °सूत्रप्रकारेण DTζ । तत्रायं विशेषः om. A ॥ 17. तथाकृते ... कृत्वा om. β ॥ 21. संपन्नः om. β । मूलकरणीनां om. A । वर्गे कृते om. N । ये om. εζ । ये + मूलकरणी (मूलकरणी om. ζ) स्थानप्रमिता एव सहजाः समद्विघात (समद्विघाता ζ, समद्विघ्नाता S) इति स्वलक्षणजा (स्वलक्षणजात BD) β । °स्तेषामेव β ॥

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2 – p. 37, 6. एवमत्र ... सर्वत्र om. A ॥ 2. निमित्ताश्च L, मिभजाश्च IM ॥ 3. °स्थित° om. γ ॥ 4. चतुर्षु] तेषु BTζ, + च LT ॥ 5. इत्यादि] वित्यादि BLT ॥ 7 – 8. योगे क्रियमाणे] Appendix #6. γ ॥ 8 – 9. भवेयुस्तावतां IMH ॥ 9. °मित° om. Tζ ॥ 11. च] °पि IM ॥ 12 – 13. °मूल° ... यन् om. L ॥ 12. °मूल° om. ζ । वा om. ζ । एवं ζ ॥ 13. °गतं सर्व° β (except L) ॥ 19. क^3 ... ४ followed by 18 – 19. क^4 ... १६^1 after 20. सर्वे° IM ॥ 18. क^4 १४४^2] १४४ क B, क ११४ IM, क ५४४ S ॥ 19. क^3 ... ४ om. S, after 20. मूलानां H ॥ 20 – 21. °नामैक्य° ... मूल°^{2} om. IM ॥ 21. रू]

क S । ७२[1]] २७ θ ॥ 24. °न्मूलंस्या° T, °न्मूलमस्या° L[1] (म add. L[1] in the right margin) । °स्या ε स्य IMS ॥

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1. क्षुरणा om. D, क्षुणा LTW, क्षुरणा I, क्षुंण M, क्षु B ॥ 4. °वर्गस्तु BDTIMS, °वर्गस्थ L ॥ 5. प्रोह्येति γ (घ्रोह्येति B, प्रो-ति D), प्रोझेति ι, प्रोझ्येति S, प्रोक्त H ॥ 9. करण° om. ζ । सूत्रद्वयमाह S ॥ 9 – 10. वर्गे ... द्वाभ्याम् om. LS ॥ 11 – 18. Verses 33a - 34d. Lθ ॥ 17. कृता L, °मतोऽपि θ ॥ 18. वर्गे + इति L ॥ 19. वा above °मथ L[1] ॥ 20 – 21. रूपाणि[1] ... °नितानि om. IM ॥ 20. रूपाणि[1] om. L (in the left margin L[1]) । °द्विशो°] °च्छो° β ॥ 21. यदि om. β । वर्गे om. ζ ॥ 23. कार्या BD । रूप° om. β । वा om. β (except L) ॥

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1. करणयो] करणयौ IM, करणौ N, करणीर् H ॥ 4. °वति°] °भवति° IM । तं नि° IM, ते नि° N, चेन्नि° R ॥ 6. एव om. β ॥ 8. जातानि om. β । °गत°] °ग° IM । ज्ञातं B, जातः εLT ॥ 9. संक्रमणाधिना R, संक्रमविधिना BS, संक्रमणीछुह्वि I, संक्रमणीछुद्धि M । योगत° ε (योगन° N) BM । रूपकृते + १ IM ॥ 10 – 14. तेषु ... ज्ञाते] तथा च शोधने यच्छेषस्य मूलं तदेवांतरं ज्ञातं A ॥ 10. शोधितेषु om. ζ । करणयंतर ζ (करणयं IM) ॥ 11. °गुणिते ζ ॥ 12. °वर्गे° DT, °वगे° B ॥ 13. °गणो[1] BL । °गुणांत्य° β (°गुणितांत्य° WH, °गणा° D) । °ग्रिमास्ति γ (°श्रिमास्ति B, °ग्रिमानि T) ॥ 14. तया° B, घातयो° IM ॥ 15. योगांतरे° A ॥ 16. °सूत्रक्रमेण] °विधिना β । °त्वे IM, °ज्ञेन T, °तने D ॥ 17. ज्ञाता A (except ε; जात R) ॥ 18. च या B, यात्र T, च εL । बहुतरा + या L[1] (या above line L[1]) । स्यात्स एवो° α । ज्ञातः A (except R) γ (except L) ॥ 19. शोध्य A (स्योध्य R) LDζ (संशोध्य H) । °मुर्वरितकरणीना° om. A ॥ 20. द्विधा om. β । °तान्यर्द्धितानि] °तानि B, °तानि तान्यर्द्धितानि

DTιS । कृत्वा + युति च कृत्वा A (except ε) ॥ 22. रूपाणि om. ζ, रूपाणी + युतो R ॥ 23. इत्युपपन्नं LD, त्याद्युपपन्नं R ॥

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1. मूनर्थं R, मूलार्धं IM, मूलार्धे S ॥ 1 – 2. अथ² ... यथा om. β ॥ 2. सूत्रावेतारो N, पूर्वत्र विचारो R ॥ 3. °शाच्छोध्य A (°ध्य om. R) ॥ 4. जातं + करणीद्वयं β ॥ 5 – 6. अथ ... क ८ om. S ॥ 5. वन्ही B, बहु ε ॥ 6. प्रकल्प्य एतस्य (तस्य R) कृ° A, प्रकल्प्य एत कृ° S ॥ 7. ४ अस्य मूलं २] स्पष्टं R । मूले BT ॥ 7 – 8. युतो° ... करणी° om. B ॥ 9. ज्ञेय° ... स्पष्टम्] करणीवर्गे मूलमानेयं β । स्पष्टम् om. R ॥ 10. धनर्णो व्य° A (except ε), ऋणात्वव्य° T । °व्यवस्थामाह LD । ऋणात्मिका चेदिति om. LS । चेति ι, वेति D ॥ 11 – 14. Verse 35a - d. Lθ ॥ 12. प्रकल्प्य L । सामध्ये S ॥ 14. °वागम्या S, °वगम्येति L ॥ 15. मूल° om. A ॥ 16. करणयो ε ॥ 17 – 22. अत्रो° ... °पपन्नम्] Appendix #7. A ॥ 19. योगेग्रे L, योगप्रे D । क्रियाविच्छिन्निरतः T, क्रियविष्टित्तितरः B ॥ 21 – 22. सिद्धौ इति γ (सिद्ध इति T) ॥ 23. श्लोकार्धे° A । त्रिसप्तमित्योर्वदेति om. S ॥

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1 – 2. Verse 36a - b. θ ॥ 3. पदं] मूलं β । वुदेति A (वुदेनि N, वद वुदेति R) ॥ 4. संबंधः om. β । क्षतगत° IM, क्षयत° N, क्षय° R, ऋणगत° H । °था च न्यासः] space in B । ३̇] ३ RDTMS, ३ B ॥ 5. ७̇ illegible in R, 7 in place of ७ in N, ७ BDM । सूत्रक्रमेण om. β । पृथग्] पूर्ववद् β (पूर्वं L) ॥ 5 – 6. स ... ८४̇] space in B ॥ 6. ८४ RDM । ऋण (ऋणगत B) करणयाः β । प्रकल्प ये रू° RB, प्रकल्पयेत् रू° D, प्रकल्पयेल्रू° N ॥ 7. तन्मध्ये β । क ३̇ om. R, क ३̇ B, क ३ DM ॥ 8. °हरणमाह β ॥ 8 – 9. द्विक° ... इति om. S, द्विकत्रिपंचेति H ॥ 10 – 13. Verse 36c - 37b. θ ॥ 14. त्वं ... तर्हि (त्वं] तर्हि कथं A । वेत्सि तर्हि] वेत्सीति (वेत्सति R) A) after 16. संबंधः

A । त्वं] ळ्कं WI, ळं M । वेत्सि IM । प्राक्कथिता β (प्राक् साधिता S) ॥ 15. कृतेः पदं च ζ । पदं + तर्हि A ॥ 16. चर्णगाः D, चर्णगता ζ, चर्णगं N, च क्षणग R । °वेकर्णगता ζ, °वेका क्षणाशा R, °वेकर्णग D, °वेकर्णकगा T । वेत्यर्थः LT, इत्यर्थः B, चेत्करणीकृत्यर्थः R ॥ 17. धनमिति] धनगता चेति γ, धनगेत्युक्तत्वात् ζ ॥ 17 – 21. स्पष्टम् ... इत्युक्तत्वात् om. ζ ॥ 18. एवमन्यत्र L, एवमन्य एवं B । क^{2} ... ५ om. R । पं] ५ ABD । क २ं om. N । २ BD । क^{5} om. B । ३ BD ॥ 21. क ४ं० om. R । ४० DMH । ६० RDMH ॥ 22. मूलार्थं om. A । °करणयोस्तु° β । धनरूपाणि β (except L, धनरूपपाणि B) । धनानीमानि om. β । रूपकृते १०० रपा° β (रूपकृतेरस्या १०० रपा° B, रूपकृतेः १०० ऋपा° H) ॥ 23. शेषस्य मूलेन] शेषं ० ग्रस्य पदेन β ॥ 23 – p. 41, 4. युतो° ... स्पष्टम्] Appendix #8. β ॥

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5. संभवाभिप्रायेण ζ (°ण om. IM), संभवाद्यभिप्रायेण L, another संभवा in the top margin L । किंचि° β । एकादिसंकलितमितेति om. S, एकादिसंकलितमिति RW, एकादीति H ॥ 6 – 13. Verses 37c - 39b. + 39c - 41a. S, Verse 37c - d. H (Verse 38a - d. after 21. °त्यादि and Verse 39a - b. after 21. स्पष्टार्थम् H) ॥ 11. च om. θ ॥ 12. प्रोह्य H ॥ 14. वर्ग° ... तदेकादि om. W । °संकल्पित° ε (°शंकल्पित° N) ॥ 14 – 15. °करणी° ... तन्मितानि om. B ॥ 15. तन्मितानि + तानि A (except ε) ॥ 16 – 17. °वर्ग^{1} ... करणी^{2} om. D ॥ 16. भावि° LS । भवति ε ॥ 17 – p. 43, 8. चेद्वर्गः ... °वशे° folio 31 om. R ॥ 17. तर्हि] तदा β (त B) । करणीयं स्यात् IM ॥ 18. क्रियमाणे om. β । °त्रितयं DTζ, °द्वयं A । स्यादित्यादि β (except T) । °मितानि° β ॥ 19. भवंतीत्यर्थः + Appendix #9. β ॥ 20. °च्छात्राणां β ॥ 21. °त्यादि + 8 – 11. Verse 38a - d. H । स्पष्टार्थम् + 12 – 13. Verse 39a - b. + Verses 39c - 41a. H ॥ 22. ग्रथ रूपकृतेरिति om. H । उक्त° + वत्° ζ । प्रोह्य BTH, प्रोस्य L, प्रोम्य D ॥ 23 – p. 42, 4. चेदन्यथेति ... भवतीत्यर्थः] Appendix #10. A ॥ 23. चेदन्यथेति om. L ॥

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1. एकस्यस्तु° IM ॥ 2. °क्रमा°] क्र om. BD । क्रमं ... °ष्वन्यथा om. ζ । सूर्यतिथ्यादिषू° B ॥ 3. रूपाणि om. γ ॥ 4. तदप्यसत्यत° ζ ॥ 5. अधो° B, अस्यो° LD ॥ 6 – 7. Verse 39c - 40a. after Verse 39b., p. 41, 13. S; after p. 41, 21. H ॥ 6. °गयात्मया S ॥ 7. यासाप्तपवर्तः S ॥ 8. °मानायाऽल्पया B, °मानयाल्पेया T, °मानाल्प्यया IM, °मानाल्पया θ, °मानया W (अल्पया in the right margin W[1]), °मानयेवमल्पया D ॥ 9. स्युः om. BH, सुः D ॥ 10 – 17. अथ ... °दित्यर्थः] Appendix #11. A ॥ 10 – 11. मूल° ... लब्धा om. B ॥ 11 – 12. Verse 40b - 41a. β (except अपवर्ते इति H), also after Verse 39c - 40a. in θ ॥ 12. नै यँदि L, यदि न B, त यदि M । तदसदिति DTιS, सदिति B ॥ 16. °कण्यो IM । न[3] om. BIM ॥ 17. °हरणकथना° A ॥ 18. °पत्तिर्यथा A । °संकालनं A (except ε), °संकलितं ζ । नामैक°] नामैकादिति (°दित BDζ, °दि L) + एकोत्तराणामेक° β । °मुत्तरं LD (त्तरो in the right margin L) । योग + विशेषः A ॥ 19 – 20. Vertical columns in A । ६ ... ९ om. A ॥ 20. १ ... ४५
२१ ४५
after p. 43, 3. मू°[1] L ॥

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1. °था om. IM ॥ 2 – 3. °त्रितये ... मूलकरणी om. D ॥ 2. करणीद्वये B । इत्यादि] इति β ॥ 3 – 4. सा ... करणी om. T ॥ 3. सा] आसीत् ζ ॥ 4. तावंत्या L, ता चेदंत्या N । त्यस्यां ζ, त्यस्या DT, यस्यां L ॥ 5 – 6. इत्येतस्य ... °निघ्ना[2] om. S । इत्येतस्य ... °निघ्ना[1] om. ιH ॥ 5. तद्यथा] तथाहि BDT, तथा Lζ ॥ 6 – 7. चतुर्गु° ... एव om. T ॥ 6. अंत्या°] अन्यवा° N, अन्या° BD ॥ 7. °र्गुणांत्यगुणिता Lθ । आसत् IM, असन् D, आसीत् S, अथतः B ॥ 7 – 8. संत्यो (संयो D, सत्यो ζ) यदि निरवशेषा β ॥ 8. Folio 32. of R begins °ष भवंति । तदैव ता रूप° β । ता ... चेत्] space + गत्वे B । शोधयेत् L, शोध्यते Dζ (शोध्यंते H) ॥ 9. °दित्युप° β ॥ 10. वर्गे ... °रिति

om. S, वर्गे यत्रेति H । सिद्धे° om. β ।। 11 – 12. Verse 41b - 42a. θ ।। 13. एवमत्र om. β । अत्र] एवमत्र A ।। 13 – 14. करणी° ... अतः om. T ।। 14. रूपकृतेरपास्य] कृत्वा A । यावन्मूलं δζ, या वर्गे करणीमूलं B । संगृह्यते LD ।। 15 – 17. सर्वासां ... भवतीति om. A ।। 15. रूपा ६४ णयपास्य BDT, रूपाणय ६४ पास्य WS, रूपाणय ६४ पास्या IM । एभ्यः om. BDT, अस्य L ।। 18. °यैवमित्यैत° R, °यैव इत्येत° N, °यैवमित्यत° B, °येत्येत° H, °मेत° ιS ।। 18 – 19. वर्गे ... °रिति om. S, वर्गे यत्रेति H ।। 20 – 21. Verse 42b - 43a. θ ।।

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1. तथा च om. β ।। 2. °द्वितय द्वि° (°स्य om.) BT । °दवाकमित° R, °दशामित° B, °दशमित° δζ ।। 3. रूपकृते १०० (१०० om. B) रपास्य (अपास्य H) β, भक्ताः सत्सा अपाप्य R । शेषं BDζ (+ ३६ अस्य ζ) । शेषस्य + ३६ L । ६ om. N, ३६ । ६ T । रूपाणि ... तेषा°] युतोनितानां रूपाणा° (शेषाणा° B) β । °मर्धे + करणयौ H ।। 4. क^1 and क^2 om. β (except LH) । अत्रोत्पत्तस्य° A (except ε), अत्रोयत्तस्य° B । क^3 om. β । क^4 om. ζ । ¿ S ।। 5. °दशमितयोरप° β । करणयो IM ।। 6. विशो° ... °त्वात्] विशोध्या इत्युक्तत्वात् + अथवा शोधितेऽपि पूर्ववन्मूलं क २ क ३ क ५ । उद्दिष्टवर्गो ह्यस्य मूलस्य (मूलं γ) न भवति यतोऽस्य वर्गोऽयं रू १० क २४ क ४० क ६० β ((रू १० ... क ६० om. D; क ४० om. IM.) यतोऽस्य ... क ६० borrowed from Kṛṣṇa's *BP* p. 80, 20-21) ।। 6 – 10. अत ... न्यासः om. D ।। 6. एतद°] एव तद° R, एव द° N, एव न ζ ।। 7. स्यादित्येत°] स्यादेत° α । अष्टौ ... °दिति om. S, अष्टाविति BTWH, अष्टाविंशति IM ।। 8 – 9. Verse 43b - 44a. θ ।। 8. कृतौ + सखे S । यत्रेति S ।। 9. रूपै° ... स्यात् om. S ।। 10. अत्र om. β ।। 11. एतत्तुल्यानि रूपा° L, तत्तुल्यरूपा° IM । शेषस्य मूलं RD, वशेषमूलं T । पूर्वलब्धं A, पूर्ववत् L ।। 13. ७ + Appendix #12. β ।। 17. नोत्पद्यते N ।। 18. अथान्यदप्याह β । चतु° ... इति om. S । सूर्य° ... °र्तव om. γιH ।। 19 – 20. Verse 44b - 45a. θ ।। 20. वदत्पदं S ।। 21. अत्र om. β ।। 22. प्रथमं

om. D । मूलं ग्राह्यम् om. β ॥ 23. अथान्यसंभ° γ (अथान्यासंभ° D), अथान्यत्संभ° ζ ॥

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1. °करण्योस्तु° A । °कृते १६९ रुपास्य BW, °कृतेरुपास्य १६९ (+ अपास्य D) DIMθ ॥ 2. शेषं ... मूलं] शेषस्य (शेष BS) मूलेन (लेन B) β । अनेन रूपाणि om. β । °नितानां (°नितानि L) रूपाणामर्धे क १ क १२ β ॥ 3. °ल्यानि रू° β ॥ 4. बहुकरणी Rβ ॥ 5. शेषस्य पदेन β । ० अनेन रूपाणि om. β ॥ 6. °नितामि R, °नितानां β । तेषामर्द्धं ε, रूपाणामर्धे β (रू B, रूपामर्द्धं L, ना above line L[1] । ५ । ५ om. B, क ५ क ५ δζ । मूलं + रा॰ IM । क[1] ... २ om. B । क[3] ... क[4] om. H । क ५[2] om. ALIM ॥
7. एवमिदमसदि°] एतदप्यसत् β (एतदेवाप्यसत् L) । °ति प्रतिभाति om. β ॥ 7 – 8. वर्गो ... °नेयम्] Appendix #13. β (from Bhāskara's *BG*, p. 25) ॥ 7. वर्गो] वर्गं ε ॥ 9. तथा चास्म°] अथ करणीनामासन्नमूलमानेतुमस्म° β (°नेतु अस्म° D, °नेतुरस्म° T) । स्वकृात° R, प्रकृति° ζ (प्र erased and स्व in the right margin W[1]) ॥ 9 – 10. आसन्न° ... उक्तः] सूत्रमुपनिबद्धं β (तत्सूत्रमु° B, तंत्रमु° D) ॥ 10. स om. β ॥ 12. °मूलमेवेति T, °मेव मूलमिति ζ ॥ 13. अस्यार्थः om. β । यस्यापि B, यया ε । कस्यापि om. B । तेन + ५ A (except ε) । एव om. A ॥ 14 – 15. भाज्यः ... यावन् om. β ॥ 15. निःशेषो भवतीत्यर्थः β ॥ 16. अत्रो° ... इष्ट°] यथेष्ट° β ॥ 17. हतः IMH । ५ (i.e. २ om.) RH । अनेन लब्धेन मूलं २ β । ९ om. Rζ । द्विभक्तं] जातं ζ (द्विभक्तं in the right margin W[1]) । ९
२ ४
om. B, ४ R ॥ 18. °रासन्नमूलेनेति] °रेवं β । °मूले° ... लब्धं om. R । कृते] क्रियमाणे β ॥ 18 – 19. मूलं ... °स्तरेण] पंचानां मूलं २/१४/१० (२२७ H /१४/१०) एवं सर्वत्र (सर्वत्र om. D) β ॥ 18 – 19. पंचानां ... इत्य°] पंचाना २/१४ मित्य° A ॥

19. २ / १४] २५ R || 20 – p. 46, 4. अथान्य° ... क २ before 9. तथा β || 20. °था° om. R, °न्या° BM । °न्यदुदाहरणमाह] °न्यद (द om. B) प्याह β । चत्वारिंशदिति om. θ, चत्वारिंशदशीतिर्द्विशती तुल्या इति ι ||

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1 – 2. Verse 45b - 46a. θ || 3. अत्र om. β । अत्रापि पूर्वव° β । क २ + उक्तवदे (+ वं L) तदपि (पि om. L, प्य DH) सदि (त् इ L) त्यर्थः β || 5. °ज्ञान° om. D || 6. °शेषभूषाविशेषे] °नंदसंदोहहेतौ β || 7. वटु om. R, बहु DTIθ || 9 – 10. इति ... °गमत् om. β || 9. °सूर्य°[1]] °सूर्येण R, °सूर्ये N ||

< 3. तृतीयोऽध्यायः >
< कुट्टकाधिकारः >

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3. श्रीगजाननाय नमः om. Nβ || 4. °लसत्°] °सत्° N, °स° R, °लस° B, °लस° L || 6. सत्सरु°] सस्तस° B, सत् शरु° D, रसत्सरु° Wθ, लसत्सरु° IM | °कलितवंड°] °वलितोरु° A (°कवलितोरु° R, °कनकवलितोरु° N) || 7. स्थलांबुरुहकुड्मलं समदसिंधुरास्यं नुमः β (रव्यांबु° B; °कुड्मल BL, °कुद्महं T; सहमद° I, संमर° H; °सिंधुनास्यं स्तुमः T, °सिंधुरास्यं नुत H) | स्थलांवुल° N, स्थलां° R | °कलाएमाल° N, °कुलायमाल° R | १ om. DH, ३ A, space in B, २ LW || 10. °त्स्वरूपं निरूपं वर्णयन्नाह N, °त्स्तूरूपन्निरूपयन्नाह R, °त्स्वरूपमाह β (°त्स्वपमाह B) || 10 – 11. भाज्यो ... °श्चेति om. LS; add. W^1 in the top margin, followed by 16. आदौ ... कुट्टकार्थं W^1 || 10. हरः TW^1 || 11. क्षेपक° om. H || 12 – 15. Verse 46b - 47b. θ || 15. क्षेपश्चेत्तद्दृ° S || 16 – 17. आदौ ... संबंधः om. H || 16. आदौ ... °पवर्त्त्य om. S | आदौ ... कुट्टकार्थं om. ι (with ζ) || 17. संबंधः + क्षिप्यते IM | भाज्यते° LDζ | तथा^{1}] क्षिप्यते (om. IM) ऽसौ क्षेपः β | तेनेति] येनेति β (येति B, नेनेति H) | तथा^{2} ... क्षेपः om. β || 18. °पवर्त्तनसंभवे B, °वर्त्तनसंभवे R | ह्यपवर्त्तः] सत्यपवर्त्तः β (सत्यवर्त्तः BD) || 20. नियमोनेति α (except TιS) | रूढि D, दृढ L || 21. कुट्टकासंभवे β (कुट्टकसंभवे DIM) | विशेषमाह] कारणमाह β (करणमाह S) | क्षेपश्चेच्छिन्नो + न भवति L ||

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1. अयमर्थः om. β || 2. तर्हि om. β || 3. अथप° DIM | के om. β | केनैवांकेन ζ | वांकेन om. B || 4. व्याकुलित° Tζ (except H), व्या + space in B || 4 – 5. परस्परं ... °रिति om. B, after Verse 49c - 50b. S || 6 – 9. Verse

47c - 48b. θ, + Verses 48c - 50b. S || 11. द्वयो° ... °ऽकः om. B । अथा° L । निमित्तभूतौ° α || 11 – 12. °पवर्त्तनं ... स्वेना° om. ζ || 12. कश्चिदित्यर्थः] कश्चित् γ । जा R, ज्ञाते γ (ज्ञाने L, ज्ञातौ T) । वि° om. IM । °भाजितौ + यौ ζ । तौ om. B || 13. द्विवनमु° A, द्विवचमु° M || 14. भाजिताः] अपवर्त्तिताः β । °संज्ञसंज्ञकत्वं] °संज्ञसंज्ञकर्तृ N, °संज्ञकर्त्त R, °संज्ञकत्व β (°त्वं B) || 15. वक्षितं D || 16 – 17. भजेत्तावित्यादि° H || 17. वृत्तैः om. β || 18 – 21. Verse 48c - 49b. θ (S has Verses 47c - 50b. together) ||

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1 – 4. Verse 49c - 50b. θ, + Verse 50c - 51b. H || 6. पूर्वं°] सर्वत्र° B, सर्वं° DT || 6 – 9. परस्पर° ... भवति om. D || 8. मिथो om. β (add. W[1] in the left margin) । °नि लब्धा° om. β । °स्थापितानि β । ततस् om. β (except L) || 9. शून्यं] सं (सं सं I, सं सं M) + शून्यं β || 10. स्वोर्ध्वो IM । च युते Tζ । च om. R । °र्वारं वारं] °र्वारं २ A (°वर्रां २ N, °र्वारं R), °र्मुहुर्वारं वारं वा β (°र्मुहुर्वा B, °र्वारं वारं वा L, °र्मुहुवारं वारं वा D, °र्मुहुर्वारं वार्यं वा M, वा om. H) । कार्यं β || 11. अत्रोपांतिमेत्य°] अत्रोपंतिमेत्य° A (except R), अत्रोपांतिमे° B, अत्रोपांतिमेनेत्य° Tζ । तिष्ठत्युपां° Nγ, तिष्टतीयुपां° R । °त्रांत्यः] °जांज्य R, यत्रायं β (वाच्चायं L, यत्रायत्रायं B, यात्रा य D, यत्रासं W, यत्रांत्यं H) || 12. तदुपरिष्ठम् γ (तदुपरितिष्टस् D) । परिशेषाव N, परिशेशेषाव R, सपरिशेषात् L || 12 – 13. तत ... हन्यात् om. A || 12. तत] अत ζ || 13. स्वांतेन L, स्यात्ते IM । स्वोर्ध्वं°] स्वोपरि° D || 13 – 14. °स्थितं ... अपरोऽधः° om. D || 13. तत] गत N, एवम् β || 14. °हरेण DWθ, °हरणे BIM || 15. सन्] सत्र ε । शेषं om. β । यत्र] अत्र L । शेषे-व S, शेषे च H || 16. तष्ट om. β । शब्दाद्य युज्यति L || 18. एवमिति om. S || 19 – 22. Verse 50c - 51b. θ (H has Verse 49c - 50b. and Verse 50c - 51b. together) || 19. यथा S || 22. तु] त S, च H ||

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1. दृढभाज° LIM । °हरयोः LD ॥ 2 – 3. उक्त° ... तर्हि om. TS ॥ 2. °क्रिययैवं ιH । लब्धि°] लब्ध° IM ॥ 4. हरः LT, हारा IM ॥ 5. प्रकृते] कृते ζ ॥ 6. °हाराविव ε, °हारौ L, space + व B । गुणलब्धो° γ (except D) ॥ 7. गुणलब्धी[1]] गुणाप्ती β (om. B, गुणाती S) । °लब्धी[2]] °लब्धौ IM ॥ 8 – p. 52, 3. अत्रोपपत्तिः ... °कुट्टककारणी° om. L (folio 26 missing) ॥ 8. °पत्तिर्यथा A ॥ 9. °हारेण (°हरेण ζ) हृतः β (हारेणाहतः B) । निःशेषो BD, भिःशेषो R ॥ 10. स] प ε । भागे om. B, भागो ε ॥ 11. भाज्यौ ε, भाज्ये B ॥ 11 – 12. न ... न om. B ॥ 12. हार°] हारेण β ॥ 13. ज्ञानार्थं om. ζ ॥ 14. °गुण°] °गुणत्व° β ॥ 15. °स्फुटफलं Wθ, °स्युढफलं IM ॥ 16. सकाशाद्य ज्ञातं ε (सकाशाय ज्ञातं N), सज्जातं β (सन संजातं B, सत् जातं DT, सत्या जातं H) ॥ 16 – 17. °गुणौ ... °लघ्वी° om. T ॥ 17 – 19. तक्षणं ... °हाराभ्यां om. B ॥ 18. मसि A (except ε) । °हत°] °हस्त° ε । °हरेण युक्तेत्येत°] °हरेणैत° β (°हरेणैवत° D) ॥ 18 – 20. युक्ते° ... °हरेण om. R ॥ 18. °त्प्रकारांतरे ζ (°त्प्रकांतरे I) ॥ 19. लब्धे° IM । अथ + तत्र β ॥ 19 – 21. यल्लब्धं ... अथ om. D ॥ 19. तमेव इष्टं A ॥ 21. भावः om. β । °द्वैये° A (except ε), °युग्मे° β ॥ 22. मुहुर्मुहुः ζ ॥ 23. गुण + भाज्ये β । °पतित्व B, °पातितत्वं T, °पालित्वं IM ॥ 23 – 24. दृष्टम् ... °पातित्वं om. IM ॥ 24. हरो ADT । भाज्याछुध्यतीति ε, भाज्यसमो भवतीति β (ति om. B) । हारांतः°] हरांतष् A (हरांतः° ε), हरांतः° γ, गुणहरांतः° ζ ॥ 25. तथात्र] यद्वा β ॥

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1. हर° α । °भक्त° om. T, °भक्तं ζ । °भाज्याल्ल°] °भाज्यादत्र ल° BTζ । °भक्त्या° ε ॥ 3. °पापत्तुप° A (except ε), °पपत्तित्युप° D । °नस्माभि° A

(except N, °नाथाभि° R) । कारिकाभिः om. β । किंचिदुच्यते β (किंदुत्न्यते B, कंविदुच्यते IM) ॥ 5. भाज्या° A (except N) ॥ 8. ते माने β (ते माणे B, ते मोने T) । °लब्ध्योः β (°ल्ध्यो D) ॥ 9. च] तु ζ । °स्तमक्षणं R, °स्तेक्षणं B, °स्तत् क्षणं D, °स्तथा क्षणं IM ॥ 11. °वोत्तरे ζ ॥ 12. द्वितीयागमनं त्विति] तदागमनहेतवे β ॥ 15. गुणज्ञाने om. β । तल्लभ्यते स्फुटम्] space in B, तस्मादथ लब्धि गुणो हरः DT, तत्स्यादथ लब्धि गुणो हरः ζ ॥ 16. om. β ॥ 17. क्षेपोनौ A (क्षेपोने R) । स्फुटः + क्षेपोनौ भाज्यहारौ स्तो गुणाप्ती एकांतरौ यदि β ॥ 18. यच्च BT, यत्रै H । कोऽपि ζ ॥

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3. Folio 26 of L starts from °भूत° । °पवर्त्तना° D, °पर्वता° IM ॥ 4. भवति कुट्टविधेरिति om. S ॥ 5 – 8. Verse 51c - 52b. θ ॥ 5. °भाजयोः S ॥ 6. °रथवा] °रपि वा θ ॥ 9. कुट्टक° δ, क्कुट्ट° IM । °द्गुणो ε ॥ 10. हरो β (हरा IM, हना S) ॥ 12. सन् गुणः om. BL । °हारकयो° ιS ॥ 13 – 14. °व्यति° ... °त्यर्थः om. L ॥ 14. °त्यर्थः + Appendix #14. β ॥ 15. अत्रो° ... युतिभाज्याव° om. B । °भाज्यावप°] °भाज्याप° ADTMS, प° B ॥ 16. हारांतः° Wθ, हरतः° L ॥ 17. गुणोऽप्यविकृत om. L । °रित्यादि अथ] °रिति अथ β, °रित्यथ R ॥ 18. कृते + कृते A (except R, + कृते कृते N) । हरा° α (except B) ॥ 19. °गुणितो°] °गुणो° β । हरस्य α ॥ 20. °गुणितो om. R, °गुणो β । च यदि अ° A, चेद° β । °वर्त्तमांकेन R, °वर्त्तनांकेन ι (°वर्त्तनांतेन IM) S ॥ 22. °धुना om. β । संभवदभिप्रायेण om. β ॥ 23. किंचिद्विशेषमाह β (किंचिद्विषयमाह B) । गुण° ... °मिति om. B । °मिति] °मित्यादितः (°मित्यादिना L) सार्धैस्त्रिभिः (सार्द्धमित्यादितः सार्द्धैस्त्रिभिः T, त्रिभिः H) β ॥

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1. Verse 52c. H, + Verses 53b., 54a., 53a., 54b., and 55a - b. H || 2. धीमता om. B, + बुद्धिमता (om. ζ) पुरुषेण δζ । फलं ... °तक्षण om. N || 2 – 3. गुण°2 ... °नंतरं] फलवल्ल्याः सकाशात् β (om. L) || 3. °गुट्टक° A (°दृढ° R) || 3 – 4. राशि° ... तष्टः om. L || 5 – 8. फलं ... °पपन्नम्] इत्यनेन राशियुग्मस्य तक्षणे फलं तुल्यमेव ग्राह्यम्। संभवदपि अधिकं न ग्राह्यम्। अतो धीमतेति विशेषणं β (इत्यनेन ... °मेव ग्राह्यम् om. L) || 7 – 8. यतो ... °पपन्नम् om. R || 9. अथो° ... °माह] अथ β । योगज इति om. S || 10. Verse 53b. S, + Verse 54a. S (S has 53b. and 54a. together; H has 53b., 54a., 53a., 54b., and 55a - b. together) || 11. गुणाप्ती ... योगजे2 om. β || 11 – 13. योगजे2 ... भवत1 om. N || 12 – 13. साधिते ... इत्यर्थः] भ + space in B || 12. स्व° om. LD || 12 – 13. दृढभाज्यहार° ... इत्यर्थः om. T || 12 – 13. °संज्ञका° ... भवत1] °तक्षणा (°लक्षणा LD) च्छोधिते (छोधितो W, छौधितौ IM, छोधितौ S) सत्यौ (सति L) वियोगजे β || 14 – p. 54, 19. अत्रोपपत्तिः ... निरूपयिष्यते] Appendix #15. β || 14. °वशेषाभू° ε || 16. °भाज्योद्भव इति] °भाज्ये भवेदिति (नवेदिति N) ε || 17. Verse 54a. θ (S has 53b. and 54a. together; H has 53b., 54a., 53a., 54b., and 55a - b. together, see line 10) । °भाजो° S । तद्व°] तद्व° θ । °भाजके + इति S || 18. धनभाज्ये A । °र्ववद्भवेदिति A || 19. Verse 53a. θ (S has 53a. and 54b. together, see apparatus criticus to Appendix #15., line 4 ; H has 53b., 54a., 53a., 54b., and 55a - b. together) । त पर्वत S ||

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1. दिति om. A (see p. 53, 18) || 2. धनक्षेपस A (except ε) || 5. Verse 54b. θ (discussed above, see apparatus criticus to p. 53, lines 1 and 19) । वर्जितेति S || 7. लब्धस्] लब्धि A (लब्धं N, लघ R) || 12 – 13. Verse 55a - b. θ (see apparatus criticus to Appendix #15., lines 35 – 36) || 13. °तात्] °तादिति S ||

15. °र्युति°] °र्यु° A (except R) ॥ 17. °युक्ते A (°युक्ते ε) । हर° A ॥ 18. हरेण A ॥ 20. °संभावमाह ε । क्षेपाभाव इति om. S ॥

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1 – 2. Verse 56a - b. θ ॥ 1. शुद्धो हरो° S ॥ 2. गण° S ॥ 3. यत्र[1] om. β ॥ 3 – 5. क्षेपः ... फलं om. B ॥ 4. अथात्रापि] अथ β । विशेष°] फलार्थ° β । क्षेपो हारहतः फलमिति] क्षेप इति β ॥ 5. च om. β ॥ 7. तत्र ... इति om. β ॥ 7 – 8. मिथो ... भजनात् om. β ॥ 8. गृह्यमाणायां om. β ॥ 9. इत्यादिना] इत्यादि + विधौ क्रियमाणे तत्र β ॥ 10. °प्रमितः om. T, °मितोऽस्ति β । शून्यगुणितः ... °तीति om. β ॥ 11. शून्यमेव β । इत्युपपन्नम्] स्यात् β ॥ 12. °वशेषो ... च om. S । तथा च] अतस्तत्रापि परिशेषात् β ॥ 13. क्षेपाभावे + वृत्ते LD, + °ष्टात्तत् ιS, + °ष्टात्त T, + °ष्ट B । गुणः शून्यमित्युचितम्] शून्यमेव गुणः इत्युचितमेव β ॥ 14. अथ कृते° B, अथ प्रकृति° L ॥ 15. शून्यं गुणः ζ । हार°] हर° A ॥ 16. °ऽपि om. ζ । हारहतः सन् om. β । हरहतः A । सन्न ε ॥ 17. हरेण A ॥ 17 – 19. भाज्यते ... °गुणिते om. B ॥ 18. भवत्येव α । °स्तत्र β । इत्युचितम् om. β ॥ 19. हर° α ॥ 19 – 20. च प्र° om. ζ ॥ 20 – 21. हारेण ... क्षेपो भाज्यः] हरभक्ते केवलः (केवलध्र्व । तथा B) क्षेप (क्षेपः D, क्षे T, क्षेपं S) एव (ए B) हरेण भाजितो भवति β ॥ 20. हरेण A ॥ 21 – 22. लभ्यत इत्युपपन्नम्] लब्धिरित्युपपन्नं β (ध्विवरित्युपनपन्नं B) ॥ 23. अथ ... °रार्थं] अथानेकधा गुणलब्ध्योरानयनार्थं β । °गमनचम° A ॥ 24. इष्टाहतस्वस्वहरेणेति om. S, इष्टाहतेति β । °स्व°[1] om. N (add. N[1] in the left margin) ॥

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1 – 2. Verse 57a - b. θ ॥ 1. युक्तं S ॥ 4 – 5. उक्त° ... स्तः om. β ॥ 5. °त्र्यादि येन LD । स्वस्वहरौ] स्वहारौ R, तौ β ॥ 8 – 11. अत्रो° ... °पपन्नम्] अत्रोपपत्तिस्तु प्राक्कारिकाभिस्तु (तु om. H) निरूपितैव β ॥ 8. °नादजातं A

(°नाद्यज्चमं N, °नाद्यंतमं R) || 9. ह्यवशेष°] ह्यशेष° ε || 11. °स्वावशेषा°] °स्वगुणाव° A (°सगुणाव° R) || 12. एकविंशतियुतं शतद्वयमिति om. S, एकविंशतीति H || 13 – 16. Verse 57c - 58b. θ || 17. गुण°] गुणक° ζ || 17 – 18. संबंधः ... न्यायात्तं] अथ तं β || 17. यत्तदो° A || 18. यद्गुण°^{2} ... °द्वयं om. T || 19. °द्भूतं + सत् β || 20. सूत्रेण om. β || 21. भाज्यो ... °द्वयमेव om. T | जातः om. β || 21 – 22. हारः ... न्यासः] space in B || 21. हारः] हारस्तथा β | एवमेतेषां] चैषां β || 22. क्रमेण om. β | न्यासः + भाज्यः हारः २२१ क्षे ६५ IM || 23. भा] भाज्यः BWS || 24. हा] हारः
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Bζ (except H) || 25. अथ तेषां BL | प्रथमं यदुक्तं L, प्रथमपद्योक्तं D, space + वक्ति B ||

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1 – 2. °र्यच्छेषं ... °भाजितयो° om. β || 3. जाता om. β || 7. भजनेन β | °स्थाप्य तदधश्च] °स्थापयेदधः β | च शून्यं (च om. L) β || 17. एवमा°] एवमत्रा° β || 18. °नामानेतुं β (°नांनेतुं L, °नानेतुं DH) || 19. °र्हारौ] °स्तक्षणौ β (°स्तक्षणे B, क्षणौ M) | १७ | १५ θ | एतावेके°] एतान्येके° γ (एतेनैके° D) | सगुण्य IM, स L || 20. °गुणलब्धि° β (°लब्धिगुण° H) || 21. त्रिकोण BD || 22. एवं ... प्रकल्प्य] अथ β || 23. शतं ... नवत्येति om. S, शतं हतं येनेति β ||

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1 – 4. Verse 58c - 59b. θ || 5. पटी°] पाटी° IM, पदा° T | °यानसि] °जानमिति ε, °यान्नसि IM | संबंधः om. β || 6. पाटीयान् IM || 7. वा विवर्जितं A | त्रि° om. Tι (add. W[1] in the right margin) S | निरग्रकं LH, विरग्रं T || 8. एवमत्र om. β || 10. ह ι, हारः θ || 11. अत्रा° ... क्रियमाणे] अत्र प्रथममनपवर्तेऽपि कुट्टक (कुट्ट IM) विधिना β || 18. १ om. Tζ (add. W[1] above

line) || 19. १० om. A (except R), १ DT || 20. ० om. A (except R) || 21. नविति° A (except ε) | प्रकल्प्य तथा] प्रकल्पनया A (कल्पनया R) D, प्रकल्प्य तया BT, प्रकल्प्या तथो IM, प्रकल्प्यानेयौ L || 22. वियोगजे om. β | सूत्रक्रमेणागते गुणलब्धी om. β || 23. पुन° om. N (add. N[1] in the top margin) β | गुणलब्धी] वियोगजे β ||

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9. ० om. A (except R) T || 10. अथ] अत्र ζ || 10 – 11. °वल्ल्यां ... १८ om. D || 11 – 12. लब्ध्या प्रयोजनं नास्ति] अत्र (om. D) लब्धिर्न लभ्यते β || 12. सूत्रे om. β || 13. °त्क्रियादर्शनार्थं] °द्विषयं (°द्विषमं BT) दर्शयितुं β || 15. भाज्यः ι || 16. हारः ι || 17. °वत्° om. IM | °फल° om. ζ || 22. तथा च] उक्तवत् β | ३०] ३ ζ || 23. यतः om. β || 24. युक्ते om. ζ, space in B || 25. °णलब्धी] °णाप्ती γ (except L) ζ | १७० + वा (कं B) β ||

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1. अथो°] अथात्रो° ζ (अत्राथो° H, अत्रार्थो° IMS), अयो° T | °सूत्रे° IM | °क्रियां° Rβ | °संदर्शनार्थं] दर्शयितुं β | पुनरुदाहरणांतरमाह] उदाहरणमाह β || 2. यद्गुणा क्षयगषष्टिरिति in the bottom margin W[1], यद्गुणेति H | यद्गुणा (यद्गुणः ιS) + 19 – 21. स क्षेपो ... स क्षेपो यदा TιS | क्षयग° ... °न्वितेति om. D, क्षयगषष्टिरिति γιS || 3 – 6. Verse 59c - 60b. H || 7. भो गणक येन गुणिता त्रिभिरन्विता वर्जिता वा सती त्रयोदशहृता च सती क्षयगषष्टिर्निरग्रका स्यात् + तं H, हे गणक + तं L[1] || 7 – 18. तं गुणं ... स्व° om. D || 7. मे पृथक् H | पूर्व° ... °योजना om. H, पूर्ववदुद्देशः कार्यः (कार्यं L) β | श्लोकथ° A (श्लोकार्थ° ε) | तथा च om. β || 8 – 17. भा ६ं० ... ०] ० ιS ||

६०
१३
०

8. ६० εB || 10 – 18. अत्र ... ९ om. H || 10 – 17. अत्र ... ० om. BT || 11 – 17. ४ N || 18. उक्तवद्गु°] उक्ते गु° N, अत्रोक्तवद्गु° BTιS. | ९ illeg. in B,

१

२

०

७ δ (corr. L[1] in the text of L) || 18 – 19. अत्र ... ११]

विषमलब्धित्वात्स्वतक्षणाभ्यां शुद्धे (विष° ... °द्धे om. H) प्राग्वद्धने (प्राग्वद्ध° om. B, प्राग्वज्जाते धन° (from Bhāskara's *BG*, part of p. 32 line 18) H) भाज्ये (भाज्ये + धने क्षेपे H) गुणाप्ती ११ | ५१ (from Bhāskara's *BG*, parts of p. 32 lines 18 and 19) β || 19 – 22. अत्र ... °लब्धी] एते स्वतक्षणाभ्यामाभ्यां १३ | ६० शुद्धे जाते ऋणभाज्ये धनक्षेपे २ | ९ | एते स्वतक्षणाभ्यां शुद्धे जाते H (from Bhāskara's *BG*, parts of p. 32 lines 18, 20, 21 and p. 33 lines 1, 4, 6). || 19 – 21. स क्षेपो ... स क्षेपो यदा] after 2. यद्गुणा TιS || 19. भाज्ये + वा ιS || 20. तदा] तथा β | गुणलब्धी] गुणाप्ती β | ऋ ९ ιS, ९ Aγ (except T) | °स्यर्णगतत्वं ζ || 21. यदा om. N, यदि δζ || 22. गुणलब्धी] गुणाप्ती L | ५१ A (१२ R) γ (९१ T) | सर्वे ε, सर्व IM || 23. धनभाज्ये भवेत्त°] धनभाज्योद्भवे (धनभाज्योद्भूते T) त° β ||

25. °दाहरणांतमाह A (except ε), °दाहरणमाह β | अष्टादश ... केनेति om. S | °दश] °देश IM | केनेति] इति H ||

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1 – 2. Verse 60c - 61a. θ || 1. हताः] गुणाः S || 3. अत्र पदार्थः सुगमः] स्पष्टं β | तथा च om. β || 5. हा ११ om. B, हा १ T, हा ११ AD ζ || 6. धनत्वे β | प्रकल्प्य] क्रियमाणे (कृते LD) सति β || 6 – 13. परस्पर° ... °ज्जातौ om. β || 13. लब्धि° ... योगजौ] गुणाप्ती ८ | १४ (८ | १४ L) योगजे β || 13 – 14. अथ ... कृते] एते (om. ζ, add. W[1] in the top margin) स्वतक्षणाभ्यां शुद्धे (द्धे L, शुद्ध T, शुद्धे शुद्धे IM) सत्यौ (सत्यो IM) β || 14. गुणलब्धी] च (om. L) β | ३ | ४ TW | अत्रोदाहरणे] अथ β || 15. ऋणत्वात्] च ऋणत्वे β | ४ NBDM, १४ R || 15 – 16. यतः ... °वोक्तम्

om. β || 15. स्वं वधे] संवंधे ε || 17. °हरणमाह β || 17 – 18. येन ... पंचेति om. S || 18. पंचेति] इति H || 19 – 20. Verse 61b - 62a. θ || 21. अत्रापि ... °मत्र om. β ||

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1 – 4. १ ... ० om. ζ (except W, in the left margin W) || 5. तथा जातं राशि°] उक्तवद्राशि° β || 6. ४६ om. ζ || 7. २३ om. ζ, २६ B || 8. °तक्षणे IM || 9. °स्तष्टे + सति ζ || 9 – 10. एवं ... °त्वात् om. B || 9. °गतं] °गतमिव + प्रतिभाति (भाप्रतिभाति IMS) β || 10. इत्युक्तत्वात्] इत्युक्तत्वादतः (इत्युक्तत्यात् अत T) + ऊर्ध्वराशितक्षणे ये नव लभ्यंते ते न ग्राह्याः (om. B) तत्रापि सप्तग्राह्याः (ग्राह्यः B)। तथाकृते (तथा B, तथाकृतं IM) गुणाप्ती २।११ (२१।१ B)। एते (एतेन IM) स्वतक्षणाभ्यां (स्वतक्ष B) शुद्धे जाते वियोगजे १।६ं (१ B, १।६ Dθ, ५।६ IM) β। अतोऽत्र ... तथा हि] अथवा β। गुणलब्धी] गुणाप्ती β। तु om. L, स्तः B, तत् D, वत् T, स्तस्तु ζ || 12. भाज्यः ι || 19. स्वतक्षणाभ्यां B, स्वहरात् ζ। शुद्धे + सत्यौ (सनौ T) β || 20. गुणलब्धी om. β || 20 – 21. लब्धिः ... वर्जिते° om. β || 21. इत्येतस्यापि γ, इत्येतस्या (इत्येतेस्या S, इत्येतस्य H) अपि ζ। सूत्रस्य om. β, सूत्र N, स्वत्रस्य R। विशेषयोः T, विशेषो ζ। दर्श्यते A (except N) L, दृश्यतौ B, दृश्यतो T। हरेण° β (except L) || 22. योग°] योगज° γ (except T), योगजा° ζ || 22 – 23. ४ जाता ... °लब्धिरियं om. BL || 22. प्राकृत ε || 23. वियोगज° β (योगज° D) || 24. विवर्जिता ζ || 24 – 25. °र्लब्धिः ... भाज्य° om. ι। °ब्धिः ... भाज्यहा° om. S। ६ं ... भाज्य° om. H || 25. °हरावेतौ ζ (°रावेतौ S), °हारौ BT। ४६ om. T, ४।६ IM। २३ om. T; after p. 63, 1. °र्जातः त° N; after p. 63, 1. °र्जातः R; २।३ IM || 26. °र्लब्धिगुणौ β। ११।२ A (१।२ N), २ं।११ T ||

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1 – 4. अत्रोप° ... °पपन्नम् om. β ॥ 5. °तत्सूत्रविषयी°] °तद्विषयी° β (°ताद्वयी° IM) ॥ 6. येन ... इति om. S, space इति B ॥ 7 – 10. Verse 62b - 63b. θ ॥ 8. ते तथा S ॥ 11. अत्र ... च] स्पष्टं β ॥ 13. ह ι ॥ 14. अत्र om. β । क्षेपाभावेति ζ (क्षेपाभावेपि H) ॥ 15. °नैक°] °ना एव (एवं B, एंव D, त above line एव L[1], एक° W) β ॥ 15 – 19. °गुणाहार° ... जातौ om. D ॥ 15. °गुण° + एव B । °हर° α । गुणाप्ती] गुणा ζ (except H, प्ती add. W[1] in the left margin) । ५।१३ AL (५ om. N) ॥ 16. अथ + द्वितीयोदाहरणे β ॥ 19. अत्र] अथ θ । हारो° ιH । शुध्येदिति Bθ, शुद्धेदिति Tι । कृत्वा om. BTζ । सन् om. β ॥ 20. फलं भवती°] फलमि° β । फलं[2]] फलत्वं IM ॥

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2. एवं ... °दानीं] अथ β । °कुट्टकसि° β ॥ 3. क्षेपं ... °मिति om. θ, क्षेपं विशुद्धिमिति γι ॥ 4 – 7. Verse 63c - 64b. θ ॥ 5. तयोर्ये] पृथग्ये H ॥ 7. °योस्ते + इति H ॥ 8. रूपं (रू५पं IM) क्षेपं β । प्रकल्प्य L, परिकल्प IM । तयोः] तया (तथा S) α । पृथग् om. β । गुणकारलब्धी] गुणाप्ती B ॥ 8 – 9. °स्ते ... °न्वयः om. T ॥ 9. पृथगभीप्सित (पृथगभीत B) β । सत्यौ + अथवा ते γ, + अवाते IMS, + अवाप्ते W, + आगते H । भवेतामिति] भवतः β । दंडान्वयः om. β ॥ 10. °गत° om. β (except D) ॥ 11. एवमत्र] अत्र β ॥ 14. तथा om. β (उक्तवत् L) । फलवल्ली β ॥ 19. उक्तवद् + गुण°[1] γ । गुणलब्धी[1]] गुणाप्ती ζ । ८।७ A (८१७ R) γ (८।१ D), ७ ζ । सत्यौ गुणलब्धी[2] om. β ॥ 21. गुणलब्धी ... १०] om. γ । गुणलब्धी निघ्ने] निघ्ने (विघ्ने IM) गुणलब्धी ८।१ जाते ζ । निघ्ने] निघ्ने N, गुनिघ्ने R । च om. Rζ । जाते ζ । लब्धिगुणौ] विशुद्धिजे ζ (विशुद्धिद्विजे IM) । ११।१०] १०।११ ζ । १० + धनक्षेपे (om. γ) लब्धिगुणौ इष्ट ५ (इष्ट ५ om. γ, अभीष्ट ५ H) निघ्नौ (निघ्नौ D, घ्नौ WIθ, घ्नौ M) (+ लब्धिगुणौ ζ) ४०।३५ (२५ T)

स्वहारतष्टौ च (च om. ζ) जातौ (क्षातौ D) लब्धिगुणौ (लब्धिगुणौ om. S) ६।५ β; (४०।३५ ... ६।५ borrowed from Sūryadāsa's *GMK*, with minor changes (see *Wai, PPM 9762*, f. 120v., 9)) || 22. कल्पिते + वा W, + भा IMS || 23. कृते om. β (except L) || 24. यथावस्थि° ζ (except W), यास्थि° R । अत्रोक्त° ζ । तौ om. BM, त I || 24 – p. 65, 6. प्राचीन° ... °पन्नम्] Appendix #16. β ||

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2. °निघ्ने ε || 3. स्वहार° ε || 4. °सिद्धेस्तथा A (°सिद्धये तथा R) || 7. इदानीं] अथ β । कुट्टक° ... तेनैव] कुट्टकेन विकलावशेषात् (विशेषात् B) β । ग्रहानयनार्थं β || 8. °र्विकलावशेषमिति] °रिति β । °वृत्तेनाह BDS || 9 – 14. Verses 64c - 65d. H || 15. अथेत्यनंतरं β (अथेत्यन T) || 16. हारः एवं] हाराव IM, हारावं S । एवं om. W || 16 – 17. अत्र ... °वोक्तम् om. β || 17. ततस्तज्ज्ञं] तत्र जातमुत्प (°मुपप° ζ) न्नं β || 18. कला°] फला° IMS । अथ] ततः β || 19. च om. LTζ । कुट्टकविधिना om. β || 20 – 22. कला ... लब्धि° om. ε || 20. कला] ला A (om. ε) । स्यात् om. H, भागशेषं β (भोगशेषं S) । अथ लवाग्रं²] पुनस्तदेव β || 21. प्रकल्प्य om. β । च om. Lζ, चे D || 21 – 22. पुनः ... भागाः] तज्ज्ञं (त + space in B) फलं लवाः (फलं लवाः om. M, ल° om. I) β || 22 – 23. गुणस्तु ... हारः om. M || 22. स्यात् om. β । अथ राश्यपेक्षायां] तदेव शुद्धिः β || 22 – 23. द्वादश ... हारः om. ζ (द्वादश भाज्यः कुदिनानि हारः add. W¹ in the bottom margin) || 22 – 23. राश्यग्रं² क्षेपशुद्धिः om. βW¹ || 23. च om. βW¹ || 23 – 24. तत्रा° ... °द्यानेयम्] Appendix #17. β (borrowed from Sūryadāsa's *GMK*, with minor changes (see *Wai, PPM 9762*, f. 121r., 5 – 8.)) ||

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1. एवमत्रो°] अत्रो° β । क्वहतो B, द्युहृतो T, क्कहृतो IM || 2 – 20. इति ... °वशेषमिति] Appendix #18. β (borrowed from Sūryadāsa's *GMK*,

with a few alterations (see *Wai, PPM 9762*, f. 121r., 9 – f. 121v., 5.)) || 7 – 8. भागा ... तावत् om. N || 8. संगुर्ण्यं ε || 18. विकलावशेषं[1] om. ε | विकलावशेषम°[2]] विकला° N, किविकलांवशेषत° R || 20. °विकिला° A (except ε) || 21. तस्य om. L, + तस्य ζ || 22. लोम्येन B, वैलोमेन L, वेलोम्येन D, वैलोम्ये T, वैलोम्य ζ || 23. भाज्य°[2]] भाग° Aγ || 24. कलावशेषं] कलाशेषं β (except B) | यतः] अतः ζ || 25. °रुत्पन्ना सा om. N | सा] ता α ||

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3. °र्थमु°] °र्थं स्वकल्पनो° β (°र्थं कल्पनो° IM, स्व add. I[1] in the left margin) | °दाहरत्वेन ζ (except H) | निरूप्येते N, निनूप्यतो T, मनिरूप्यते I, मनिरूप्यते M || 3 – 4. तत्र ... साध्यते om. β || 4. एवमत्र] तथा चात्र β || 5. ग्रह° ... °ल्पिताः] कल्पिता अभीष्टग्रहभगणाः β | अथ + द्युचरचक्रहतो दिनसंचयः इति β | ज्ञातो ε | भगणा°] यगणा° N, भगरमा° R, भगण° T, भागा IMS || 7 – 11. ० ... ४३ om. W, add. W[1] in the right margin || 7. ० om. ND, १ Rζ (except H) || 11. ४१ ε || 12. विकलाशेषं NDTH, कलाशेषं ιS, विशेषं B || 13. ० om. DT | ७ BD, ३ H, ३ S || 15 – 20. अत्र ... वियोगजे] प्राग्वज्जातौ लब्धिगुणौ β || 20. ४३ ... अत्र om. B | ४३] ४ | ३ IM | ८ om. S, १८ R || 21. ४३ ... °लाः] जाता विकलाः ४३ β || 22. कला°[1]] कल्प° IMS | °नयनार्थं] °ज्ञानार्थ R, °ज्ञानार्थं β | हा ११ om. εβ | °स्तु ... ८ om. B | कलाशेषं ζ (कालाशेष S) | ८ om. L | इमां β | प्रकल्प IM || 23. यष्टिर् N, षष्टिश्च L | इत्यादिवा R, इत्यादि IM || 26. अत्र om. β | लब्धिः[2]] लब्धिरियं ३२ (अ T) β | जाताः om. β | गुणो हि] गुणस्तु β ||

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1. इमं] दअं N, तमेव β । भाज्यं + कुदिनानि हारं (हरः D) β । प्रकल्प्य^{2}] कृत्वा β ॥ 2. ६० R, ६ NBDIMH ॥ 4. °ज्जातौ Rβ । लब्धिगुणौ २४।९ β । जाता om. εβ । इमां LDζ (except S), एवं B ॥ 5. शुद्धि प्रकल्प्य β (शुद्ध + प्रकल्प IM) । द्वादशमितं β । पुनः + कुट्टकार्थं β ॥ 8 – 12. अत्र^{1} ... इति om. β ॥ 11. यो ε ॥ 12. अत्र β । ९^{1} om. DT, illeg. M, ९ं H । ९^{2} om. DH, ९ं N । इमां BLζ, इयं R, इसं T ॥ 13. तथा om. β । °भगणा β । भाज्यं कुदि° β । प्रकल्प्य] कृत्वा β । च om. Rβ ॥ 14. ९ εβ (except LT) ॥ 16 – 21. फल° ... ० om. β ॥ 19. ० A ॥ 22. उक्तवल्लब्धिगुणौ (°गुणैः S) ०।३ (व ३ B, ३ D) β । ०^{2} om. RBS । गुणो° ... ३ om. A ॥ 23 – 25. अत्रो° ... °मित्य° om. β ॥ 25. °स्त्वधि°] °लब्धि° ε ॥ 25 – 26. °मित्यल्पमति°] एवमल्पमति° β (एवमति° B, एवमल्पमिति° S, ९वमल्पमिति° IM) ॥ 26. °विल्पेण R, °विस्तारेण Lι ॥

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2. °श्चेदिति β (i.e. गुणकौ विभिन्नौ om. β) ॥ 3 – 6. Verse 66a - d. H ॥ 7. यदाहाको N, चेदेको β (except L) । च om. γ (except T) । °रैक्यं + स्यात् (स्यात् om. L) स (स om. BLD, व T) β ॥ 8. °क्यम°^{1}] °वयम° IM । क्षेपं] शेषं β (except LD, corr. W^{1} in the text of W) । °ऽनु°] °चनु° ε (°चतु° R), ह्यनु° γ, स्वनु° ζ (ह्यनु° W, space + नु° H) ॥ 9. ज्ञेयः om. IM । °संज्ञे ε, °संस D, °संज्ञकः Tζ । स्फुट° om. β ॥ 10. संयोग° (संयोगि° IM) °विशेषः (°विषमः L) β ॥ 11 – 12. गुणका° ... °वोक्तम्] Appendix #19. β ॥ 13. अत्रो° ... इति part of Appendix #19. ॥ 14 – 17. Verse 67a - d. H ॥ 18. स्पष्टम् part of Appendix #19. ॥ 19. तथात्र om. β ॥ 20. श्रे^{1,2}] क्षे^{1,2} β ॥

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1. सूत्रेण β । क्षेपं ... प्रकल्प्य] च (om. B) ऋणक्षेपं (+ च D, हणक्षेपं IM) कृत्वा (om. B) β ‖ 1 – 3. न्यासः ... ६३ om. B ‖ 2 – 3. भा ... ६३ add. A in the bottom margin (except ε) ‖ 2. भा om. S, गु DT । २१ A (११ R), ११ D ‖ 3. हा ६३ om. BS ‖ 5. ७ BD, ० R ‖ 6. २१] ६३ β (except L, ११ D) ‖ 7. ३] ३३ ι (except W), २३ θ । ३ + अत्र (तत्र H) गुणः (गुणकः ζ) १४ अयमेव राशिः अस्मिन्नालाप° (°न्नाफला° B) °द्वयं (°द्वय IM) घटते (घटतः T) श्री (om. γH) β । एवमनेकधा om. β ‖ 8. °त्मकजकवि° (except R) ‖ 9. °शेषभूषाविशेषे] °नंदसंदोहहेतौ (°नंदसंदोहहोतौ B) β ‖ 10. सम्य IM । °बटु°] °पटु° N, °वटु° Rγ (°षदु° D) ‖ 11. जाता ζ ‖ 12 – 13. इति ... °मगात् om. β ‖ 12. श्रीदैवज्ञ° ε ‖

CHAPTER IV

APPENDICES

TO

THE SANSKRIT *TEXT ALPHA*

The Apparatus Criticus (Chapter III) for the *Text Alpha* refers to a total of nineteen appendices. The first section of the present chapter consists of the texts for these appendices. The second section contains an apparatus criticus for each appendix. The place in Chapter III where the first reference to a particular appendix is made is indicated at the beginning of the apparatus criticus to that appendix.

1. *Appendices*

#1. यद्वा अंतरं हि योगाद्विपरीतम्। यतः पूर्वं स्वयोरस्वयोर्वा योगे युतिस्तथा धनर्णयोगेऽंतरमभूदिदानीं तद्वैलोम्येन स्वयोरस्वयोर्वांतरेऽंतरमेव भवति। धनर्णयोश्चांतरे क्रियमाणे योगो भवत्यतः <u>संशोध्यमानं स्वमृणत्वमेती</u>त्याद्युपपन्नम् ॥

#2. अत्रोपपत्तिः। यस्य कस्याप्यंकस्य शून्ययोगे वियोगे च कृते धनर्णत्वं यथा पूर्वमेव तिष्ठति। अथ <u>च्युतं शून्यत</u> इति। "संशोध्यमानं स्वमृणत्वमेती"त्युक्तत्वाद्धनर्णवैपरीत्यं प्राप्नोतीत्युपपन्नम् ॥

#3. अथ द्वित्र्यादिकानामिति। तत्र समयोर्द्वयोर्घाते वर्गस्तथा त्रयाणां घाते घन इति प्रसिद्धमेव। तथा भावितं तु भिन्नवर्णगुणनोपलक्षणं यथाज्ञातकोटिभुजे समचतुरस्रे क्षेत्रे भुजकोटी वर्णमाने प्रकल्प्य तत्र फलं भाविताख्यमुत्पद्यते। तदेव तत्स्वरूपमित्युपपन्नम् ॥

#4. अत्रोपपत्ति⌊स्तु⌋ गुण⌊न⌋सूत्रवैलोम्येन ज्ञेय⌊ा स्पष्ट⌋त्वान्न लिखिता ॥

#5. धनर्णयोरंतरमेव योगः इत्यंतरमेवेत्युपपन्नम् ॥

तथा च

करण्योः समयोर्योगे एका कार्या चतुर्गुणा।
तयोरेवांतरं शून्यम् ॥

इत्यस्मद्गणितोक्तसूत्रक्रमाद्यासामंतरमुपपन्नं ताः करण्यो गताः ॥

#6. योगं कृत्वा वर्गं संस्थाप्य ता एकादिसंकलितमिताः करण्यः स्युः। ननु यत्र द्वित्रिचतुरादिस्थानस्थितानां तुल्यकरणीनां वर्गे क्रियमाणे ये सहजा वर्गराशयस्तावतामेव मूलैक्यं रूपाणि प्रकल्पयेदित्यर्थः। निमित्तजास्तु करण्य एव कल्प्याः। तासां यथासंभवं योगे क्रियमाणे

#7. अत्र क्रियानिर्वाह[मात्रमेवो]पपत्तिः। अन्यथा "न मूलं क्षयस्यास्ति तस्याकृतित्वादे"तत्सूत्रमनर्थकं [स्]यादित्यर्थः ॥

#8. युतोनितान्यर्धितानि च कृत्वा जातं करणीद्वयं क २ क ८ । अनयोर्महती ऋणं ८̇ । एतान्येव रूपाणि कृत्वोक्तवज्जातं करणीद्वयं क ५ क ३ । अत्रैका महत्यृणम्। एवं क्रमेण प्रथमोदाहरणे जातमूलकरणीनां न्यासः क २ क ३ क ५̇ ॥

अथ द्वितीयोदाहरणे मूलकरणीनामानयनार्थं वर्गोऽयं रू १० क २४ क ४०̇ क ६०̇ । अत्रापि प्राग्वदृणकरण्यौ क ५ क ५̇ । तत्रैकर्णी प्रकल्प्या। तत्र या धनगता सा मूलकरणी यर्णगता सा रूपाणि कृत्वा न्यासः क ५̇ क २४ । अत्रापि पूर्ववज्जाते ऋणकरण्यौ क ३̇ क २̇ । यत ऋणोत्पन्ने करणीखण्डे ऋण एव । एवं मूलस्य न्यासः क २̇ क ३̇ क ५ ॥

#9. अथ यद्युदाहरणे मूलार्थमुपदिष्टे करणीवर्गराशावेकादिसंकलितमिति सूत्रनियमितानि करणीखण्डानि <न> भवंति यथोक्ताधिकन्यूनानि च स्युस्तदा संयोज्य करणीं विश्लेष्य वा तावंति खण्डानि कृत्वा मूलं ग्राह्यमित्यर्थः ॥

#10. चेदन्यथा स्यात्संभवदभिप्रायेण तन्मूलं न लभ्यते तर्हि तत्र सत् दुष्टोद्दिष्टमित्यर्थः ॥

#11. अथापवर्त्तादपि या लब्धा करण्यस्ता यदि मूले करण्यो न भवंति तदा शेषविधिना कार्याः ता अपि न चेत्स्युः तदा मूलं सन्न भवतीत्यादि

#12. परं तु शेषविधिना मूलकरणयौ नोत्पद्येते, किंतु अन्यदेव मूलं लभ्यते। तदेतत् क २ क ३ क ५ । एवमुद्दिष्टवर्गो ह्यस्य न भवतीति कृत्वा अपवर्त्तादपि लब्धं मूलकरणयो भवति। ता नोचेदित्येतदुदाहृतं भवतीत्यर्थः ॥

#13. वर्गोऽयं रू २३ क ८ क ८० क १६० । एतादृक्स्थले करणीनामासन्नमूलान्यानेयानीत्याह। एवंविधेष्विति ॥ स्पष्टम् ॥

#14. तथा च युतिभाजकापवर्त्तपक्षे लब्धिस्तु यथावस्थिता। गुणो न पारमार्थिकः। स चापवर्त्तनेन गुणः सन् पारमार्थिको भवति। तथैव युतिभाज्यापवर्त्तपक्षे गुणस्तु यथावस्थितः। लब्धिर्न पारमार्थिकी। सा चापवर्त्तनसंगुणा सती लब्धिर्भवतीति ज्ञेयम्॥

#15. अथ धनभाज्योद्भवे स्यातां। धनभाज्यवदुद्भवो ययोस्ते धनभाज्योद्भवे। भाज्यभाजकक्षेपाणामृणगतत्वेऽपि धनगतत्वं प्रकल्प्य गुणलब्धी साध्ये इत्यर्थः ॥

हरतष्टे इति ॥

<हरतष्टे धनक्षेपे गुणलब्धी तु पूर्ववत् ॥५३a॥>

धनक्षेपं हरेण विभज्य यच्छेषं तन्मितं क्षेपं कृत्वा पूर्ववद्गुणलब्धी साध्ये ॥

अथ क्षेपतक्षणेति ॥

<क्षेपतक्षणलाभाढ्या लब्धिः शुद्धौ तु वर्जिता ॥५४b॥>

हारेण क्षेपस्य तक्षणे कृते यो लाभो लब्धिस्तया कुट्टकागतलब्धिर्युता सती लब्धिर्भवति। अथ शुद्धौ ऋणक्षेपे सति तया लब्ध्या कुट्टकागतलब्धिर्वर्जिता सती लब्धिर्भवतीत्यर्थः ॥

अथवेति ॥

अथवा क्षेपभाज्ययोर्भागहारेण तष्टयोः सतोः प्राग्वद्गुणो भवति। क्षेपभाज्यौ हारेण विभज्य शेषमितौ च क्षेपभाज्यौ प्रकल्प्य पूर्ववत्कुट्टकविधिना गुण एव भवति। गुणकेन हतात्क्षेपेण युताद्धारेण हृताच्च भाज्याल्लब्धिरानेयेत्यर्थः ॥

अत्रोपपत्तिः। तत्र पूर्वोत्पन्नराशियुग्मे दृढभाज्यहाराभ्यां तष्टव्ये। सममेव लब्धमुभयत्र ग्राह्यम्। यतः "इष्टाहत स्वस्वहरेणे"त्यादिना गुणलब्धिसाधने भाज्यहारावेकेनैवेष्टेन गुणयित्वा योज्येते। अथ यदा राशियुग्ममितौ लब्धिगुणौ भवतस्तदा तक्षणलब्धिरेवेष्टः कल्पितो भवति। तेनोभयत्र तुल्येनैव भाव्यम् ॥

अथ योगज इति। "उपांतिमेन स्वोर्ध्वे हत" इत्यादिना ऋणक्षेपेण फलवल्ल्या गुणने क्रियमाणे यद्राशियुग्मं तदृणगतमेव स्यात्। अतो लब्धिगुणावृणगतावेव स्याताम्। तत "इष्टाहत स्वस्वहरेणे"त्यादिना स्वतक्षणयोगे क्रियमाणे "धनर्णयोरंतरमेव योग" इति कृत्वा लब्धिगुणौ स्वतक्षणाभ्यां शोधितौ भवतः। अथ "धनभाज्योद्भवे तद्वदि"ति। भाज्यभाजकक्षेपाणामन्यतमस्य ऋणत्वे फलवल्ल्या ऋणत्वं भवति। तथैव गुणलब्ध्योरपि। एवं कृते महानायासः स्यात्। अतो धनक्षेपाभिप्रायेण लब्धिगुणावानीय ततः स्वतक्षणाच्छोध्यौ। तथा च भाजके ऋणे सति लब्धिर्ऋणा कल्प्या "भागहारेऽपि चैवं निरुक्तम्" इत्युक्तत्वात्। तदुक्तम् ॥

ऋणभाज्ये ऋणक्षेपे धनभाज्यविधिर्भवेदिति ॥

अथ "हरतष्ट" इति विशिष्टक्षेपगुणितायाः फलवल्ल्याः सकाशाद्यद्राशियुग्मं तत्राधः स्थितो राशिर्हरेण तष्टः क्रियते। अथवा क्षेपमेव हरतष्टं विधाय तत्र यच्छेषं तन्मितेन क्षेपेणोक्तविधौ क्रियमाणेऽपि गुणे साम्यमेव भवति। अथ "क्षेपतक्षणे"ति पूर्वक्षेपाद्यद्गुणो हरः शोधित आसीत् तावन्न्यूना लब्धिरागता। अतः क्षेपतक्षणेन यत्फलं तेन चेल्लब्धिर्योज्यते तर्हि लब्धिर्भवति ॥

अथवेति ॥

<अथवा भागहारेण तष्टयोः क्षेपभाज्ययोः।

गुणः प्राग्वत्ततो लब्धिर्भाज्याद्धतयुतोद्धृतात् ॥५५॥>

गुणस्य हरांतःपातित्वाद्धरे त्वविकृते गुणोऽप्यविकृत एव। अथ भाज्यस्य विकृतत्वाल्लब्धिरपि विकारं प्राप्ता। अतः सा च गुणकादानीयत इत्युपपन्नम् ॥

#16. चेदभीष्टक्षेपेण गुणयेते तर्ह्यभीष्टक्षेपजौ भवतः। अतोऽत्र फलितोऽनुपातः। यद्येकमितक्षेपेणैतौ लब्धिगुणौ तर्ह्यभीष्टेन काविल्यत्राभीष्टक्षेपनिघ्नौ यौ तावेव लब्धिगुणौ भवतः। परं तु तल्लघ्वीकरणार्थमुक्तं स्वहारतष्टे इति। सर्वमुपपन्नम् ॥

#17. तज्जं फलं राशयो गुणो भगणशेषम्। तदेव शुद्धिः कल्पभगणा भाज्यः कुदिनानि हारः। तज्जं फलं भगणाः गुणोऽहर्गणः स्यादित्यादि। एवममुना प्रकारेण तदूर्ध्वमहर्गणादूर्ध्वमधिमासावमाग्रकाभ्यां अधिमासावमशेषाभ्यां रवींद्वोर्दिवसाः साध्याः। यथा कल्पाधिमासा भाज्यो रविदिनानि हारोऽधिमासशेषं शुद्धिर्लब्धिर्गताधिमासा गुणो गतरविदिवसाः। एवं कल्पावमानि भाज्यश्चंद्रदिवसा हारोऽवमशेषं शुद्धिः। फलं गतावमानि। गुणो गतचंद्रदिवसा इति ॥

#18. इत्यादिना इष्टग्रहसाधनार्थमिष्टाहर्गणात् ग्रहभगणहतात् कुदिनभक्तात्फलं भगणाः। शेषं द्वादशभिः संगुण्य कुदिनैर्भजनेन लब्धं राशयः। शेषं त्रिंशता संगुण्य पुनः कुदिनैर्लब्धं भागाः। शेषं षष्ट्या संगुण्य कुदिनैर्लब्धं कलाः। शेषं षष्ट्या संगुण्य कुदिनैर्लब्धं विकलाः। तत्र यच्छेषं तद्विकलावशेषम्। तस्मात्कुट्टकविधिना ग्रहः साध्यते। तत्र कलावशेषं षष्ट्या संगुण्य कुदिनैर्भक्तं लब्धं विकलाः। तत्र शेषं विकलावशेषं जातमासीत्। अथास्मादेव विकलाकलावशेषयोः ज्ञानापेक्षायामिदमेव ऋणक्षेपकः कल्पितो यतः पूर्वभाज्याद्भाजकेन भागे ह्रियमाणे विकलावशेषत्वेनाधिकर्तुमुर्वरितमासीत्। इदानीमस्मिन् गुणकगुणिताद्भाज्याच्छोधिते भाज्यो हि भागहरणे निःशेषो भवतीत्यत उक्तं कल्प्याथ शुद्धिर्विकलावशेषमिति ॥

#19. यत्र गुणको ज्ञातस्तत्र भाज्यो ह्यज्ञातस्तत्र कुट्टकविधिना यो गुण उत्पद्यते स एव भाज्यो भवति। यतो भाज्यादिष्टगुणे हरे शोधिते सति यदवशेषं तत्प्रकृते ज्ञातमस्ति। तच्चेदिष्टगुणो हरे योज्यते तर्हि गुणकभाज्ययोर्घातो ज्ञायते। स चेद्गुणकेन ह्रियते तर्हि भाज्यो ज्ञायते। अथ विलोमप्रकारेण गुणकमेव भाज्यं

प्रकल्प्य शेषं च शुद्धिं कृत्वा कुट्टकविधिना यो गुणः स एव भाज्यः। यथात्रैव हारः ६३ शेषं ७ गुणः ५ । अथ शेषमेकगुणं हरे योज्य जातो गुणकभाज्ययोर्घातः ७० । अस्मिन् गुणकेन हृते लब्धो भाज्यः १४ कुट्टकादयमेकगुणः स्यात्।

अथवा हारः ६३ शेषं १४ गुणः १० । अथ शेषं द्विगुणं हारे योज्य जातो भाज्यगुणकयोर्घातः १४० । अस्मिन् गुणकेन हृते लब्धो भाज्यः। स एव १४ । अत्रापि कुट्टकात्स एव गुणः १४ । अतो लाघवार्थं गुणयोगो भाज्यः कृतः। शेषयोर्योगः शुद्धिः। हारस्तु सम एव। तत्र कुट्टकेन गुणश्च स एवेत्युपपन्नम् ॥

अत्रोदाहरणमाह। कः पञ्चनिघ्नः इति ॥

<कः पञ्चनिघ्नो विहृतस्त्रिषष्ट्या
सप्तावशेषोऽथ स एव राशिः।
दशाहतः स्याद्विहृतस्त्रिषष्ट्या
चतुर्दशाग्रो वद राशिमेनम् ॥६७॥>

स्पष्टम् ॥

2. *Apparatus Criticus to the Above Appendices*

Appendix #1. (See App. Crit. for p. 10, 4 of Text Alpha.)

Line 1. यद्वा ... योगे om. B || Line 2. °योगेऽन्तर°] °योगोऽन्तर° B, °योर्योगोऽन्तर° L, °योर्योगेऽन्तर° T | °स्स्वयोवर्ति°] °स्स्वयोरंत° D, °वर्ति° Lθ (except W) ||

Appendix #2. (See App. Crit. for p. 12, 17 – 18 of Text Alpha.)

Line 1. कृते om. ζ ||

Appendices #3–4. (See App. Crit. for p. 17, 5 – 7 and p. 19, 15 – 22 of Text Alpha.)

Nil apparatus criticus.

Appendix #5. (See App. Crit. for p. 33, 8 of Text Alpha.)

Line 1. °मेवोत्युपपन्नं IM, °मेवेप्रपन्नं W || Line 3. °र्योग DT | कार्या + च LT || Line 5. °द्रूणनोक्त° BT, °द्रुणनोक्त° ζ (°द्रुरुणोक्त° H) | °मुत्पन्नं LD | ताः करणयो गताः om. M ||

Appendix #6. (See App. Crit. for p. 36, 7 – 8 of Text Alpha.)

Line 1. योगं om. B | वर्गे δ | °संकल्पित° T, °संमूलित° D || Lines 2 – 4. ये सहजा ... क्रियमाणे om. L || Line 2. ये om. T, जे B ||

Appendix #7. (See App. Crit. for p. 39, 17 – 22 of Text Alpha.)

Nil apparatus criticus.

Appendix #8. (See App. Crit. for p. 40, 23 – p. 41, 4 of Text Alpha.)

Line 1. कृत्वा + न्यासः रूप ५ क २४ । पूर्ववत्करणयो क ३ क २ । अथवा न्यासः रू १० क २४ क ४० क ६० । अत्रापि तुल्यानि रूपाणि रूपकृतेरपास्य शेषं ३६ । अस्य मूलेन रूपाणि युतोनितानि अर्धितानि च कृत्वा L || Line 2. ८ BDM || Lines 2 – 3. °द्वयं ... °करणी° om. B || Line 3. ३] ५ δζ || Line 4. ५ BDM || Lines 5 – 6. क ४० क ६० β || Line 6. °न्वद्गुण° T, °न्वद्गुणक° L, °न्वदगा° B | °कर° + °णयोस्तुल्यानि रूपाणि १०० एतानि रूपकृते १०० रपास्य शेषं ० अस्य पदेन रूपाणि युतोनितान्यर्द्धितानि च कृत्वा जाते कर° L | ५̇] ५ BDM || Line 7. क^{1} om. B | ५ BDM || Line 8. ३ DM, इ B | २ BDM | यतोत्पन्ने D, यतत्पन्ने T | ऋण] ऋणे BLH || Line 9. क २ क ३ BDM ||

Appendix #9. (See App. Crit. for p. 41, 19 of Text Alpha.)

Lines 1 – 2. अथ ... भवति यथा om. LT || Line 1. यदि] णदि ζ (य in the left margin W^{1}) || Line 2. न भवति यथोक्ता°] space in B | न in the left margin W^{1} (from Kṛṣṇa's *BP*, p. 77, 5) | भवंत्यथ उक्ता° ζ || Lines 2 – 3. भवंतीत्यर्थः (i.e. यथो° ... ग्राह्यम्° om.) D | °तदा ... °त्यर्थः from Kṛṣṇa's *BP*, with a few additions (see p. 77, 5-6) || Line 3. ताव + space in B, यावंति ζ (corrected to तावंति W^{1}) ||

Appendix #10. (See App. Crit. for p. 41, 23 – p. 42, 4 of Text Alpha.)

Nil apparatus criticus.

Appendix #11. (See App. Crit. for p. 42, 10 – 17 of Text Alpha.)

Line 1. मूले] मूल N ||

Appendix #12. (*See App. Crit. for p. 44, 13 of Text Alpha.*)

Line 1. परं शेषविधिना मूलकरणयौ नोत्पद्येते from Bhāskara's *BG*, p. 24, 17 । नोपद्येते ι, नोपपद्यते S, नोपपद्येते H ॥

Appendix #13. (*See App. Crit. for p. 45, 7 – 8 of Text Alpha.*)

Line 1. वर्गोऽयं ... १६० from Bhāskara's *BG*, p. 25, 10-11 ॥ Lines 1 – 2. २३ ... °विधेष्वि° om. B ॥ Line 2. °नीयाह IM । एवंविधेष्विति] एवंविधे वर्गे करणीनामासन्नमूलकरणेन मूलान्यानीय रूपेषु प्रक्षिप्य मूलं वाच्यमिति तद्रूपसंख्याकाः करणयो मूलमित्यर्थः (from Kṛṣṇa's *BP*, p. 82, 20-22) S; एवंविधेष्विति + एवंविधेषु वर्गेषु करणीनामासन्नमूलकरणेन मूलान्यानीय रूपेषु प्रक्षिप्य मूलं वाच्यं (from Bhāskara's *BG*, p. 25, 12-14 or from Kṛṣṇa's *BP*, p. 82, 20-21; with slight modifications) + अत्र महती रूपाणीत्युपलक्षणं ततः क्वचिदल्पापि (from Bhāskara's *BG*, p. 25, 14-15, with slight modifications) H । स्पष्टम्] शेषं स्पष्टम् (from Kṛṣṇa's *BP*, p. 82, 22) S ॥

Appendix #14. (*See App. Crit. for p. 52, 14 of Text Alpha.*)

Line 1. तथा च ... गुणो om. B । तथा च ... °कापवर्त्त° om. L ॥ Lines 1 – 3. यथावस्थिता ... गुणास्तु om. T ॥ Line 2. चापवर्त्तने WS । भवतीति ζ ॥ Line 3. °पवर्गपक्षे IM । °मार्थिका γ (°मार्थिकः T) ॥ Lines 3 – 4. °पवर्त्तनसंगुणा ... ज्ञेयम् om. B ॥

Appendix #15. (*See App. Crit. for p. 53, 14 – p. 54, 19 of Text Alpha.*)

Line 1. अथ om. BT । धनभाज्योद्भवे[1]] दृढभाज्योद्भवे (space + भाज्योद्भवे B) + तद्वदीति (तद्वदति तद्वदति B) । तद्वत् ऋणभाजके सति तथा हरतष्टे धनक्षेपे सति (ध + space + त B) गुणलब्धी पूर्ववत् (पूर्वत्व B) + धनभाज्योद्भवे BT ॥ Line 2. °मृणागत°] °मृण° W (गते add. W[1] in the top margin), °मृणागते° IM ॥ Line 3. हरतष्टे इति om. S ॥ Line 4. Verse 53a. S, +

54b. S (S has 53a. and 54b. together; H has 53b., 54a., 53a., 54b., and 55a - b. together, see Apparatus Criticus for *Text Alpha*, p. 53, line 1) । त पर्वत S ॥ Line 5. धन° ... साध्ये om. B । क्षेप²] पेक्षं IM ॥ Line 7. Verse 54b. θ (see line 4 above) । वर्जितेति S ॥ Line 9. °क्षेपे] °पेक्षे IM ॥ Line 11. अथवेति] अथ शुद्धौ अणाक्षेपे सति तथा लब्ध्या कावेति ιS ॥ Line 12. °र्भागहरेण L, °र्भागहरणे B ॥ Line 14. हृताच्च] भक्ताच्च W, भक्ता IM, भक्त S, भक्तात् H ॥ Lines 15 – 18. अत्रोपपत्तिः ... भाव्यम् borrowed from Sūryadāsa's *GMK* (see *Wai, PPM 9762*, f. 119r., 1-4) ॥ Line 16. °हरे°] °हारे° BD, °हर° IMS ॥ Line 17. योज्यते TW ॥ Line 24. ऋणात्वं] गुणात्वं γ । °लब्धे° IM । माहानाया।।स IM ॥ Line 25. °छ्योध्ये β (°छ्योध्यो T, °छेध्ये IM) ॥ Line 26. °र्ऋणा] °र्ऋणां β । प्रकल्या L, कल्पा IM ॥ Line 28. ऋणे भाज्ये ऋणे क्षेपे β ॥ Lines 29 – 33. अथ ... °र्भवति borrowed from Sūryadāsa's *GMK* (see *Wai, PPM 9762*, f. 119r., 4-7) ॥ Line 30. °हारिण LD ॥ Line 32. हारः γ । असीत् + व IM ॥ Line 34. अथवेति om. S ॥ Lines 35 – 36. Verse 55a - b. S (H has 53b., 54a., 53a., 54b., and 55a - b. together) ॥ Line 36. °तात्] °तादिति S ॥ Line 38. विकृत°] विकृति° Wθ ॥

Appendix #16. (See App. Crit. for p. 64, 24 – p. 65, 6 of Text Alpha.)

Lines 1 – 2. चेद° ... लब्धिगुणौ om. IM ॥ Line 1. °क्षेपेण गुणयेते तर्ह्यभीष्ट° om. ζ । गुणयते BT । तर्हि स्वाभीष्ट° L ॥ Lines 1 – 2. °जौ ... °त्राभीष्टक्षेप° om. B ॥ Line 1. अतोऽत्र] यतोऽत्र LD, यतोत्तत्र T । फलितानुपातः LT । फलतोनुपातः D ॥ Line 2. °क्षेपेण तौ H । लब्धिगुणयै S । तर्हि ह्यभीष्टेन LD । कावित्य°] कश्चित्य° T, कात्य° S ॥ Line 3. तु om. D । तल्लघ्वी°] लघ्वी° B, तत् लब्धि° D, घ्वी° T, तल्लब्धी° IMS । स्वहार°] स्तहार° D, स्वहा° I, स्वाहा° M ॥

Appendix #17. (See App. Crit. for p. 65, 23 – 24 of Text Alpha.)

Lines 1 – 2. तज्जं ... तज्जं om. M || Line 1. तज्जं ... शुद्धिः om. ζ (add. W^1 in the bottom margin) || Lines 1 – 6. तदेव ... इति borrowed from Sūryadāsa's *GMK* (see *Wai, PPM 9762*, f. 121r., 5-8) || Line 1. कल्प°] कल्प्या LH, कल्प्य DTS || Line 2. एवमगुना D, एवममुक ιS, एवमुक्त H || Line 4. कल्पाद्धिवमासा B, कल्प्याधिमासा L, कल्पाधिकल्पाधिमासा ι, कल्पाधिकल्पामासा S | भाज्याः ζ (except H) | गणे B, गणो DTIMS || Line 5. कल्प्यावमानि LT, कल्पाग्रभानि B || Line 6. गतश्चंद्र° β (except BDH) ||

Appendix #18. (See App. Crit. for p. 66, 2 – 20 of Text Alpha.)

Line 2. संगुणय[1]] दशंगुणय B, संगणय IM || Line 3. शेषं[1] ... षष्ट्या[2] om. B || Lines 3 – 4. शेषं[2] ... विकलाः[1] om. T || Lines 4 – 5. °लावशेषम् ... लब्धं om. B || Line 4. °वशेषम्] °वशेषो T, °शेषं ζ | तस्मात्] तस्मा IM || Line 5. ग्रहः] अग्रं WIS, अर्त्तं M, अत्र θ (S in repetition) | कलाशेषं Tζ | °र्भक्ते H || Line 6. विकलावशेषयोः β (except L, विकलावशेष H) || Line 7. °क्षपकः B, °क्षेपः LD || Lines 7 – 8. °तो यतः ... °मुर्वरि° om. ζ || Line 8. गुणकगुणिताद्भाज्या°] गुणकगुणिताभाज्या° BD, गुणकगुणितभाज्या° H || Line 9. भाज्यो] भाज्ये ζ | उक्ते IM | कल्पाथ L, कल्प्यात I, कस्यात M | °विकिला° D ||

Appendix #19. (See App. Crit. for p. 69, 11 – 12 of Text Alpha.)

Line 1. गुणको ... ह्यज्ञातः from Sūryadāsa's *GMK*, with slight change (see *Wai, PPM 9762*, f. 122r., 4) || Lines 2 – 3. शोधिते ... हरे om. B || Line 3. °भाज्य° om. B, °भाजक° DTζ || Line 6. °मेक°] °मेव° ζ (except W) || Line 8. अथ] अत ζ (except H) | हारे om. ζ, हरे B || Lines 9 – 10. °योर्घातः ... गुण°[2] om. B || Line 10. अत्रा° ... १४ misplaced after line 11. कुट्ट° ζ (after line 11. कुट्टक° S, after line 11. एव H) | गुणक्योगो Tζ || Line 11. तत्र

कुट्ट° om. H । कुट्टकेन] कुट्टश्न ζ (कुट्टकश।न S, श्न H) । गुणः ζ ॥ Line 12. अत्रो° ... इति from Sūryadāsa's *GMK*, with little variation (see *Wai, PPM 9762*, f. 122r., 6) । °निघ्नः] °त्रिघ्न IM ॥ Lines 13 – 16. Verse 67a - d. H ॥ Line 17. स्पष्टम् from Sūryadāsa's *GMK* (see *Wai, PPM 9762*, f. 122r., 6) । स्पष्टार्थं L ॥

CHAPTER V

TRANSLATION OF THE *TEXT ALPHA*

THE *SŪRYAPRAKĀŚA*

Written by the venerable astrologer,
the scholar Sūryadāsa

(A commentary on the *Bījagaṇita* of Bhāskarācārya)

1. *Principles and Conventions Followed in the Translation*

A. *A Literal Translation*

A translation as literal as possible has been provided. It is conventional among most historians of the mathematical sciences in Sanskrit, and in Arabic, Akkadian etc. to translate literally so that the reader can get a better idea of how the mathematicians thought about their material and thereby know what the basis is for interpretation of that material in terms of modern mathematics.

Modern mathematics has a formal, logical structure which medieval Indian mathematics does not have. The modern mathematical meaning cannot be given in translation, because the Sanskrit expresses something different. As a historian, one has to be aware of the difference between what the Indian author actually says and how it might be expressed in modern mathematical terms, if it is so expressible. This is why we provide a commentary. It gives our interpretations and justifications in terms of the historical context in which the text was written. Giving a modern non-literal translation is imposing an interpretation on the original text of the author. This interpretation is what we are supplying and not what the author is writing.

If one were to translate directly from Sanskrit poetry or prose into mathematical formulae, one would lose the possibility of conveying to the non-Sanskritist the arbitrariness of any particular interpretation.

There are three particular problems with the Indian mathematical texts:

(i). Style—When only the part of the rule is given by the author and the rest has to be supplied by the reader. An example of this in our thesis is Āryabhaṭa I's (b. 476 A. D.) verses 32-33 in which he describes the method of kuṭṭaka (see Chapter VI, section 4.F.).

(ii). The Use of the Technical Terms Which Have No Correspondence in Modern Mathematics—For example, the term karaṇī. This term is not the same as the square-root, because on the one hand, Bhāskarācārya says that the square-root of a negative number does not exist because it is not the square of a number (*BG*, 7d, p. 4). But on the other hand, in the rules for karaṇī (*BG*, 27c-28b), Bhāskarācārya says that the square-root of a negative karaṇī is a negative number (*BG*, 28a-b, p. 13). For example, karaṇī –25 *yields* the number –5. So the text itself differentiates between karaṇī and square-root. Thus there are ambiguities.

(iii). The Use of the Same Term in Different Mathematical Meanings—For example, the term rupa. On the one hand, rupa means number 1 and on the other hand, it means any number.

Thus we have given a literal translation of the *Text Alpha* and have provided a commentary.

B. *Other Principles and Conventions*

In order to facilitate the reading, complete verses from the mūla have been translated, though Sūrya, except in a few cases, mentions only the lemmas.

The remarks in parentheses have been supplied by the translator. Quotes have been used to replace 'iti' (see e.g. p. 11, 17) of the *Text Alpha* or to define some expressions which have no iti (see e.g. p. 9, 22 "svam").

Note that in the preceding paragraph "p. 9, 22" means page 9, line 22. This format is followed throughout the entire thesis. In the present section, all such references are to the Devanāgarī page numbers of the *Text Alpha*.

The dot representing a negative number in Sanskrit has been transliterated as the usual negative sign, –. Furthermore, though the manuscripts of the *Sūryaprakāśa* use no sign for plus, the sign + has been introduced in the commentary wherever 'plus' or 'sum' is intended in the *Text Alpha* (see e.g. the commentary on verse 36c-37b).

The daṇḍa has been replaced by "(and)" when it appears between two numbers or expressions (see e.g. pp. 9, 16; 19, 6; 34, 6-7). At one or two places the daṇḍa has been replaced by "(or)," as, for example on page 36, 5. Furthermore, the daṇḍa between two numbers which are written in the sexagesimal system, has been replaced by a semi-colon; e.g., on p. 24, 11.

If there is no separation sign between two mathematical expressions in the text, then "(and)" has been used in the translation (see e.g. pp. 26, 3; 28, 20). Also, in many places in the text the senses of "addition" and "(and)" are interchangeable, thus we may find "(and)" in the translation while "+" may also be implied (see, e.g., pp. 34, 1-2; 36, 18-19).

The horizontal line between the numerator and denominator of a fraction is not found in our manuscripts (see e.g. p. 45, 17), but has been used in the translation. A fraction in sexagesimal system has been translated using a semi-colon and commas (see our translation and commentary to the *Text Alpha* (p. 45, 18). The coefficients of the unknowns which appear to their right in Indian mathematics have been transliterated as they appear in the manuscripts. For example, (p. 20, 17) या ३̇ रू २̇ has been translated as yā –3 rū –2.

The reader is reminded that integers (rūpas) greater than one are in the plural in Sanskrit (see e.g. Verse 44a, p. 44, 9). Furthermore, the original form of phrases like "a quartet of ones," which means simply the number four, has been kept in the translation (see e.g. Verse 6c, p. 11, 8).

Finally, a glossary of technical terms has been compiled for the convenience of the reader.

2. <*Text Alpha*, First Chapter>

<*Preface*>

Obeisance to Gaṇeśa. Obeisance to Sarasvatī. Obeisance to the elders.

1. On whose forehead is the Moon, at the opening of whose pair of lovely eyes are the eight perfections; on the throat of whom the son of one (Śiva) whose throat (is clung to) by Śrī, is the gleam of the excellent jewels on the hoods of the serpents (like the gleam of) the jewel of the day (i.e., the Sun); at the edge of the seat which is his (Śiva's) lotus-like feet sits he who has a swarm of bees on the surface of his head (like) Brahmā and so on, whose light has endless power—may he, called Gaṇapati, protect us here.

2. Saying "Oh Kṛṣṇa" (*or* "Oh dark-blue one") I adore that certain source (*or* algebra) which wears the unknown (yāvat) as (*or* just) a garment splendid with the colours black and blue, yellow, white, (or) red, and (wears) a string, a type of necklace, as if with laughing lower (lips) (*or* a chain having a sort of division as if by means of easy lower (numbers)), which is an origin called the unmanifest which is known by those whose purification is constant (*or* a root called the unknown which is known by means of equal subtractions), or which is to be understood by the intellect from an excellence of discipline (yoga) (*or* from a type of addition).

3. I, who have gone to the lofty further bank of the treatises on arithmetic, the pulverizer, and algebra (bīja) because of my understanding that was produced by a small particle of grace from his lotus-like feet, and who know the meanings of the teachings on meters, rhetorical adornment, poetry, drama, and music, praise him, my own father who has the highest virtues, my teacher, Jñānarāja.

4. The sun-rise of good understanding which destroys the night of confusion as if it were the union of two ruddy geese (*or* of two disk-like sky-goers (*i.e.*, the Moon and the earth's shadow)) in a circle, which gladdens the lotus-like mouth of the poet in the distinction of the two meanings of *gaṇita* (*i.e.*, arithmetic and algebra) (*or* of two counted meanings), which fills the direction of (the god) Indra (*i.e.*, the East) of the gods (*or* of the learned) reddened as if by (*or* devoted to) the sentiments such as that of love, and constantly firm in Viṣṇu's place (*i.e.*, the sky), is victorious.

5. Thinking: "Let there not be a burden of toil for the bewildered whose minds have departed (*or* who are dead) and who desire a crossing (*or* seek emancipation) in the glistening ocean of algebra (*or* in the ocean which is the source of the manifest) whose waters are deep with (*or* fordable by) various contrivances," I, Sūrya the calculator, whose mind is attentive and compassionate, construct at once this measured (brief) boat of a commentary.

6. The meaning of the first syllable of a mantra (bījākṣara) (*or* of the symbolic syllables used in algebra) is hard to grasp at first; how is an idea (*or* a demonstration) in this matter to be considered? Nevertheless, I, Sūrya, of lofty intellect, make intelligible bīja (algebra) together with its origin.

By Brahma took on a body as a favour to all the worlds which exist in the middle of the three-world (universe) which glitters with the charm of various rituals within the temple which is the Brahmāṇḍa which is arranged in the form of different beautiful regions where wide-spreading night is shattered by the discharge of a mass of unfragmented burning hot rays from the disk of the excellent Sun (*i.e.*, Bhāskara and Sūrya), (by Brahma) who desires the usefulness of the excellence of the ritual actions that yield the fruits of the other world and of this one Jyotiḥśāstra was created, the foremost of all the sciences (āgamas) and aṅgas (of the Veda). That Bhāskara (*or* Sun), whose body was revealed in order to lift this world up when it had been destroyed by the power of the time of the Kali (yuga), in order to help it when it had been struck down by the darkness of ignorance, having written the manifest mathematics (arithmetic) in accordance with his expressed plan, desiring to expound this exceedingly difficult mathematics of the unknown which is algebra (*or* the origin), at first, with the wish to accomplish what had been begun, effected by himself the auspiciousness which is in the form of a reverential salutation to a deity (Gaṇapati) in accordance with the standards of behaviour of the cultured, the necessity of making which (gaṇita *or* maṅgala) was made known to him by his hearing what is inferred from the behaviour of the learned that is distinguished by its being the special cause of the removal of the obstacles which impede it. Having joined it by means of words having several meanings with usefulness to students, he ties it together with the Upendravajrā meter in a verse (beginning): "The generator."

> <1a-d. The generator of the intellect I praise, which the wise men (*or* Sāṅkhya philosophers) declare to have been imbued by the existing Puruṣa, the unique source of all that is manifest, the unmanifest lord, and the numbered.>

"I praise the lord of the intellect," this is the (grammatical) connection of the verbal action and the instrument of action. The meaning is: "I praise, (i.e.) I salute, the lord of the intellect, (i.e.) Gaṇādhipati. Here his (Gaṇapati's) lordship of success and intellect is really established from the evidence of the traditional doctrines and the teachings. Surely in this paying homage to (him) just as the lord of the intellect if (one asked) "what is the cause?," (the answer is that) it is not (only as such). Since, because this science of the unmanifest is uniquely feasible through the intellect, such a god (as the lord of the intellect) is to be asked by us (for assistance), having this in view from the beginning, the teacher shall speak.

> "Algebra (bīja) is indeed thought accompanied by various colours. (The thought) which, for (its) usefulness in awakening the dull (witted), has been spread about by their and other teachers who are Suns to the lotuses that are computers, arrives at the state of being called 'bījagaṇita'."

Suspecting that, in the non-obviousness of his bowing to his chosen deity, because of ignorance of (the answers to such questions as) Who is he? and of what form?, this obeisance to him would lack authority, he indicates the authority by a distinctive statement: "The generator." Of what sort is he? The generator of all that is manifest (*or*: of all arithmetic). Kṛtsnasya (means) "of all," vyaktasya (means) "of a solid effect such as the earth and a mountain," and utpādakam (means) "maker." Furthermore, from the definition "an intelligent person (saṅkhyāvān), a learned man (paṇḍita), a poet," sāṅkhyāḥ (means) "wise men." They declare that that which is unmanifest is imbued by him, the existing Puruṣa, by which existing Puruṣa the unmanifest (*i.e.*) the "formless," the sky and so on, is imbued (*i.e.*) "pervaded." This is its meaning. The reasoning is that an object that is coming into existence implies a maker. So this also with its being an object as the cause will result in having a maker by the example of a pot. In this case the maker happens to be the highest lord, because only he is regarded by us as having attained the state of having the attributes of Vighneśa by means of an object. The application of this in the case of its being in the state of having a maker is well known in Nyāyaśāstra. It is that: "The earth and so on has a maker because it is an object like a pot." By the (statement) beginning "The unmanifest is inhabited" is indicated the all-pervasiveness of him who pervades time and so on. From the meaning, "this is accompanied by eternity," (his) eternity also (is known). Of what sort is the numbered? He is numbered because, being without number, he has become a multitude—that numbered one (I praise). The intention is that, even though he is the lord himself, modifying himself by appearing to be Maheśa (= Śiva) because of the characteristics of his actions, having appropriated the state of being the overseer of multitudes with the appearance of being Gaṇādhīśa for the sake of producing the activities of others (and) for the sake of manifesting greatness in the characteristic of his own form, he agreed to exist as master and as servant in unity. Again, considering "of what sort (do I serve)," he says "ekabīja," *i.e.* he for whom there is one bīja, i.e. syllable. So one must meditate on the fact that this was said with the sense of the one-syllable mantra of Gaṇapati. And so the highest meaning of the verse is Gaṇapati.

Now, because of the force of the tradition that:

> "Whose highest devotion is to a god, and to his teacher as to his god, of that great-souled person all the purposes are illuminated,"

with this verse (which begins:) "The generator" he bows down to his teacher, who taught him this science of the unmanifest, Maheśvara, his father. The (grammatical) connection there is: "aham utpādakaṃ vande." A father is a generator because he generates. The meaning is: "I praise—i.e., I bow down to—that generator—i.e., (my) father." Surely (he operates) with the etymology: "he is a guru who gṛṇāti—i.e., teaches" and with the fact that "father" is a meaning of the word "guru" from his being incited (bodhana) to pay obeisance by his memory of him. But here, fearing that bowing down to his father as the generator appears to be inappropriate as if it were because of affection, he says: "Buddheḥ" (of the intellect). What sort of generator? A lord of the intellect also. Because of the force of the meaning of the fifth (genitive case) of the word "buddheḥ" in the sense of "obtaining control of," he is a lord (of the intellect) because of his knowledge. The ācārya, having it in view that, if his being a guru is established by his being the cause of knowledge, then bowing down to him is appropriate, will make manifest that his father is a guru (teacher) at the occasion of the conclusion of the book as follows:

"He who was known on the earth as Maheśvara has attained the epithet, "best of the ācāryas of the wise." Having obtained a minute quantity of knowledge from him, his son, Bhāskara, made algebra (bījagaṇita) easy."

Now wondering what would be the superiority in paying obeisance to him on the occasion of speaking about the computations of the unmanifest, he reveals its superiority by the device of a second connection (with the word) "sāṅkhyāḥ." Saṅkhyā (means) enumeration, counting. Those habituated to this are Sāṅkhyas, i.e. followers of Jyotiṣa. They say that the calculation of the unmanifest called "bīja" is imbued by the existing Puruṣa. The intention is that in that with respect to which occurs the activity of the author of the book it is necessary because of its being possible because He is its imbuer. "For those dull-witted ones who think that, although doubt about this does not arise because of the illumination of the cleverness of the calculation of the unmanifest, in one of two possibilities is the obstruction of the other, was there or was there not an experience of his manifestation?"—in order to dispel this doubt he specifies the unmanifest with (the word) "vyaktasya." Of what sort is the unmanifest? It is the unique source of everything that is manifest; that is to say it is the unique source, that on which it depends, of the manifest i.e. of the calculation of the manifest, whose other synonym is Pāṭīgaṇita (arithmetic). The intention is: "The wise composed this which depends on that."

Thus is the second meaning of this verse, relating to the supremacy of the guru (teacher/father). Indicating that he himself understands Sāṅkhya philosophy by the pre-eminent character of his devotion, revealing his cleverness by means of a pun, joining two meanings with words such as "avyakta," with this verse (which begins): "Utpādakam" he salutes his chosen (deity) who is also the deity of the science. The application is: "I praise this reality which is called "the unmanifest" (avyakta) whose other synonyms such as being the cause of the equilibrium of the guṇas are not expounded." Thinking "what is this?," he proclaims (the phrase beginning): "Yad." The Sāṅkhyas call that which is the unmanifest imbued by the Puruṣa the generator of the intellect, which is a mahattattva ("great reality"). (This means): "The Sāṅkhyas are so-called because they teach the śāstra called Sāṅkhya, (the science) which treats of twenty-four tattvas (of which one is intellect)." Because of (the rule): "He studies that, he knows that," (the suffix) aṇ (is applied to the word "saṅkhyā" to produce the word "sāṅkhya").

The meaning is this: The Sāṅkhyas believe that the creation (of the world) is from just the binding together of the Paramātman and Prakṛti, another synonym of unmanifest, by means of the production of the realities (tattvas) beginning with the intellect. So has it been said by Bhāskarācārya in the *Siddhāntaśiromaṇi* that he wrote (with the verse) beginning:

> "From which came into being the great (intellect) from Prakṛti and Puruṣa
> when they were agitated, (and) in its interior self-awareness."

So (it was said) by the feet of our father in the *Siddhāntasundara* (with the verse) beginning:

> "The tattva that is intellect (comes) from the union of Prakṛti and Puruṣa."

Wondering what the means of knowledge might be in this case since it is unmanifest, he states: "of the manifest." Of what sort is the unmanifest? It is the unique source of everything that is manifest; the unique source, that is, the cause, of what is manifest, that is, of what has attained manifestation, the earth and so on. The meaning is that the unmanifest is known by its being the cause of everything, of the whole, that is manifest. Again of what sort? The meaning is: a lord, that is, one who is powerful because he accomplishes such actions.

Now, since computation is praiseworthy and has the form of the lord, he praises computation also with the figure of a pun with the same words (beginning) "Utpādakam."

The connection is: "I praise that mathematics called "avyakta" (algebra) whose other synonym is "bīja." Thinking "what is that?," he proclaims (the phrase beginning) "Yad." The Sāṅkhyas proclaim that what is imbued by the existing Puruṣa is the generator of the intellect. By the existing Puruṣa, that is, by the Puruṣa who has qualities like pervasiveness, imbued, that is resorted to, is the generator of the intellect. The meaning is that the Sāṅkhyas, that is, (those who) do "saṅkhyā," counting, those Sāṅkhyas who are calculators, proclaim (it). (The suffix) ka (is applied because of the rule Pāṇini 3, 1, 136): "Also after (a root ending in) ā in an upasarga (a word in which there is a prefix)." Again of what sort is it? The unique source of everything that is manifest. (That means): the unique source, i.e. the cause, of the manifest, that is of Pāṭīgaṇita (arithmetic). Again of what sort is it? The lord (īśa). The meaning (of the word īśa) occurs that it is the one in whom (any) desire is unopposed.

Having established the auspiciousness characterized by paying obeisance to his chosen deity with the first verse, now, beginning the book, the teacher, praising bīja with the dodge of telling the usefulness of beginning it, with one śālinī-verse tells (the verse beginning): "Previously mentioned."

> <2a-d. Previously mentioned (in the *Līlāvatī*) was the manifest whose source is the unmanifest. Since generally questions cannot be very well understood by the dull-wited without the application of the unmanifest (algebra), therefore I tell also the operation of the bīja (algebra).>

The sequence is: "Previously mentioned was the manifest. Therefore I tell the operation of the bīja." Thinking: "Why therefore?," he says (the phrase beginning): "Yasmāt." From which cause generally questions cannot be very well understood by the dull-witted, that is, by those of little intelligence, without the application of the unmanifest (algebra). The meaning is that (the questions) are excessively difficult to understand. The manifest of what sort? Avyaktabījam. That is avyaktabījaṁ whose source is the unmanifest. The meaning is that the calculation of the unmanifest has become the cause of the manifest.

3. <*Text Alpha*, Second Chapter>

<*The Chapter Concerning the Six-Fold (Operation)*>

<A. *The Six-Fold (Operation) of Positive and Negative (Quantities)*>

Now in connection with describing what is to be explained in the treatise, with respect to all (operations) such as multiplication and division, because of its priority he speaks of the addition and subtraction of positive and negative (quantities) by means of half an Upendravajrā verse (beginning): "In addition, the sum occurs."

> <3a-b. In the addition of two negative (quantities) or of two positive (quantities), (their) sum occurs; the addition of a positive and a negative (quantity) is (their) difference.>

The (syntactic) connection is: "Of two negative (quantities) or of two positive (quantities) in the addition the sum occurs and of a positive and a negative (quantity) in the addition the difference occurs." "Kṣayau" (means) a negative (quantity) and a negative (quantity). Of those two negative (quantities, i.e.) of two that have become negative, and likewise of two positive (quantities, i.e.) of two which have become positive, when the addition is made, just the sum is the addition because of the mutual homogeneity of those two (quantities). And here it should be understood that the addition of two positive (quantities) is a positive, the addition of two negative (quantities) a negative. So in the addition of a positive and a negative number, there is just the difference because of the non-homogeneity of these two (quantities).

The demonstration in this case is (as follows). For instance, in the *Grahagaṇita* (the computation of (the longitudes of) the planets), when the correction due to the half-equation of daylight and (that) due to the difference in risings is made for the sake of computing the true (longitude) of the Sun, if both were negative, then it is accepted that first (that due to) the difference in risings is to be subtracted from (the longitude) of the Sun, and then (that due to) the half-equation of daylight is to be subtracted from that. Now, for the sake of easiness, even when the sum of the two (quantities) is subtracted, the result is the same. Therefore it is clear that the sum of two negative (quantities) is negative and the sum of two positive quantities is their sum. In this case, (the correction due to) the half-equation of daylight is seen to be negative and (that due to) the difference in risings positive. And so it is demonstrated that, when first (the correction due to) the difference in risings is added to (the longitude of) the Sun because it is positive, afterwards when (the correction due to) the half-equation of daylight is subtracted because it is negative, then, because of their non-homogeneity there is left (their) difference as if they were camphor and fire.

When the addition of a positive and a negative (quantity) has been made in this way, in whatever remains the state of being positive or the state of being negative is to be known as that which pertains to the larger number. And it is said:

> "In the addition of two positive (quantities) occurs a positive, and in (the addition of) two negative (quantities), a negative. In the addition of a positive and a negative (quantity), it is like the larger number."

The demonstration in this case is (as follows). It is clear that in that (previous) case the sum of two positive (quantities) is a positive. So when it is asked by someone: "Subtract from ten, four and three," when at first four are subtracted from ten, then the remainder is measured by six. From that also, three again are subtracted. Then the remainder is measured by three. Now, because, for the sake of easiness, when the sum of four and three is subtracted, then also the remainder is just three; when one computes (thus) it occurs that in the addition of two (numbers) that have become negative there is negativity. Now, (with regard to the phrase): "as pertains to the larger number," whatever is known as the difference in the addition of a positive and a negative (quantity) by its being (their) remainder, it must be known of which (quantity) it is the remainder, of the positive one or of the negative one. It is similar (in sign) to that (quantity) of which it is the remainder. So it was demonstrated in the *Grahagaṇita* that, when the subtraction of the latitude and declination which are in different directions is made, the remainder is the accurate declination; in this case, the direction of the remainder is (the same as) that of whichever (quantity) is the larger.

Now, for the sake of teaching students, he enunciates an example here, with the previous verse (beginning with): "A triad of ones."

> <3c-4b. A triad of ones and a quartet of ones are together (both) negative or (both) positive or separately positive and negative or separately negative and positive. Tell me quickly (if) you know the addition of the two positive and negative (quantities).>

Since here whichever (numbers) have become positive remain just as they were, and "whichever have become negative have dots above them," having established the sign of positivity or negativity in this way, one should compute the addition and subtraction. Since the matter in question is computed thus, the (first) layout is: –3 (and) –4. Here by the procedure of the sūtra: "In the addition of two negative or of two positive (quantities), (their) sum occurs," the sum –7 is produced. Now again the layout is: 3 (and) 4. In (their) addition, 7 is produced. Again, the layout is: 3 (and) –4. In (their) addition, –1 is produced. Again, the layout is: –3 (and) 4. In (their) addition, 1 is produced.

Having described thus the addition of positive and negative (quantities), now he speaks of the subtraction of positive and negative (quantities, with the verse that begins): "That is going to be subtracted."

<4c-d. A positive (quantity) that is going to be subtracted attains negativity; a negative (quantity) positivity. The addition of those (two) is as has been described (previously).>

"Svam" (that is, a positive (quantity)), that is going to be subtracted attains negativity. So "kṣaya"—i.e., a negative (quantity)—that is going to be subtracted, "eti" (that is, attains) "svatva"—i.e., positivity. Then the addition of those (two) is as was described (previously). The meaning is that it is as (in the verse) beginning: "In the addition there is the sum."

The demonstration in this case is (as follows). In that (verse), there is a succession (of the terms) "the state of being one that is going to be subtracted" and "the state of one that has become negative." Therefore, the negativity of a positive (quantity) that is going to be subtracted is easily accomplished. Now, when the negativity of a (number) which has become negative is being accomplished, by the rule that: "in the absence of non-being (there is) the necessity of being," (therefore), by exclusion, there is positivity. Otherwise, in the addition of two negatives there would be no sum. Therefore, it has been demonstrated that a negative (quantity) that is going to be subtracted attains positivity.

Here he enunciates an example (with the verse that begins): "A pair from a triad."

<5a-b. When one subtracts a pair (of ones) from a triad (of ones), (either) a positive from a positive or a negative from a negative, and the reverse, (in each case) say the remainder quickly.>

The whole has a clear meaning, and is understood from the book.

Thus, having described the addition and subtraction of positive and negative (quantities), now he enunciates a karaṇasūtra on the multiplication of positive and negative (quantities, with the verse beginning): "Of two positive (or) of two negative (quantities, the product is) positive."

> <5c-d. In the multiplication of two positive (and) of two negative (quantities, the product is) positive, (but it is) negative in the multiplication of a positive and a negative (quantity). But it is also explained in the same way in the division (of positive and negative quantities).>

The (syntactic) connection is: "Of two positive or of two negative (quantities) in the multiplication (the product) is positive. So in the multiplication of a positive and a negative (quantity) it is negative. "Ca" (means) but. In division also in the same way is it explained." The meaning is that in the multiplication of two positive (quantities)—i.e., of two which have become positive—(the product is) positive. So in the multiplication of two negative (quantities)—i.e., of two which have become negative—(the product is) positive. So in the multiplication of a positive and a negative (quantity), (the product is) negative. And in the division of two positive or of two negative (quantities) the result is positive. And in the division of a positive and a negative (quantity) the result is negative.

The demonstration in this case is (as follows). It is known that in the multiplication of two positive (quantities) (svayoḥ)—i.e., of two positives (dhanayoḥ)—(the product is) positive (sva)—i.e., positive (dhana). Since in the multiplication of two negative (quantities) also (the product) is positive, when in this case (a quantity) which has become positive is to be divided by (a quantity) which has become negative, the quotient is (a quantity) which has become negative. Then again, when the multiplication of a quotient which has become negative and a divisor which has become negative is being accomplished, the dividend is (a quantity) which has become positive. Otherwise, because of the homogeneity of the two negative (quantities) in division, addition would also occur in division.

But here, in an example, the dividend (is) 6, the divisor –3. In this case on account of (the line) beginning: "But it is also explained in the same way in division," the quotient in the division is –2. So it has been demonstrated that when this divisor –3 is multiplied again by this negative quotient, as the result produced is the previous dividend, this 6 (is produced).

So here the demonstration (of the line): "But it is also explained in this way in division" is (as follows). When division of a negative dividend is being carried out by a negative divisor, because of the procedure of the sūtra: "That is the result by which the divisor when multiplied is subtracted from the dividend (without remainder)," (the quantity) by which the divisor when multiplied is subtracted from the dividend (without remainder) is just positive. So it has been demonstrated that just that is the result in division.

Here he enunciates an example (with the verse beginning): "A positive by a positive, a negative."

<6a-b. A positive pair is multiplied by a positive triad, (or) a negative by a negative, (or) a positive by a negative. What is (the result)?>

And (he enunciates another example with the verse beginning): "An octet of ones by a quartet of ones."

<6c-7b. A positive octet of ones is divided by a positive quartet of ones, (or) a negative by a negative (or) a negative by a positive, (or) a positive by a negative. Say quickly what this (quotient) is (in each case) if you understand (computation) thoroughly.>

It has a clear meaning. It is also exemplified in the demonstration.

Thus he enunciates a sūtra which has obtained its order immediately after the multiplication and division of positive and negative (quantities) for the sake of (taking) the square of positive and negative (quantities), (with a verse that begins): "The square of a positive and of a negative (quantity)."

> <7c-d. The square of a positive and of a negative (quantity) is positive. The two square-roots of a positive (quantity) are positive and negative. The square-root of a negative (quantity) does not exist because it is not a square.>

The grammatical construction is: "Of a positive and of a negative (quantity) the square is positive." Here (by the term) "svarṇayoḥ" is to be understood "of two positive (or) of two negative (quantities)." So he describes the condition of being positive or negative when the square-roots of squares which have been produced are being taken (with the words): "The two square-roots of a positive (quantity)." The meaning is that the square-root of the square of a positive number is positive and that (of the square) of a negative (number) is negative.

The demonstration in this case is, however, to be understood as like the demonstration in the multiplication of positive and negative (quantities). Now he describes the condition of the square-root of a square which is negative (with the words beginning): "No square-root." The square-root of a negative (square) does not exist. There is the non-existence of the square-root of a negative—i.e., of a square which is negative. (In an answer to the question:) "For what reason?," he says: "Because it is not a square." This is the meaning: because that negative square is unassailed by the characteristics of a square.

Three are negative and three are positive: –3 (and) 3. In the multiplication of both there is the non-existence of a square because of (their) being unequal. The idea is that (this is so) because of the application of the characteristic of a square (according to the words): "The product of two equal (quantities) is a square."

In this case he enunciates an example (with a verse beginning): "Of a positive (triad) of ones."

<8a-d. Oh friend! Tell me quickly the square of a positive triad of ones and (that) of a negative. And quickly tell (me) separately the square-root of nine having a positive nature and having a negative nature.>

It is clear.

Thus the six-fold (operation) of positive and negative (quantities).

<B. *The Six-Fold (Operation) of Zero*>

Having described in this way the six-fold (operation) of positive and negative (quantities), now he investigates the six-fold (operation) of zero (with the verse beginning): "In the addition of zero."

<9a-b. In the addition of zero or in the subtraction (of zero) a positive or negative (quantity remains) as it was. (But) when it is subtracted from zero it goes to its opposite.>

In the addition and in the subtraction of zero a positive or a negative (quantity remains) as it was. The meaning is that, when addition and subtraction are being accomplished by means of "kha"—(that is,) zero—a positive or negative (quantity) remains "tathaiva"—(that is,) as it was determined because in the addition and subtraction of any number whatsoever by zero, the zero does not change the form (of the number). So when it is subtracted from zero, it goes to its opposite. The meaning is that a positive or negative (quantity) when it is subtracted from zero attains reversal; because it is said that: "A positive (quantity) that is going to be subtracted attains negativity."

Here he proclaims an example (with the verse beginning): "A positive triad of ones."

<9c-d. There are a positive and a negative triad of ones, and there is a zero. Tell (me) what (each) will be when it is added to zero and when it is subtracted from zero.>

It has a clear meaning.

Now he describes multiplication by zero (with the verse beginning): "In the multiplication and so on."

<10a-b. In the multiplication and so on of zero (by a quantity the result is) zero. In the multiplication (of a quantity) by zero (the result is) zero. And a quantity divided by zero becomes (a quantity) having zero as its divisor.>

In the multiplication and so on of kha—(that is,) of zero (by a quantity), a kha—(that is,) a zero—is (i.e. results). The meaning is that (it is a fact) that, when zero is multiplied by any number whatsoever, zero is (the product) because a number multiplied by zero is zero because of the non-existence of its being in the sphere of counting by reason of its independence. Here by the word "ādi" it is to be known that division, square, and square-roots are the same. In this way Nārāyaṇa also has defined this incidentally by means of a poetic utterance in his algebra as follows:

"On account of multiplication by zero a quantity goes to the state of being zero. But, when it is divided by zero, it does not return to its previous condition (non-zero finite quantity) because it is absorbed in that (infinite) just as a serious yogī who has attained the unique bliss-giving place of Brahma which consists of pure thought because he is pervaded by the ātman does not (return) to the path of saṃsāra (finite world)."

So a quantity divided by zero becomes one having zero as its divisor.

Here he proclaims an example (with the verse beginning): "Multiplied by two."

<10c-d. Tell me (the results when) zero is multiplied by two (and) divided by three, (when) three is divided by zero, and the square and square-root of zero.>

It has a clear meaning.

Now he shows that in the science of computation there is another name, infinite, for the number which has a zero as its divisor. Then he skillfully defines the infinity of this (with the verse beginning): "In this."

<11a-d. In this quantity also, which has zero as its divisor there is no change even when many (quantities) have entered into it or come out (of it) just as at the time of destruction and of creation, when throngs of creatures enter into and come out of (him, there is no change) in the infinite and unchanging one (i.e., Viṣṇu).>

In this quantity which has zero as its divisor, even when many numbers have entered into or come out of (it), there is no change. The meaning is that of whatever (quantity) the divisor is zero, when a fractional number is being combined with that (khahara) which has the same denominator, there is zeroness in the denominator and the numerator. If (it is asked): "Surely, since one sees change in the quantity having zero as its divisor at the beginning of its combination with a number divided by one, two, three, and so on, how is it said that no change occurs?," it is true that since it follows from the meaning of the words, there is no change of its state of being (a quantity) which has zero as its divisor in the quantity which has zero as its divisor. Or else the word "aṅkeṣu" (i.e., in numbers) here is to be understood "in non-fractional (numbers)."

Now he shows how wonderful his poetry is by confirming the infinity of (the quantity) which has zero as its divisor by the example of Viṣṇu because of the sameness of (his) infinity (with the lines beginning): "Just as." "Just as at the time of destruction and creation when many throngs of beings enter into and come out of (him), there is no change in the infinite and unchanging (Viṣṇu), so (there is no change in the khahara)." The idea is that, when at the time of destruction, beings enter into Viṣṇu and at the time of creation come out of Viṣṇu, there is no change (in him) since he is infinite. This has been stated in the *Bhārata* in the Śāntiparvan in a conversation between Bhīṣma and Yudhiṣṭhira:

> "From whom all beings are born at the coming of the first yuga and in whom they go to destruction again at the end of the yuga."

Thus the six-fold (operation) of zero.

<C. *The Six-Fold (Operation) of One and More Than One Colours*>

Having thus described the six-fold (operation) of zero, now, with reference to the colours of the unknowns in this operation involving (quantities) that are unknown, wishing to speak of their six-fold (operation), at the beginning he enunciates the names of the unknowns (which are) imagined by the appearance of their being colours (with the verse beginning): "An unknown (yāvattāvat)."

> <12a-d. An unknown is the colour black, another blue, yellow and red. (Colours) beginning with these have been imagined by the best of teachers

> as the names of the measures of the unknowns, in order to accomplish their calculation.>

First "an unknown," then "black," immediately thereafter "blue," then "yellow and red." Surely it is proper to imagine them to be the names of the unknowns since the colours "black" and so on are well known. But if (one asked): "what is the reason for imagining "yāvattāvat," the unknown, to be the name of unknowns?", it is not (proper) because it (yāvattāvat) is among the other synonyms of "measure" because of the saying of Amara: "Yāvattāvat (is used) in the meanings of "totality," "limit," "measure," and "restriction"." So, why does he say "kartum" (to accomplish) (in the phrase beginning): "tat?" "To accomplish their calculation." With the word "tat" (he refers to): the unknowns. The meaning is: "To accomplish their saṃkhyāna—i.e., calculation."

Then he describes the addition and subtraction of unknowns (with the verse that begins): "The sum and difference."

> <13a-b. Among these (unknown quantities), the sum and difference of two having the same character (is as usual), but (for the sum and difference) of two having different characters putting them separately (is required).>

Among these (unknown quantities) the sum or the difference of two having the same character is to be accomplished (as usual). The meaning is that among these colours, the sum and difference of colours having the same character is to be accomplished mutually. And a putting down of two having different characters separately is to be done. When the sum of colours with rūpas (numbers) is being accomplished, then a putting down of rūpas separately is to be done. And it is easy to put down separately the squares of unknowns when they are being summed with simple unknowns.

In this case he enunciates an example (with the verse beginning): "A positive unknown."

<13c-14b. One positive unknown together with one one and a pair of positive unknowns diminished by eight ones. Oh friend! Tell (me) quickly what is (the result) in the summing of these two sides? And what is (the result) in the summing (of these sides) if one reverses (their) positive and negative (signs)?>

It has a clear meaning.

Now he proclaims an example for the sake of making known that, in the addition of squares of unknowns and of simple (integer) unknowns, they must be put down separately (with the verse beginning): "A triad of the squares of a positive unknown."

<14c-d. A triad of the squares of a positive unknown together with three ones is combined with a pair of negative unknowns; what is (the result)?>

And, again confirming (this) for the teaching of students, he enunciates (the verse which begins): "From a pair of positive unknowns."

<15a-b. From a pair of positive unknowns subtract six negative unknowns together with eight ones; tell (me) quickly the remainder.>

It all has a clear meaning.

Having described thus the addition and subtraction of unknowns, now he speaks of a special property in the multiplication of unknowns (with the verse that begins): "There is in a rūpa (number) and a colour."

<15c-16b. There is, however, in the multiplication of a rūpa (number) and a colour, a colour (as the result). But, in the multiplication of two, three, and so on (unknowns) which have the same character, there are their squares, cubes, and so on (as the results). In the multiplication of (unknowns) which have different characters, (the result is) their product.>

In the multiplication of a rūpa and a colour (the product) is a colour. Here the rūpa is a known number and the colour is an unknown. In the multiplication of those two, (the product) certainly is an unknown. If (it is asked): "What is the reason for the rule that, in the multiplication of a known and an unknown (the product) is an unknown?," in that case let it be heard that, since the unknown is the large(r number, i.e., more important) as it is a root with respect to the known because an unknown is made to be known, but a known is not made to be unknown since it is self-evident as a known, so (the product) is similar to that which is larger (i.e., more important).

A demonstration in this case is (as follows). When rūpas are multiplied by a simple unknown, an unknown is produced. Again, when division of it is made by a simple unknown, rūpas are the result because, from the procedure of the sūtra for division which is about to be enunciated (which begins): "After being multiplied by whatever colours and by whatever rūpas," if the divisor, a simple unknown, is multiplied by colours, then there results a square of an unknown. But, considering that it is not subtracted (without remainder) from the dividend which has the characteristics of a simple unknown, as one computes, having been multiplied by rūpas, it is to be subtracted. So, by whatever (rūpa) when multiplied the divisor is subtracted from the dividend (without remainder), that is the result. Hence in the matter under discussion rūpas are the result. So it has been demonstrated that, if those (rūpas) are again multiplied by a simple unknown, then again there results an unknown. In this way, the meaning of this (verse which begins): "There is, however, in the multiplication of a rūpa and a colour, a colour (as the result)," has been achieved.

Now, "in the multiplication of two, three, and so on (unknowns) having the same class, (the products) are their squares, cubes, and so on." Those (numbers) of which two and three are the beginnings are such as four, five, and so on. In their multiplication in succession squares, cubes, and so on result because it is well known that in the multiplication of two equal (quantities) a square, and in the multiplication of three (equal quantities) a cube (is produced). Here in both places with the word "ādi" the meaning is that in the multiplication of four, five, and so on equal (quantities), squares of squares and cubes of cubes etc. result. And in the multiplication of (unknowns) having different classes that occurs and the product occurs. In the multiplication by each other of an unknown (yāvattāvat), black, blue, and so on having different classes that occurs. Here there is a triad of colours since with the word "tat" (the meaning is that) there is an akṣara in the name of that by which it is multiplied, and an akṣara in the name of the multiplicand, and the product. In that case "bhāvita" (the product) is called a type of designation which is brought about by its being a metaphor for the multiplication of different colours. This is the meaning: when blacks are multiplied by the unknown (yāvattāvat), there results "yākābhā;" and, when blue is multiplied by black, there results "kānībhā." The intention is that in this way it is to be written by one who makes the akṣara of the multiplier first.

Now he discusses a characteristic in an operation with unknowns when fractional numbers appear (with the lines that begin): "Division and so on."

> <16c-d. Division and so on (of fractions are to be accomplished) just as in the case of rūpas. The remaining (explanation) in this case is that which was mentioned in known calculation.>

An operation beginning with division is to be known (to be) just as in the case of rūpas. The meaning is that it is to be accomplished as in the case of known numbers. So also, (any) remaining—i.e., left over—operation is to be understood (to be) the same here as (that) which is mentioned in (the mathematics of) the known—i.e., in Pāṭīgaṇita. The meaning is that squares, cubes, common denominators, rule of three, series, areas, and so on is all accomplished with the meaning of Pāṭīgaṇita.

Having described in this way a special property in division and so on, now he enunciates an operational rule in multiplication (with the verse that begins): "The multiplicand separately."

> <17a-b. The multiplicand which is equal to the parts of the multiplier is to be entered separately. When it has been multiplied in order by the parts, it is combined according to the (previous) statement.>

The multiplicand which is equal to the parts of the multiplier is to be entered—i.e., to be established—separately just as it is. Then, when it has been "hata"—i.e., multiplied—by those parts in order, to which parts (the multiplicand) which has been entered is equal, immediately afterward it is to be combined according to the (previous) statement. Here by this (word) "yathoktyā," it is indicated that the addition of two (numbers) of the same (class) or two of different (classes), of a known and an unknown, or of a positive and a negative (quantity) is to be accomplished by the method described (previously). The meaning is that, when multiplication has been accomplished in this way, the result is (correct).

A clear demonstration in this case is said for the sake of increasing the intellect of slow-witted students. There multiplication is, indeed, a kind of addition which consists in the repetition of the multiplicand (for a number of times) measured by the enumeration of the number that is the multiplier. So division is a kind of subtraction from the dividend which consists in the repetition (of the subtraction for a number of times) measured by the enumeration of the number which is the divisor. In this way here in the multiplication of unknowns, whatever is the multiplicand, there are different colours such as the unknown (yāvattāvat). So for one operating (with the rule): "also in multiplier" there occurs multiplication of the parts. When multiplication one by one has been accomplished by one who has made (there to be) as many parts of the multiplicand as there are parts of the multiplier (and) when it has been combined as (previously) mentioned, a (correct) result is obtained. As, when twelve are multiplied by twelve, (the product) is one hundred and forty-four. Now it has been demonstrated that (here), by one who has made twelve parts (and) has multiplied each by twelve, as soon as they are made into one, the same result is obtained.

Now he enunciates a special property in the multiplication of surds which will be spoken of incidentally and in the squaring of unknowns (with the verse that begins): "Squares of unknowns."

> <17c-d. In the multiplications of squares of unknowns and of surds, the method of multiplying parts as described in (the mathematics of) knowns is to be thought of.>

In the multiplications of squares of unknowns and of surds, here, the method of multiplying parts as described in (the mathematics of) knowns is to be thought of. The unknowns are the yāvattāvats and so on. The meaning is that, when the squaring of them and the multiplying of surds are being accomplished, the rule is to be followed by the order of the sūtra mentioned in the *Pāṭīgaṇita*:

> "The multiplicand is multiplied, one below the other, by the parts equal to the parts of the multiplier and combined."

Now he enunciates an example of multiplication (with the verse which begins): "Five yāvattāvats."

<18a-d. Multiplying five yāvattāvats diminished by one rūpa by three yāvattāvats plus two rūpas, tell (me) quickly, (my) learned (fellow), (the product), or else (tell me the product) if you imagine the positive and negative multiplicand and multiplier to be reversed.>

Thus here the multiplicand and multiplier (are respectively) yā 5 rū −1 ($=5x-1$) (and) yā 3 rū 2 ($=3x+2$). Now from multiplying with the order of the sūtra: "The multiplicand which is equal to the parts of the multiplier is to be entered separately," the result is produced: yā va 15 yā 7 rū −2 ($=15x^2+7x-2$).

Then he describes an operational sūtra in division (which begins with): "The divisor from the dividend."

<19a-d. The divisor, after being multiplied by whatever colours and by whatever rūpas each in its own place in order, having been subtracted from the dividend, is without a remainder; these (colours and rūpas) here are the quotients in division.>

The (syntactic) connection is: "The divisor, by whatever colours and by whatever rūpas after being multiplied, from the dividend having been subtracted, is without a remainder. These here in division are the quotients." The dividend is capable of being divided. The meaning is that the cheda—(that is,) the divisor—when (i.e. after being) multiplied by whatever colours or rūpas, is subtracted from that without remainder until there are quotients.

The demonstration in this case is (as follows). Previously, in the sūtra for multiplication, when one had placed the parts of the multiplicand (in the number of places) equal to the parts of the multiplier, the sum of them when they have been multiplied by the parts of the multiplier, was produced as the result of the multiplication. Now the dividend is imagined to be the result of the multiplication, (and) the multiplier is made the divisor. And that divisor, multiplied by the colours existing in the previous multiplicand, is subtracted without a remainder from the dividend. Hence, (after) being multiplied by whatever colours, it is subtracted without a remainder. That is the result, and that is characterized as a colour. The multiplicand multiplied by the previous rūpas was combined. Now it is demonstrated that when it is multiplied by the rūpas it is subtracted without a remainder.

Now in order to instruct students this is explained clearly by there being an example. For here result (i.e. product) of the previous multiplication is this dividend: yā va 15 yā 7 rū –2 ($=15x^2+7x-2$). And the multiplier is this divisor: yā 3 rū 2 ($=3x+2$). Thus this divisor, (after) being multiplied by whatever colours or rūpas, is subtracted without a remainder from the dividend, these are the quotients. So here, the three existing in the divisor being multiplied by five are subtracted without a remainder from the dividend. And, so the setting out is: yā va 15 yā 7 rū –2 (and) yā 3 rū 2. Here, when three multiplied by five are subtracted without a remainder (from the dividend), what is obtained is yā 5. The remainder is yā 7 rū –2. Now, since the divisor exists in two places because it has the nature of a colour and a rūpa, even though division is carried out by colours, it must be divided by rūpas. Therefore, because of the operation: "Division is made by rūpas multiplied by that (quantity) by which when multiplied division is made by colours," the pair of rūpas in the divisor, after being multiplied by five, is to be subtracted without remainder from (the quantity) above. When it has been done thus, there are ten below. Since they are to be subtracted, from the order of the sūtra: 'a positive (quantity) which is going to be subtracted becomes negative,' they are to be subtracted. And so these are negative. When the difference is taken, since both the subtrahend and the dividend, which are measured by (the numbers) ten and seven, are positive (or) negative, there results

yā −3 rū −2.

yā 3 rū 2.

Now, when it is cast out again, because the dividend is in the place of the rūpa, the divisor being multiplied by whatever rūpas, is subtracted without remainder from the dividend. When it is done thus, there is produced −1. The divisor multiplied by a rūpa is subtracted without remainder. Hence, when the divisor is multiplied by a negative rūpa, since the divisor which was positive becomes negative because (of the rule): "in the multiplication of a positive and a negative (the result is) negative," there is produced:

yā −3 rū −2.

yā −3 rū −2.

Thus, in this case, because of (the rule): 'a negative (quantity) that is going to be subtracted attains positivity,' when (the calculation): "the difference of a positive and a negative" is performed, what is obtained is the multiplicand, yā 5 rū −1. In this way, (is the procedure) in every case.

Now he enunciates an example of the squaring of an unknown (with the verse that begins): "Of those which are diminished by six rūpas."

<20a-b. Oh friend! Tell me the square of four unknowns diminished by six rūpas.>

Here the squaring of the unknown is accomplished by the sūtra:

"In the multiplications of squares of unknowns and of surds, the method of multiplying parts as described in (the mathematics of) knowns is to be thought of."

Then he enunciates an operational sūtra on the square-root of a square (which begins with): "Having taken (the square-roots) from the squares."

> <20c-21b. Having taken the square-roots from the squares one should subtract from the remainder the product of each two of them multiplied by two. If there are rūpas, one should take the square-root of the rūpas; the rest is the same.>

The (syntactic) connection is: "From the squares, the square-roots having taken, of each pair the product multiplied by two from the remainder one should subtract." The meaning is that, whatever are the squares in the square-quantity (of which the square-root is to be extracted), having taken the square-roots from them, one should subtract the product of each two square-roots multiplied by two from the remaining number. (In reply to the question:) "Having done what?," he says: "If there are (rūpas)." (The connection is:) "If there are rūpas in the square-quantity, then for the sake of the square-root, having taken the square-root of the rūpas." For the sake of the square-root: the meaning is: "for the sake of the square-root of the square-quantity."

The demonstration in this case is (as follows). As when the squaring of an unknown is being performed, in the matter under discussion, a pair of parts is (known) to exist in the unknown quantity. When multiplication by the pair of parts in a pair of places has been accomplished in this way, by whichever part of the unknown the first number was multiplied, its square is produced there. So, when rūpas are multiplied by that part, there is a colour. Now, when the number in the second place is multiplied by the second part which has the nature of a rūpa, (then) there is the square of a rūpa there. So there is a "yāvattāvat." (Considering) that in this way in the square-quantity one square of an unknown is produced and the second is the square of a rūpa, a pair of squares is produced. Therefore (the statement:) "Having taken the square-roots from the squares" is appropriate. Now, (because of the statement:) "of each two," whatever in the square-quantity is called the "yāvattāvat," when multiplied by a pair of square-roots in a pair of places, was again multiplied by two because it was combined with itself. Therefore it has been demonstrated that "One should subtract from the remainder the product of each two multiplied by two."

Thus having described the six-fold (operation) of one colour, now desiring to describe the six-fold (operation) of many colours, he speaks of (their) addition and subtraction (with the verse that begins): "The (unknown) "yāvattāvat" and "black"."

> <21c-22b. The (unknown) "yāvattāvat" and the colours "black" and "blue" (are respectively) positive three, five, and seven. How many are they when combined with (or) diminished by the negatives two, three, and one?>

The meaning is that in this case the sum or difference is to be accomplished by the procedure of the sūtra:

> "Among those (unknown quantities), the sum or the difference of two (quantities) of the same sort (is normal), but two (quantities) of different sorts are set down separately."

The remainder is clear.

So he enunciates an example for the sake of multiplying them (with the verse that begins): "The (unknown) "yāvattāvats" are three."

> <22c-23b. The (unknown) "yāvattāvats" are negative three, there are two negative blacks and a positive blue; they are increased by a rūpa. When just these (quantities) are multiplied by the same (quantities) multiplied by two, what is the result produced by their multiplication? And what is that

divided by the multiplicand? Tell the square of the multiplicand and the square-root of this square.>

Here the multiplicand is yā –3 kā –2 nī 1 rū 1. And the multiplier, doubled with reference to the multiplicand, is yā –6 kā –4 nī 2 rū 2. Now by (the sūtra) beginning with: "The multiplicand separately," the multiplicand is to be placed in four places. (Then) one should multiply (it) by the four parts of the multiplier. There, in (performing) the multiplication as described previously (according to the sūtra):

> "In the multiplication (of quantities) of the same sort (the results are) their squares; and so, in the multiplication (of quantities) of different sorts, (the result is) their product,"

and in combining (them) according to their places, the result from multiplication is produced: yā va 18 kā va 8 nī va 2 yā kā bhā 24 yā nī bhā –12 kā nī bhā –8 yā –12 kā –8 nī 4 rū 2. Thus, as described (previously), division, squares, and square-roots are to be understood. The remainder is clear and is understood from the treatise.

Here ends the six-fold (operation) of colours.

<D. *The Six-Fold (Operation) of the Surd*>

Thus having examined the six-fold (operation) of unknowns, now wishing to speak of the six-fold (operation) of surds at the beginning he speaks of their addition and subtraction (with the verse that begins): "The sum of two (given) surds."

> <23c-24b. Assuming the sum of two (given) surds to be the great(er surd) and the square-root of the product multiplied by two to be the small(er surd), the sum and the difference of these two are (treated) like rūpas, (but) one should multiply and divide a square by a square.>

The (syntactic) connection is: "Of two surds the sum assuming to be the great(er surd) and likewise of the product of two surds the square-root multiplied by two assuming to be the small(er surd), of these two the sum and the difference are like rūpas." Whatever the two surds are in the statement of the problem, one should assume the (technical) term "mahatī" ("great(er surd)") for their sum. Then whatever is the square-root of the product of the two surds, having multiplied it by two, one should assume the (technical) term "laghu" ("small(er surd)") for it. The meaning is that then one should effect the sum or the difference of those two, the great(er surd) and small(er surd), as rūpas, (that is) as known numbers (vyaktāṃkas). Now under the guise of stating the method of multiplication of this surd, having ascertained its form, he says here: "By a square." The meaning is that one should multiply a square by a square, (i.e.) by a square number, and one should divide a square just by a square, but one should not multiply or divide a square by a (non-square) rūpa. By this the state of being a surd is indicated to be the state of being a number considered to be in the state of being a square. It is stated by Nārāyaṇa:

> "But of whatever quantity the square-root is to be taken, the name of that is "karaṇī" (surd)."

Now he enunciates the addition and subtraction of surds by another method (with the verse that begins): "Of it divided by the small(er surd)."

<24c-25b. The square-root of the great(er surd) divided by the small(er surd), increased by one (or) diminished by one, (each) multiplied by itself (and then) multiplied by the small(er surd)—(these) are their sum and difference in order. Or, if the square-root (of the above quotient) does not exist, it is put down separately.>

The (syntactic) connection is: "The square-root of the great(er surd) divided by the small(er surd) is to be taken. When one has put that in two places, in one place it is increased by one, (and) in the other place diminished by one; in both places (the result) is multiplied by itself and (then) multiplied by the small(er surd); (the products) are the sum and the difference (respectively)." Multiplied by itself (means) made into a square. The meaning is that here there is smallness and greatness, but not the state of being a smaller or a greater number as before. Now, (in answer to the question): "If, when the sum or difference is being formed, the square-root of the great(er surd) divided by the small(er surd) is not possible, then what is to be done?", he says: "If the square-root does not exist, it is put down separately."

If (it is asked): "Surely, whatever sum or difference is attained by (the sūtra) which begins: "Assuming the sum of the two surds to be the great(er surd)," in that case even though there is a similarity in such things as multiplication in accordance with what was said (previously), how does there come to be a discrepancy in the sum and the difference?," a demonstration is enunciated. In this case, the sum and the difference of the two surds which are to be spoken of, which are measured by two and eight, in accordance with what was said (previously) are 18 (and) 2. Here "two and eight" are assumed to be squares. As much as is the square of the sum of the square-roots of these two (numbers), the sum measured by that must exist. And so, when the sum is being effected in accordance with what was said, there is produced a number that is measured by the square of the sum of the square-roots. So, for instance, the surd 2. Its square-root is 1;25. And the surd 8. Its square-root is 2;51. The sum of these two is 4;16. Its square is 18;12. This is the sum of the two surds. So the difference of the two roots is 1;26. The square of this is 2;3. It has been demonstrated that this is the difference.

Now the demonstration of the sūtra for surds is from (the rule) that begins: "The sum of two surds." In that, the sum of the squares of the two square-roots (of the given surds) being increased by twice the product of the square-roots becomes the square of the sum of the square-roots. (Wondering): "In this way, however, in the case under discussion where there is ignorance of the two square-roots, how is the square of their sum known?," the teacher composed a sūtra for computing the square of the sum of the square-roots in another way (with the words): "The sum of two surds." In this case, even though there is ignorance of the square-roots, the two squares of the square-roots (of the given surds) are known with the form of being surds. Because their sum (i.e. the sum of the given surds or mahatī) is the sum of the squares of the two square-roots, therefore it was said: "(Assuming) the sum of the two surds (to be) the great(er surd)." Now the square-root of the product of these two squares is equal to the product of the square-roots. Because the sum of the squares (of the square-roots) increased by twice that (product of the square-roots) is the square of the sum of the square-roots, therefore (the verse) which begins: "And the square-root of the product multiplied by two as the small(er surd)" was enunciated. In this way, the square of the sum of the square-roots is the sum of the surds, and the square of the difference of the square-roots is the difference of the surds. Therefore it is said: "The sum and the difference of these two are (effected) as in the case of rūpas." Now because the product of two surds is equal to the square of the product of their square-roots, therefore it is said: "One should multiply a square by a square." Also, because whatever is the result in the mutual division of (two) surds—is equal to the square of the result from the division of their square-roots, therefore it is said: "And one should divide (a square by a square)." All has been demonstrated.

Now, in order that slow-witted students may understand (this) clearly it is repeated by being made an example. So here the two surds are imagined to be 9 (and) 4. In their addition as described (previously) 25 is produced, and in (their) difference 1. So the square-roots of the two surds are 3 (and) 2. The square of the sum of these two is the sum of the two surds, 25. So the difference of the two square-roots is 1. The square of this is the difference of the two surds, 1. So the product of the two surds is 36, which is equal to the square of the product of the (two) square-roots, 36.

Now for the sake of the division of two surds, two other quantities are assumed, 16 (and) 4. Here the result from division is 4. Now the two square-roots of the two surds are 4 (and) 2. The square of the quotient (obtained) in the mutual division of these two is 4. It is sufficient (to say) that the result of the division of the (two) surds is equal to this.

So here is the demonstration (of the words): "Divided by the small(er surd)." In that case, the operation is seen, that whatever is the result in the mutual division of both square-roots, the square of that (after this result has been) increased by one (and) when multiplied by the square of the square-root of the small(er surd) is the sum of the (two) surds, and, the square of the result of the division of the (two) square-roots (after this result has been) diminished by one (and) multiplied by the square of the square-root of the small(er surd) is (their) difference. Thus, because in the case under discussion whatever is the quotient in the mutual division of the two squares of the square-roots, which are known by the form of being surds because there is ignorance of the square-roots, is equal to the square of the result of the division of (their) square-roots, therefore it has been demonstrated that what was said, beginning: "The square-root of the great(er surd) divided by the small(er surd)," is correct.

Here too, for the sake of an example, the two surds are assumed to be 16 (and) 4. The two square-roots of these two are 4 (and) 2. In the mutual division of these two the result is 2. The square of this (after it has been) increased by one is 9. This multiplied by the square of the square-root of the small(er surd), 4, is the sum of the (two) surds, 36. And the square of the result of the division of the square-roots (after it has been) diminished by one, is 1. This multiplied by the square of the square-root of the small(er surd), 4, is the difference, 4. In this way the sum and the difference are 36 (and) 4.

Or else, by the procedure of the sūtra the setting out is 16 (and) 4. Here the square-root of the great(er surd) divided by the small(er surd) is 2. Having put this (result) in two places, increased by one and diminished by one becomes 3 (and) 1. Squared in order they are 9 (and) 1. Multiplied by the small(er surd) they become the sum and the difference, 36 (and) 4. Thus all is irreproachable.

Now he mentions an example for the sake of the addition and the subtraction of surds (with the verse that begins): "Of the two (surds) measured by two and eight."

> <25c-26b. Tell (me) separately the sum and the difference of two surds measured by two and eight, and of two (others) numbered three and twenty-seven, and, oh friend, thinking for a while, of (another) two measured by three and seven, if you know the six-fold (operation) of the surd.>

The (syntactic) connection is: "Oh friend! If of the surd you know the six-fold (operation), then tell (me) the sum and the difference." Now (in answer to the question) "of which two?," he says (the words) beginning: "Of the two measured by two and eight." It is clear.

And so the setting out is: ka 2 (and) ka 8. Here by (the sūtra) that begins: "Assuming the sum of the two (given) surds to be the great(er surd)," the great(er surd) is

10. Then the product of the two surds is 16. Its square-root is 4. (This) multiplied by two is the small(er surd), 8. As in the case of rūpas, the sum and the difference of these two are 18 (and) 2. These are the surds of sum and difference, ka 18 (and) ka 2.

Or else the setting out is: ka 8 (and) ka 2. The square-root of the great(er surd) divided by the small(er surd) is 2. Having put this in two places (by one) making it increased by one in one place, in the other place diminished by one, the setting out is (made): 3 (and) 1. Multiplied by themselves (they are) 9 (and) 1. Multiplied by the small(er surd), (one is) the sum and (the other) the difference, ka 18 (and) ka 2. In this way (is the procedure) in every case.

Because of the fact that in the pair of examples (given in the lines) "of two (surds) measured by two and eight, and of two (others) numbered three and twenty-seven" the square-root of the product is possible, the sum joins in also by means of a pair of methods. Then for the sake of demonstrating that, "if there is no square-root, it is put down separately," he enunciates (the lines beginning): "Of (another) two measured by three and seven." The remainder has a clear meaning.

Now he proclaims an example for the sake of the multiplication of surds (with the verse that begins): "The numbers two, three and eight are the multiplier."

> <26c-27b. The numbers two, three, and eight, (which are) surds, are the multiplier, and the number three combined with (i.e., plus) the rūpa five is the multiplicand. Tell (me) quickly the product, when the multiplicand is diminished by the rūpa five (and the multiplier is) the two surds measured by three and twelve.>

Here the multiplier is ka 2 ka 3 ka 8. So, because the multiplicand is the number three, (which is) a surd, combined with the rūpa five, the setting out is: ka 3 rū 5. Here, in the case of the multiplier, since the sum of the two surds measured by two and eight is possible, when one has effected (that) sum, the setting out is: ka 18 ka 3. And rūpas are observed in this multiplicand, ka 3 rū 5. Having made their square(s), the state of being a surd is to be brought about because it has been said previously: "One should multiply and divide a square by a square." When it has been done in this way, there is produced ka 25 ka 3. Now by the procedure of the sūtra:

> "In the multiplications of squares of unknowns and of surds, just the method of multiplying parts as described in (the mathematics of) knowns is to be thought of,"

from multiplication there is produced ka 54 ka 450 ka 9 ka 75.

Then in the second example, assuming the negativity of the rūpas in the case of the multiplicand, he declares: "Diminished by five rūpas." Or else the multiplier is assumed to be the two surds measured by three and twelve while the multiplicand is diminished by five rūpas. But the multiplicand has been described previously. There (he says): "Tell (me) the product." The remainder, which is clear, is understood from the treatise also.

Now he speaks of a special property in the rule for negative surds and rūpas being squares or square-roots (with the verse that begins): "Should be negative."

> <27c-28b. The squaring of a negative rūpa should be negative if it is achieved for the purpose of its being a surd. Likewise the square-root of a surd having the nature of a negative, is negative for the sake of the creation of rūpas.>

The (syntactic) connection is: "That squaring of a negative rūpa, if it is achieved for the purpose of its being a surd, then it is negative." The meaning is this. "Kṣayarūpāṇi" (means) negative rūpas. If their squaring is achieved for the sake of obtaining its being a surd, then it is "kṣaya," (that is) negative. Here it is to be known that the strangeness of this is made clear by the fact that while, because of the method of the sūtra previously considered, that "the square of both a positive and a negative is positive," the square of a negative obtains the state of being positive, it is (still) negative.

Now, investigating the strangeness of this (considering that) "the square-root of a negative does not exist because it is not a square," he says: "Of one having the nature of a negative." The meaning is that since, if the square-root of a surd having the nature of a negative is achieved for the purpose of establishing rūpas, then it is negative, when the square-root of negative surds is taken for the purpose of (its) being a rūpa, it is negative because it has been stated that the squaring of a negative rūpa is negative.

The demonstration in this case is (as follows). When the square-root of a negative surd is being taken by means of (the rule in the *Līlāvatī*) which begins 'having subtracted the (greatest) square from (the number in) the last odd place,' whatever square of a negative rūpa is produced with the form of being a surd, that is subtracted because the square of a negative rūpa is a negative surd. And, when this (negative surd) is being subtracted from the square-quantity, the negative quantity which is going to be subtracted, becomes positive. So, by this rule of the sūtra: "The addition of a positive and a negative (quantity) is (their) difference," the square-quantity is subtracted. And so, of whatever rūpa the square was subtracted, that rūpa is the square-root. Therefore, it was said:

> "The squaring of a negative rūpa should be negative if it is achieved for the purpose of its being a surd."

But here is an example. The square-quantity (is) ka –25. Then with respect to its square-root, since it is subtracted with the form of being a surd by means of (the rule) that begins: "Subtracting the square from the (number in the) last odd (place) one should double it," the square of five negative rūpas is subtracted. This is –25. So, since the difference is computed as previously because of (the rule) that begins: "A positive which is being subtracted becomes negative," the square-root that is obtained is rū –5. If (it is asked): "Since the difference of two surds is being computed, here, surely it must be accomplished by the difference because of the rule of the sūtra which begins: "Assuming the sum of two surds to be the great(er surd)"," we reply "no." Since the sum of two surds is computed for the sake of imagining the great(er surd) in the case of "imagining the sum of two surds to be the great(er surd)," there is excessive occurrence in what is obtained from this sūtra. Why? In the case under discussion, in what is obtained from the sūtra, since the remainder is zero when the difference of two equal surds (is computed). And it has been said:

> "It is in all cases a certainty that in the summing of two equal surds one (surd) is to be made fourfold, and in their difference is zero,"

and so on. Everything has been demonstrated.

Now the setting out of a second example useful as an example of this sūtra is. In this case, the multiplicand is ka 25 ka 3 (i.e. $\sqrt{25}+\sqrt{3}$), and the multiplier is ka 3 ka 12 rū –5 (i.e. $\sqrt{3}+\sqrt{12}-5$). Here in the multiplicand there are rūpas. Therefore, the meaning of the sūtra has been established: "When, for the sake of establishing the state of being a surd of negative rūpas, (their) squaring is being performed by the procedure of the sūtra "one should multiply and divide a square by a square," negativity (results) there." When this has been done, the multiplier which is produced is ka –25 ka 3 ka 12 (i.e. $-\sqrt{25}+\sqrt{3}+\sqrt{12}$). Here also having made the sum of the two (surds) measured by three and twelve, there is produced ka –25 ka 27 (i.e. $-\sqrt{25}+\sqrt{27}$). When the previous multiplicand ka 25 ka 3 is multiplied by this (multiplier), there is produced the result of the multiplication, ka –625 ka 675 ka –75 ka 81 (i.e. $-\sqrt{625}+\sqrt{675}-\sqrt{75}+\sqrt{81}$). Here there are two square quantities, ka –625 (and) ka 81. When the square-root of these two are taken, the meaning of this sūtra, "Likewise the square-root of a surd having the nature of a negative, is negative for the sake of the creation of rūpas,"

is established. And so the two square-roots are rū −25 (and) rū 9. The sum of these two is (their) difference, −16. Its square is −256. (Thus) the difference of these two surds, ka −625 (and) ka 81, has been produced. Also in the previous number (i.e., $-\sqrt{625}+\sqrt{675}-\sqrt{75}+\sqrt{81}$), the two remaining surds are ka 675 (and) ka −75. The difference of these two is produced in accordance with what was said (previously), ka 300. Thus, here is the setting out of the two surds of difference in order: ka −256 ka 300 (i.e. $-\sqrt{256}+\sqrt{300}$).

Here ends the multiplication of surds.

Now for the sake of dividing surds, (when one) assumes the result of the previous multiplication (from the first example) to be the dividend, and assumes its multiplier to be the divisor, making the great(er) surd to be first, the setting out is: Dividend: ka 450 ka 75 ka 54 ka 9 (i.e. $\sqrt{450}+\sqrt{75}+\sqrt{54}+\sqrt{9}$). Divisor: ka 2 ka 3 ka 8 (i.e. $\sqrt{2}+\sqrt{3}+\sqrt{8}$). Here (when one) has formed the sum of the two surds measured by two and eight in the divisor, the setting out is: ka 18 ka 3 (i.e. $\sqrt{18}+\sqrt{3}$). Now when the division is carried out by means of the method (of the verse that begins): "The divisor when subtracted from the dividend is without a remainder," what is obtained is the (previous) multiplicand rū 5 ka 3 (i.e. $5+\sqrt{3}$).

Now in the second example (the setting out is): Dividend: ka 675 ka −625 ka 81 ka −75 (i.e. $\sqrt{675}-\sqrt{625}+\sqrt{81}-\sqrt{75}$). Divisor: rū −5 ka 3 ka 12 (i.e. $-5+\sqrt{3}+\sqrt{12}$). In this case, for the sake of easiness, (when one) has made the sum of the two surds measured by three and twelve in the divisor and has obtained the state of being a surd of the rūpas, the divisor becomes ka 27 ka −25 (i.e. $\sqrt{27}-\sqrt{25}$). When the division is carried out by this (divisor) as before from the dividend, what is obtained is the (previous) multiplicand, rū 5 ka 3 (i.e. $5+\sqrt{3}$).

Now because it has been stated: "Multiplication and division are to be effected (when one) has made for the sake of easiness the sum, as it is possible, of two surds or of (more than two) surds (present) in the multiplicand or in the multiplier, in the dividend or in the divisor," in the second example, the setting out for the sake of the division of the surds contained in the result of multiplication is: Dividend: ka 81 ka –625 ka 675 ka –75 (i.e. $\sqrt{81}-\sqrt{625}+\sqrt{675}-\sqrt{75}$). Divisor: the previous multiplier, rū –5 ka 3 ka 12 (i.e. $-5+\sqrt{3}+\sqrt{12}$). Here, when for the sake of easiness, the sum of the surds in the dividend is being made, because it has been stated: "The addition of a positive and a negative (quantity) is (their) difference," the difference results. Therefore, when the difference of the first and the second, and of the third and the fourth (surds) is made by means of the method "(assuming) the sum of two surds to be the great(er surd)," the two surds which are produced are ka –256 (and) ka 300. Or else, the difference of these two surds ka 81 (and) ka –625 (is made) also by means of the method because they are in the state of being square-quantities as follows. When the square-root of these two is taken again to be rū –25 (and) rū 9, in the addition of these two (in view of) "the addition of a positive and a negative (quantity) is (their) difference," the difference is produced; this is the sum, rū –16. Its square, the difference of those two surds, is produced, ka –256.

Now for the sake of the division of these two surds of difference the setting out is: Dividend: ka –256 ka 300 (i.e. $-\sqrt{256}+\sqrt{300}$). Divisor: this multiplier, ka –25 ka 3 ka 12 (i.e. $-\sqrt{25}+\sqrt{3}+\sqrt{12}$). In this case also, when the sum as mentioned of the two surds measured by three and twelve is made, the divisor is produced, ka –25 ka 27 (i.e. $-\sqrt{25}+\sqrt{27}$). When the division of the dividend is carried out by this, when it is performed by the methods of sūtras such as:

> "The divisor, after being multiplied by whatever colours and by whatever rūpas each in its own place in order, having been subtracted from the dividend, is without a remainder,"

it is seen that the divisor, after being multiplied in this way by the two surds ka 25 ka 3 (i.e. $\sqrt{25}+\sqrt{3}$) which occur in the previous multiplicand, (having been subtracted) from the dividend, is without a remainder as follows. When this divisor ka –25 ka 27 (i.e. $-\sqrt{25}+\sqrt{27}$) is multiplied by these two surds ka 25 ka 3 (i.e. $\sqrt{25}+\sqrt{3}$) which occur in the previous multiplicand, because it has been stated:

> "In the multiplications of squares of unknowns and of surds the method of multiplying parts as described in (the mathematics of) knowns is to be thought of,"

when the method of multiplying parts: "The multiplicand which is equal to the parts of the multiplier separately is to be entered," is carried out, in that case the multiplicand is indeed the divisor. And this is ka –25 ka 27 (i.e. $-\sqrt{25}+\sqrt{27}$). The previous multiplicand is its multiplier. And this is rū 5 ka 3 (i.e. $5+\sqrt{3}$). Here, because it has been stated: "One should multiply and divide a square by a square," (when one) has established the state of being surds of the rūpas, the multiplier ka 25 ka 3 (i.e. $\sqrt{25}+\sqrt{3}$) is produced. Here, (considering) that in the multiplier there is a pair of parts, (when one) has put in two places the new multiplicand which is in the form of the divisor, the setting out is:

ka –25	ka 27 (and)
ka –25	ka 27.

When this is multiplied by these two surds ka 25 ka 3 which occur in the (new) multiplier which is in the form of the previous multiplicand, there is produced

ka –625	ka 675 (and)
ka –75	ka 81.

Then, since it has been stated: "When it has been multiplied by these parts in order, it is combined according to what was said," when the sum of these (products) is made, since "the addition of a positive and a negative (quantity) is (their) difference," the difference results.

Now for the sake of their difference, the setting out in order is: ka –625 ka 81 ka 675 ka –75 (i.e. $-\sqrt{625}+\sqrt{81}+\sqrt{675}-\sqrt{75}$). In this case, when the difference of the first and the second, (and) of the third and the fourth (surds) is made as (already) described, in the divisor a pair of surds ka –256 ka 300 is produced. (When one) has caused (it to happen) that this, being subtracted from the dividend, which is the result of the previous multiplication, this ka –256 ka 300, by the method (of the sūtra): "A positive (quantity) that is going to be subtracted attains negativity," is without a remainder, what is obtained is the previous multiplicand; this is rū 5 ka 3. So it is to be known by a wise person in every case.

Now, for the sake of the division of surds by another method, he enunciates a sūtra by means of two (verses, beginning): "The reversal of the positivity or negativity of a chosen (surd)."

> <28c-29b. Having established repeatedly the reversal of the positivity or the negativity of a chosen surd in the divisor, by such a divisor one should multiply the dividend and divisor until there is only one surd in the divisor.>

> <29c-30b. By that (surd) the surds (which are) in the dividend are to be divided. If the surds which are obtained are arising from addition, they are to be made separately by the separation-sūtra as they are desired of the inquirer.>

The (syntactic) connection is: "In the divisor, of a chosen surd the reversal of the positivity or the negativity repeatedly having established, by such a divisor, the dividend and the divisor so long one should multiply." So long in what manner? Until in the divisor, i.e. the "cheda," there is only one surd. In this way by that one surd the surds in the dividend are to be divided. In that division whatever surds are obtained, if they are arising from addition, then by the separation-sūtra which will be mentioned they are to be made separately in that manner. How "in that manner?" It is clear that (it is) as they are desired of the inquirer, i.e. "praṣṭuḥ."

The demonstration in this case is (as follows). When the reversal of the positivity or the negativity of the divisor has been made, some third number is produced other than the dividend and the divisor. The dividend and the divisor are multiplied by that and, when added or subtracted as described (previously), become small since among a positive dividend and divisor which are multiplied by a negative divisor, or else a negative dividend and divisor which are multiplied by a positive divisor, because of the positivity and negativity some surds whose difference (must be taken) have been produced. Because when their difference is taken in this way, smallness is produced, therefore it has been said: "The reversal of the positivity or the negativity of a chosen (surd)." Or else, after one has multiplied the dividend and the divisor by any number whatsoever, when the division has been carried out, the result is like the previous result. So the dividend and the divisor are assumed to be 16 (and) 4. Here, from division the result is 4. So, it has resulted that, after one has multiplied the dividend and the divisor by two, when the division has been carried out again, the same result is obtained.

(Considering) that they are to be made separately by the separation-sūtra, because of the obscurity of the separation-sūtra wondering what it is, he proclaims (the verse beginning): "By a (certain) square."

<30c-31b. The surd of addition when divided by a (certain) square is reduced (without a remainder). When one has made the parts of its square-root as desired, the squares of those (parts) multiplied by the previous quotient become separately the surds.>

The (syntactic) connection is: "The surd of addition by what square when divided is reduced (without a remainder), of its square-root as desired parts when one has made, the squares of those by the previous quotient being multiplied these surds separately are." By what square—i.e. square quantity—when divided a surd of addition is reduced—i.e., becomes without remainder, of its square-root—i.e., of the square-root of the square-quantity which has become the divisor—parts are to be made as desired. Then the squares of those parts, when multiplied by the previous quotient, become separately the surds. Here the meaning of "by the previous quotient," is whatever is the quotient where a surd of addition has been divided by a (certain) square.

The demonstration in this case is (as follows). Here the (present) sūtra has been composed by the inversion of this sūtra: "The square-root of the great(er surd) divided by the small(er surd)." As for instance, in this case, when, after the addition of two surds is effected by this addition-sūtra, it is multiplied by itself (and then) multiplied by the small(er surd), then in that case, considering that (each) of the two square-roots, (one) increased by one (and the other) diminished by one, is "multiplied by itself," the square of the square-root (which was) increased by one was multiplied by the small(er surd). Now if this is again divided by this square, then by whatever small(er surd the square was) multiplied, it will arrive at the result. Thus, by this (procedure) from the surd of addition the small(er) surd is known. Hence it has been enunciated: "The surd of addition when divided by a (certain) square." Now by whatever square the surd of addition is divided, that square was made the square of the square-root increased by one (or) diminished by one by means of (the rule) beginning: "(Each) multiplied by itself." If by one who has taken its square-root its parts are made, then that square-root increased by one or diminished by one arises. Therefore it has been enunciated: "The parts of the square-root of that square." Now if the square of this is effected, then the quotient of the great(er surd) divided by the small(er surd) is known. So it has been demonstrated that if that quotient from division by the previous square is multiplied by the small(er surd), then it becomes the surd of addition.

Now, in order to see that it is with respect to the operation of the sūtra beginning: "The reversal of the positivity or the negativity of a chosen (surd)," the setting out of the previously described dividend and divisor is: Dividend: ka 9 ka 450 ka 75 ka 54. Divisor: ka 18 ka 3. Then by the procedure of the sūtra: "The reversal of the positivity or the negativity of a chosen (surd)," assuming the negativity of the surd measured by three in the divisor, the setting out of the divisor is ka 18 ka –3. In this case, there is a pair of parts in the divisor. Therefore, when the dividend is multiplied by those two (parts) in two places, there is produced ka 162 ka 8100 ka 1350 ka 972 (and) ka –27 ka –1350 ka –225 ka –162. Now, when, for the sake of easiness, the sum of these surds is made, in our mathematics whose tradition still exists (it has been said) as follows:

> "It is a certainty that, in the summing of two equal surds, one (surd) is multiplied by four; (and), when their difference is to be taken, there is zero."

And so in the difference of equal surds zero is produced. Therefore they have disappeared.

Now the setting out of the surds remaining (after the difference has been effected) is: ka 8100 ka –225 ka 972 ka –27. In this case, in the difference of the first and the second (and) of the third and the fourth there is produced a pair of surds, ka 5625 ka 675. Now in accordance with what was said (previously), when the divisor has been multiplied in two places, the setting out is: ka 324 ka 54 (and) ka –54 ka –9. Here in the disappearance of the two equal surds and in the difference of the two remaining ones because of (the instruction): "until there is only one surd in the (resulting) divisor," there is produced one surd in the divisor, ka 225. When the division of this dividend, ka 5625 ka 675, by this (surd) has been carried out, what is obtained is this previous multiplicand ka 25 ka 3.

In this way it is also produced by this method in the second example.

Just as the result of multiplication is the dividend of this, ka 81 ka –625 ka 675 ka –75, so this multiplier, ka –25 ka 27, is the divisor. So here, having made as mentioned the difference, (which is) the sum, of the first and the second surds and of the third and the fourth in the dividend for the sake of easiness, there are produced in the dividend in order two surds of difference, ka –256 ka 300. And likewise, in this divisor ka –25 ka 27 if one assumes the positivity of the surd twenty-five, the setting out of the divisor is: ka 25 ka 27. Here in the divisor there exists a pair of parts. Therefore, when the dividend has been multiplied by those two (parts) in two places, there is produced: ka –6400 ka 7500 (and) ka –6912 ka 8100. Then, when the addition of these surds has been made for the sake of easiness, since "the addition of a positive and a negative (quantity) is (their) difference," the difference is produced.

Thus, in this case, for the sake of (finding) the difference the setting out of the surds in order is: ka 8100 ka –6400 ka 7500 ka –6912. In this case, in (taking) the difference of the first and the second (surds, and) of the third and the fourth there is produced a pair of surds, ka 100 ka 12. Now, when the divisor (has been multiplied) in two places as was described, the setting out is: ka –625 ka 675 (and) ka –675 ka 729. Here too, when the disappearance of the two equal surds and the difference of the two remaining ones (have been made), because of (the instruction): "Until there is only one surd in the divisor," there is produced one surd in the divisor, ka 4. When the division of this dividend, ka 100 ka 12, by this (surd) has been carried out, what is obtained is this previous multiplicand, ka 25 ka 3.

Now when, (as) in the previous (i.e. this last) example, the result of the multiplication is made the dividend and the multiplicand the divisor, the setting out is: Dividend: ka 9 ka 450 ka 75 ka 54. Divisor: ka 25 ka 3. Here too, if one assumes the negativity of the surd measured by three, the setting out of the divisor is: ka 25 ka –3. Thus, here in the divisor there exists a pair of parts. Therefore, when the dividend has been multiplied by those two (parts) in two places, there is produced: ka 225 ka 11250 ka 1875 ka 1350 (and) ka –27 ka –1350 ka –225 ka –162. Then, when the addition of these surds has been done for the sake of easiness, since "the addition of a positive and a negative (quantity) is (their) difference," the difference is produced. In that case, when the difference of the equal surds has been made by the method: "(Assuming) the sum of two surds (to be) the great(er surd)," just a zero occurs. Therefore, those which are equal surds are abandoned.

In this way the setting out of the remaining surds is: ka 11250 ka –162 ka 1875 ka –27. Here also, when the difference of the first and the second and of the third and the fourth has been made as was described, in the dividend a pair of surds is produced, ka 8712 ka 1452. Then, when the divisor has been multiplied in two places as was described, the setting out is: ka 625 ka 75 (and) ka –75 ka –9. Here too, when the disappearance of the two equal surds and the difference of the two remaining ones has been made, there is produced one surd in the divisor, ka 484.

The setting out of the dividend and the divisor which have been produced in this way is: ka 8712 ka 1452 (and) ka 484. In this case, what is obtained in division is the multiplier, ka 18 ka 3. Here, in the previous multiplier there was a triad of parts. Therefore this is imagined to be a "surd of addition." Therefore the separation is obtained by the rule of the separation-sūtra. And so the surd of addition is just this ka 18. This when divided by the square measured by nine, (i.e.) 9, reduces without a remainder. In that (division) what is obtained is 2. Now the square-root of nine, which has become the divisor, is 3. Its two parts are 1 (and) 2. Their squares are 1 (and) 4. When they are multiplied by what was obtained before, by this 2, two surds are produced separately, ka 2 (and) ka 8. Thus, the multiplier is produced in order, ka 2 ka 3 ka 8.

Here ends the division of surds.

Now he enunciates an example concerning the square of (a sum of) surds (with the verse beginning): "Measured by two, three, and five."

> <31c-32d. Measured by two, three, and five are the surds. Tell me quickly, oh learned one, the square(s) of these, and of two numbered three and two separately, (and) the squares of (those) measured by six, five, three and two (and) measured by eighteen, eight and two oh friend, and the square-roots of the squares.>

Here the meaning of the verse is easy to obtain.

Now for the sake of (taking) the square, the setting out of the first example is: ka 2 ka 3 ka 5. In this rule for the squares of surds it has been stated by the teacher that (by one) remembering the procedure for (taking) the square which is told in the *Pāṭīgaṇita*:

> "The square of the last (digit counted from the right) is to be placed, (then the other digits) multiplied by twice the last,"

the square is to be made. (But) in that there is this difference. Since, when one has carried out (the rule): "multiplied by twice the last," (it is observed) that, when it has been done so, there is no square, because one remembers the special (rule):

> "One should multiply and divide a square by a square,"

having made a square in order to make the (number) two, which has the nature of rūpa, into a surd, one should multiply the other numbers. And so, the meaning of the sūtra: "multiplied by twice the last" becomes in this case: "multiplied by four times the last." It is to be known that, when the square of the root-surds has been taken, (then) whatever are the square-quantities, one should imagine the sum of their square-roots to be rūpas.

Thus in this matter, in the square of (a sum of) surds, it is seen that there are square-quantities which are "sahajās" (produced by multiplication with themselves) and "nimittajās" (produced by multiplication with an unequal quantity). In that, sahajās are produced because there is the multiplication of two equal (quantities) when they are in odd places. When they are in even places nimittajās are produced arising from the multiplication of unequal (quantities); so, when four are multiplied by four, the "16" that are produced are sahajās, (and) when eight or thirty-two are multiplied by two, the "16" (or) "64" that are produced are nimittajās. Thus the meaning is that here, when the square of (a sum of) surds is computed, whatever square-quantities are sahajās, one should imagine the sum of the square-roots of that many to be rūpas; but the surds are assumed to be nimittajās. When their sum is computed as far as possible, since they are produced from their own qualities (by the rule): "The multiplication of two equal (quantities)," all the sahajās become square-quantities. Hence it is true that the rūpas in the square of a surd are the sum of the square-roots of all (these), as many as there are, not the surds measured by (numbers) such as one. In that case there is no breaking off of the operation. Therefore, since it has been stated by the author of the treatise: "When one has computed for the sake of easiness as far as possible the sum of two surds (or) of (more than two) surds, (their) square and square-root should be computed," so when at any time, after the square of the given root-surd has been computed, all the square-quantities are sahajās or nimittajās, likewise whatever is the sum of their square-roots is the sum of all the surds in the square-roots. When the sum of the root-surds has been produced in this way, when one has separated (them) as desired by the separation-sūtra, they become root-surds. Just as the root-surds ka 18 ka 8 ka 2, when multiplied by themselves by the method (of the sūtra):

> "In the multiplications of squares of unknowns and of surds the method of multiplying parts as described in (the mathematics of) knowns is to be thought of,"

become: ka 324 ka 144 ka 36 (and) ka 144 ka 64 ka 16 (and) ka 36 ka 16 ka 4. All these sahajās and nimittajās have been produced as square-quantities. Therefore, the sum of the square-roots of all of them is rū 72. The square of the surds is produced as rū 72. This is the sum of the root-surds, ka 72. Considering that there was a triad of parts of the root-surds, computing (according to the sūtra):

> "The surd of addition when divided by a (certain) square is reduced (without a remainder),"

it is reduced by thirty-six; what is obtained is 2. The parts of 6, the square-root of thirty-six, are 3 (and) 2 (and) 1. The squares of these are 9 (and) 4 (and) 1. Multiplied by 2, the previous quotient, they become the root-surds, ka 18 ka 8 ka 2.

Or else, even in such a place, if one assumes the sum of the square-roots of the sahaja squares which are measured in the places of the root-surds to be rūpas, yet the other squares are surds. When one has combined them as far as possible, in the square are the surds which are referred to by (the words): "The sums of (the natural numbers) beginning with one." Therefore the square-root is possible by the method: "When one has subtracted from the square of rūpas." In this way (is the procedure) in every case. When it has been computed in this way, there are produced in order the squares: rū 10 ka 24 ka 40 ka 60 (and) rū 5 ka 24 (and) rū 16 ka 120 ka 72 ka 60 ka 48 ka 40 ka 24.

Having described the square of the surds in this way, now he enunciates a karaṇasūtra on the square-root of surds with two (verses beginning): "If in a square, of one surd or two surds."

<33a-34d. If in a square one should subtract from the square of rūpas the rūpas equal to one surd or two surds or many (and if) the rūpas are separately increased or diminished by the square-root of the remainder, in their half there is a pair of surds. Now if in the square there are surds remaining, whichever is the great(er) surd of those two in the square-root these are made rūpas (to be computed) after that.>

One should subtract from the square of the rūpas rūpas equal to one surd, two surds, or many surds—four or five—in a square—(i.e.), a square quantity. Whatever is the remainder there, the rūpas are separately to be increased or diminished by its square-root. In their half there is a pair of surds. Now, assuming that, when it has been computed in this way, if there are surds remaining in the square, then whichever is the great(er) surd in the square-root, these are made rūpas, (one realizes) that one must keep on computing until the square is without a remainder. If (it is asked): "Is the phrase "equal rūpas" meaningless because the objective is attained by this (rule): "One should subtract one surd or two surds or (more) surds from the square of rūpas"? ", (the answer is) surely "no." In subtracting the surd(s) from the square of the rūpas there is excessive occurrence in what is obtained from the sūtra:

"The square-root of the great(er surd) divided by the small(er surd)."

The taking of equal rūpas is for the sake of its removal.

The demonstration in this case is (as follows). Here when the square of the surd has been taken as described (previously), the squares of just as many (surds) occur as there are surds in the square-root quantity. Thence there are other surds also which are multiplied by four times the final surd. As many as are the square-quantities produced in this way in it, the collection of the square-roots of so many is assumed to be rūpas. Whatever rūpas have been produced in this way, that is known as the sum of the surds in the square-root. Now by the method of concurrence, for the sake of separating the surds in the sum, with respect to the difference rūpas equal to the remaining surds are subtracted from the square of the rūpas. When they have been subtracted, the square of the difference of the surds will be left. The square-root of that is known to be the difference of the surds because when the product of (two) quantities multiplied by four is subtracted from the square of the sum of two quantities, the square of the difference of the (two) quantities will be left. In the case under discussion, whatever are the rūpas equal to the surd, that is four times the product of a pair of root-surds which is going to be produced because the first multiplied by four times the last is four times the product of those two. When the square of the sum is less than that, the square of the difference will be left. When the difference is known in this way, one who has set in two places the rūpas known by their being the sum of surds by the procedure of the concurrence-sūtra:

> "The sum diminished and increased by (their) difference, when halved, is those two (quantities),"

and who has caused them to be diminished and increased by difference (and) halved, obtains a pair of surds. There whatever is the smaller surd, becomes one root-surd. And likewise whichever is the greater becomes the sum of the remaining surds. Then, when one has made them rūpas (and) again for the sake of knowing the difference has subtracted the remaining surd from the square of the rūpas, the square-root of the remainder is the difference of the remaining surds. Also, again, when one has made the rūpas (set) in two places to be diminished and increased by that (difference) and these to be halved, again a pair of surds is obtained. Thus (one continues) until the square is without a remainder. Hence what was said:

> "If in a square, the rūpas are equal to one surd or to two surds,"

and so on, has been demonstrated.

Now the setting out for the sake of the square-root of the first square is: rū 10 ka 24 ka 40 ka 60. Here the form of the sūtra is as follows: In this square the rūpas equal to the two surds measured by twenty-four and forty are these: 64. When one has subtracted (them) from the square of the rūpas, from this 100, the remainder is 36. Its square-root is 6. When one has set the rūpas in two places (and) made them to be diminished and increased by this, there are produced 16 (and) 4. In the halving of these two a pair of surds is produced, ka 8 (and) ka 2. Now in the square one surd is remaining. For the sake of its square-root, of the pair of surds in the square-root, the great(er) is this surd, ka 8. When one has assumed these to be rūpas (and) has subtracted from the square of this, from this 64, the rūpas equal to the surd sixty, the remainder is 4. Its square-root is 2. When one has made the rūpas to be increased and diminished by this and to be halved, again a pair of surds is produced, ka 3 (and) ka 5. Thus, the square-root of the surd is, in order, ka 2 ka 3 ka 5. "It is to be understood in this way elsewhere also." The remainder is clear.

Now he describes the special usage of positivity and negativity in the square of a surd (with the verse beginning): "If having the nature of a negative."

> <35a-d. If a surd in a square has the nature of a negative, when one has assumed it to have the nature of a positive, the two surds in the root are to be obtained. Of these two, the chosen one is to be understood by an intelligent person to have the nature of a negative.>

If there is in a square a surd having the nature of a negative, then when one has assumed that to have the nature of a positive in order to obtain the square-root, there come to be two surds in the root. Between these two one (surd) is to be understood by an intelligent person to have the nature of a negative.

The demonstration in this case is (as follows). If in the square of a surd there exists a surd which is negative, then when the subtraction of the rūpas which are equal to that (surd) from the square of the rūpas has been completed, the sum of both takes place (by the rule that) 'a negative (quantity) that is going to be subtracted becomes positive.' And when the sum has been made, there is a breaking off of the operation. Consequently, positivity or negativity is to be assumed in the square-root as (in the sūtra) beginning:

> "The square of a positive and of a negative (quantity) is positive; the two square-roots of a positive (quantity) are positive and negative."

But in a square is the state of having the nature of a positive. Hence it has been demonstrated that there is a superiority of "(the reading) sādhye."

Here he proclaims an example with half a verse (beginning): "Tell (me) of the two (surds) measured by three and seven."

> <36a-b. Tell me the square of the difference of the two surds measured by three and seven and the square-root from the square.>

The (syntactic) connection is: "Oh friend! Of the two surds measured by three and seven the square of (their) difference tell (me) and also from the square the square-root tell (me)." The square of the difference (means) the square of that which is negative. And so the setting out is: ka −3 ka 7 (and) ka 3 ka −7. When the square of these two has been computed separately by the procedure of the sūtra, the square is the same. That is as follows: rū 10 ka −84. When one has assumed the positivity of the negative surd in this square, since one of those two root-surds which have been obtained 'is chosen to be negative,' there results ka −3 ka 7.

Now, with the imagining of more (surds), he again enunciates another example (with the verse beginning): "The surds measured by two, three and five are positive, positive and negative (respectively)."

<36c-37b. The surds measured by two, three and five are positive, positive and negative (respectively) or have the positives and the negatives reversed. Tell (me) oh friend, their square and the square-root from the square, if you know the six-fold (operation) of a surd.>

The (syntactic) connection is: "Oh friend! If you of a surd the six-fold (operation) know, then the previously mentioned surds measured by two, three and five—when one has assumed (them) to be positive, positive and negative (respectively), or when one has assumed (them) to have the positives and the negatives reversed—their square and the square-root of the square tell (me)." "Svasvarṇagāḥ" (means) positive and positive and negative. The meaning is that there are two positive surds and one negative. So by this (term) "vyastadhanarṇagā" it is clear that there are two negative surds and one positive.

Thus in this case, the setting out is: ka 2 ka 3 ka –5 (and) ka –2 ka –3 ka 5. The square of these two is the same, because it has been stated that:

"If in a square a surd has the nature of a negative"

(and so on). And so the square is this: rū 10 ka 24 ka –40 ka –60. In this case, rūpas equal to the two negative surds are for the sake of the square-root positive, these 100. When one has subtracted these from the square of the rūpas (and) has made the rūpas to be increased and diminished by the square-root of the remainder, half of them is produced, 5.

Now, when one has subtracted rūpas 64, (which are) equal to two positive surds, from the square of the rūpas, the two halves of the rūpas which have been increased and diminished by the square-root of the remainder, this 6, are ka 2 (and) ka 8. When one has assumed the negativity of the great(er) of these two (surds and) has made them rūpas in accordance with what was said (previously), the two surds are ka 3 (and) ka –5. Thus in this case the negativity of the great(er surd) is to be assumed. The rest is clear.

Now, with respect to what is possible in the square of surds, he proclaims some rule (with two verses beginning): "Measured by the sums (of the natural numbers) beginning with one."

<37c-39b. In a square-quantity there are surd-parts measured by the sums (of the natural numbers) beginning with one. When one has subtracted from the square of rūpas rūpas equal to two surds in a square having three surds, to three in one having six surds, to four in (one having) ten, and to five in (one having) fifteen, the square-root is to be taken. If in some case it is otherwise, it is not possible.>

In a square-quantity there are surd-parts measured by the sums (of the natural numbers) beginning with one. "Ekādi" is that of which the beginning is one. The meaning is that: "(The series) beginning with one; and the sum of that (series); the surd-parts measured by these (sums)." Since in the square of chosen surds, there is necessarily a rule for the existence of rūpas, so that in the square of one surd rūpas occur, and if the square of two surds is made, then rūpas (and) one surd would occur, and also, when the square of three (surds) has been made, rūpas and a triad of surds would occur, the meaning is that there are surd-parts measured in order by the sums of one, two, three, four, five, and so on (natural numbers).

Then for the sake of removing the doubt of ignorant students in extracting the square-root of the square of a surd, he enunciates a rule for equal rūpas (with the verse) beginning: "In a square having a triad of surds." The meaning is clear.

Now when one has taken away, i.e. subtracted, from the square of the rūpas rūpas equal to the (previously) described surd-parts, (by the rule): "From the square of the rūpas," the square-root is to be taken. Now if the square-root is taken by a rule different from that mentioned, "if in some cases otherwise," that is impossible. But this is the rule as described: When one has first subtracted rūpas equal to three surds in (a square) having six surds from the square of the rūpas, then (rūpas) equal to two (surds), then to one (surd), the square-root is to be taken. When one has left aside the method, (i.e. when one uses an) order other than (the order) described in this way, (as) sometimes in examples such as "twelve and fifteen multiplied by four," the square-root is to be taken otherwise. The meaning is that, as—first rūpas equal to one surd, then to two, then to the rest—when one has made (the computation) in this way, the square-root is taken, that is impossible because the square of the square-root does not exist.

Now (the verse which begins): "By the one which is going to be produced."

<39c-40a. Of whichever (surds) division is possible by four times the small(er) root-surd which is going to be produced in this manner, are to be subtracted from the square of the rūpas.>

By the small(er) surd which is going to be produced, which is also multiplied by four, of whichever (surds) division is (possible), only those are to be subtracted from the square of the rūpas.

And now he proclaims the knowledge of the root-surds by another method.

40b-41a. Whatever (surds) are obtained in the division, they too are certainly root-surds. If they are not produced by the method of the remainder, then that square-root is impossible.

The meaning of this (verse) is: in the division whatever (surds) are obtained, (i.e.) have attained numeration, all of them are root-surds. The meaning is that all the root-surds become known by such an operation. The meaning is that, if they, when they are known by the method of the remainder which is mentioned (by the verse):

> "That which is the great(er) surd of the two in the square-root is (turned into) rūpas,"

are not root-surds, do not agree, do not attain agreement, then this square-root is impossible. We will explain everything on the occasion of an example.

The demonstration in this case is (as follows). There the sum (of the natural numbers) beginning with one is the sum of (the numbers) one, two, three and so on in increasing progression. That is as follows.

1	2	3	4	5	6	7	8	9
1	3	6	10	15	21	28	36	45

Here, since the sum of one is one, then of two three, of three six, of four ten, (and) of five fifteen, therefore it has been said: "in a square having a triad of surds" and so on.

Now when the square-root of the square of a surd is to be taken (by the rule beginning): "By (the surd) which is going to be produced," whatever root-surd is going to be produced is just the last surd. When it is grasped that, when that is known, the others are also to be known, a sūtra was composed by inverting this sūtra: "The square of the last (digit) is to be placed (above itself; then the other digits) are multiplied by twice the last (digit)." That is as follows: by this (rule): "The square of the last (digit) is to be placed (above itself; the other digits) are multiplied by twice the last (digit)," which here, in the square of surds, (is to be interpreted): "multiplied by four times the last (digit)," the other surds (which are) different from the last were multiplied by the last multiplied by four. Now just those, when they are divided by the last multiplied by four, come to have the remaining (surds) in their own squares. Thence, if, after one has subtracted them from the square of the rūpas, the square-root is taken as before, then, since the root-surds result, therefore what was said (in the rule) beginning: "The division of which is possible," has been demonstrated.

In this place he has mentioned an example (with the verse that begins): "In a square where the surds (are measured) by thirty-two (and) twenty-four."

> <41b-42a. In a square where the surds are measured by thirty-two, twenty-four (and) eight, (and) are augmented by ten rūpas, tell (me), oh learned man, what is its square-root?>

Thus in this case the setting out is: rū 10 ka 32 ka 24 ka 8. Here there is a triad of surds in the square. Therefore since, when one has subtracted the rūpas equal to a pair of surds from the square of the rūpas, as the square-root is taken as described (previously) it is not obtained, when one has computed rūpas equal to all (the surds), 64, (and) when one has subtracted (these) from those (i.e., the square of the rūpas), the remainder is 36. The two halves of the rūpas increased and diminished by its square-root, 6, are the two surds, ka 8 (and) ka 2. The meaning is that, since this is not the square of this square-root, this is indicated to be faulty.

Now he enunciates an example pertaining to the subject of this (rule which begins): "By (the surd) which is going to be produced in this manner" (with the verse that begins): "In a square where the surds (are equal to) fifteen, thirteen and three."

> <42b-43a. In a square where the surds are equal to fifteen, thirteen and three multiplied by four, (and) are united with ten rūpas, tell (me), what is its square-root?>

And so the setting out is: rū 10 ka 60 ka 52 ka 12. Here in the square there exists a triad of surds. Therefore, when one has subtracted the rūpas equal to a pair of surds measured by fifty-two (and) twelve, these 64, from the square of the rūpas, this 100, the square-root of the remainder is 6. The rūpas are increased and diminished by this. The two halves of these are ka 2 (and) ka 8. In this case the small(er) surd which is going to be produced is this ka 2. When one has made what is equal to four times this, ka 8, usable as rūpas, because division of the two surds measured by fifty-two and twelve does not proceed, those two surds are not to be subtracted. The meaning is that, since it has been stated: "of whichever (surds) division is possible are to be subtracted from the square of the rūpas," therefore this is impossible.

Now he states an example where there is the possibility of this (enunciation) "of whichever (surds) division is possible" (with the verse that begins): "Eight, fifty-six."

> <43b-44a. In a square where there is a triad of surds, eight, fifty-six, (and) sixty, augmented by ten rūpas, tell (me): what is its square-root?>

In this case the setting out is: rū 10 ka 8 ka 56 ka 60. Here the first pair of parts is ka 8 (and) ka 56. When one has subtracted the rūpas equal to this, these 64, from the square of the rūpas, the square-root of the remainder is 6. By this, as before, a pair of surds is obtained, ka 2 (and) ka 8. Here by the division of this pair of parts by four times the small(er surd), by this 8, two parts are obtained, 1 (and) 7. Here is the manifestation of the sūtra: "they are obtained from division." If the surds which are obtained from the previously described division are not root-surds, then they are to be computed by the method of the remainder, which begins: 'By the square-root of the remainder.' If they do not come into being by even that, then the square-root is not possible, is not correct; or the reading is "āsannam" (approximate). Thus in the case under discussion, in this pair of parts, 1 (and) 7, the two surds are not produced by the method of the remainder. The meaning is that therefore those two are not to be subtracted.

Now he proclaims another example (with the verse that begins): "Four times twelve, fifteen, five, eleven, eight and six."

<44b-45a. In a square where the surds are four times twelve, fifteen, five, eleven, eight and six together with thirteen rūpas, tell (me) its square-root if you are respected for (your) cleverness in algebra.>

In this case the setting out is: rū 13 ka 48 ka 60 ka 20 ka 44 ka 32 ka 24. In this (square) having six surds, when one has first subtracted rūpas equal to three surds from the square of the rūpas, the square-root is to be taken. When one has afterwards done the same for two, then for one, there is an absence of the square-root. Now with the intention of (finding it) possible in another way, when one has subtracted rūpas equal to the first surd, these 48, from the square of the rūpas, from this 169, the remainder is 121. Its square-root is 11. The rūpas are increased and diminished by this. Their two halves are 1 (and) 12. Here there is a great(er) surd. When one has made it to be rūpas (and) has subtracted rūpas equal to the preceding pair of surds as was described (previously), again there are two surds, ka 2 (and) ka 10. Here too, when one has made the great(er) surd to be rūpas (and) has subtracted rūpas equal to the triad of surds preceding that, these 100, from the square of the rūpas, this 100, the square-root of the remainder is 0. The rūpas are increased and diminished by this. Their two halves are 5 (and) 5. In this way the square-root is in order: ka 1 ka 2 ka 5 ka 5. The meaning is that it seems that this is impossible in this way because it is not its square. Thus in the case of the square of a surd of this kind an approximate square-root is to be obtained.

So the method of obtaining the approximate square-root has been described by our father's feet in the chapter on algebra in the *Siddhāntasundara* that he composed. It is as follows:

"The (imagined approximate) square-root is increased by that (quotient) which is obtained from its square divided by the approximate square-root (and the sum) is divided by two; that is a near(er) square-root. Then, (if one repeats this) again and again, the (nearly) accurate square-root results."

The meaning of this is: for whatever square quantity or non-square quantity the (imagined) approximate square-root is to be taken, by that (approximate square-root) its own square is to be divided. There whatever (quotient) is obtained, the (imagined) approximate square-root is to be combined with that, and that divided by two becomes the near(er) square-root. The meaning is that one should proceed in this manner again and again until the square is without a remainder.

Here, however, is an example. The chosen quantity is 5. Its imagined square-root is 2. By this its own square is divided: $\frac{5}{2}$. This (imagined) square-root, 2, is combined with that which was obtained: $\frac{9}{2}$. It is divided by 2: $\frac{9}{4}$. This is the near(er) square-root. Now, when one has again computed (by the rule): "by the approximate square-root," the square-root of five is obtained: 2;14. That is enough because (it becomes) excessively long.

Now he proclaims another example (with the verse that begins): "Forty."

<45b-46a. If, in a square there are surds equal to forty, eighty, and two hundred, (and) they are combined with seventeen rūpas, tell (me): what is the square-root in that case?>

Here the setting out is: rū 17 ka 40 ka 80 ka 200. The square-root (obtained) as described (previously) is in this case ka 10 ka 5 ka 2.

In the excellent commentary on the *Bīja(gaṇita)*, the *Sūryaprakāśa*, which is distinguished by all the adornments of the virtuous and the wise, which is capable of destroying the darkness in the hearts of unintelligent students, and which has been enunciated by the poet, calculator, and teacher named

Sūrya, the son of the astrologer Jñānarāja, a collection of operations with positive and negative (quantities) and with one and more than one colours and of the six-fold (operation) with surds has been produced.

Thus in the commentary on Bhāskara's *Bīja(gaṇita),* called the *Sūryaprakāśa,* composed by the astrologer, the Paṇḍita Sūrya, the chapter concerning the six-fold (operation) has come to a conclusion.

4. <*Text Alpha*, Third Chapter>

<*The Chapter Concerning the Pulverizer*>

Salutations to Him who has an elephant's face.

1. I salute (Gaṇeśa) who has a pendulous belly, for whom laughter and mirth are produced by the humming of the gracious wives of the honey-drinking (bees) as they playfully frolic on his cheeks beautiful with the circle of his large trunk, the region of whose hot temples is covered with flowing, straight (streams of) ichor, and who (wears) garlands of hybiscus and kalāya-flowers.

<A. *The General Pulverizer*>

Thus having told of the six-fold (operations) beginning with unknowns and ending with surds, having now begun the pulverizer because it is useful for such things as the several colours (varṇa) of the *vargaprakṛti* which will be spoken of (presently), just describing its nature he says: "The dividend, the divisor, and the additive."

<46b-47b. For the sake of the pulverizer, the dividend, the divisor, and the additive are to be reduced (by a common measure) by a certain (number), if (there is) the possibility in the beginning. (If) the additive is not (divisible) by that by which the dividend and the divisor are divided, that is indicated to be faulty.>

The (syntactic) connection is: "If there is the possibility in the beginning, by a certain number for the sake of the pulverizer the dividend, the divisor, and the additive are to be reduced." That dividend is divided and the divisor is divided "by that," and the additive is added. Thus wherever these three are, just there the pulverizer is possible. Here, by this (phrase) "if there is the possibility," when there is a possibility of division,

division is to be performed, when that is not possible, the rule of the pulverizer is to be carried out with the additive, divisor and dividend just as they are. But (by the words) "just to be reduced" is indicated "necessarily." Here "pulverizer" is a conventional word. So he speaks of the distinction between the possibility and the impossibility of a pulverizer when reduction has been performed (with the words beginning with) "by what." By what number the dividend and the divisor are divided, if divided by just that the additive is not subtracted (without remainder), then this is indicated to be faulty. This is the meaning: by what number reduction of the dividend and the divisor was accomplished, if reduction of the additive by just that does not succeed, then this is indicated to be faulty. The meaning is: "It was asked fraudulently."

So, when there is uncertainty about the reduction, thinking "let there be no doubt for ignorant students whose minds are confused" (as they ponder:) "What are the dividend, the divisor, and the additive, or by what number are they to be reduced?," he speaks a sūtra for the sake of knowing the reduction-number: "Of two (quantities) mutually divided."

> <47c-48b. Whatever is the (last non-zero) remainder of two (quantities) which are mutually divided, that is their reducer. Whatever two (quantities) are divided by that reducer, they are the dividend and the divisor called "confirmed.">

Of two (quantities) which have been mutually divided whatever is the (last non-zero) remainder that is their reducer. The meaning is: "of the two (quantities), the dividend and the divisor." Here the meaning is: "the reducer is a certain number that has been measured for division without a remainder." When the reducer has been produced in this way, whatever dividend and divisor are divided by their own reducer are called "confirmed." Here the dual "dividend and divisor" is used elliptically. The meaning is:

"the dividend, the divisor, and the additive, when divided by that, are called 'confirmed'." Here the state of being called with the name "confirmed" means unchangeableness.

So, for the sake of achieving the meaning of what is to be explained concerning the pulverizer, he enunciates a sūtra with three verses from the beginning: "One should divide the confirmed dividend and divisor mutually."

> <48c-50b. One should divide the confirmed dividend and the confirmed divisor mutually until there is one (rūpa) here in (the place of) the dividend. The results (are placed) one below the other; below them is to be entered the additive. Then zero is at the end. When (the number) above it (i.e. the antepenultimate) has been multiplied by the penultimate and combined with the final, one should subtract the final. Thinking: "it should be (done) again and again" (one finds) a pair of quantities. The upper is divided (taṣṭa) by the confirmed dividend; (the remainder is) the result. The other, (is divided) by the (confirmed) divisor, (the remainder) is the multiplier.>

The (syntactic) connection is: "One should divide these two, the confirmed dividend and the confirmed divisor, mutually to such an extent." With this (word) "these two," the meaning is: "whichever two have been obtained by the previous sūtra." In what way (is meant) "to such an extent?" Until there is the number one (*rūpa*) here in (the place of) the dividend. The meaning is: "in mutual division one should divide until, in the place of whatever quantity is obtained by its being the dividend, the remainder is *rūpa*, one." The results obtained thus in this mutual division are to be placed one below the other. Then the additive is to be entered below them; and at the end, below them all, zero is to be placed. Thus it should be written so as to become a chain of results. When it has been done in this way, when (the number) above it (i.e., the antepenultimate) has been multiplied by the penultimate and (the result) has been combined with the final (number), one should abandon (that) final. In this way it is to be done "muhu" i.e. again and again until there is a pair of quantities. Here with this (word) "penultimate" (the definition is): "Penultimate (means) that it stands "upa" i.e. next to the final." So here the final (number) is zero, the penultimate is above it; consequently it is just "the additive." Then, ascending (the chain link) by higher (link), one should multiply (the number) above it (i.e., the antepenultimate) by whatever is the penultimate. Then, when a pair of quantities has resulted in accordance with what was said (previously), whichever quantity is higher (in the chain) being divided (*taṣṭa*) by the confirmed dividend, the remainder is the result (*phala*). Likewise the other quantity which is lower (in the chain) being divided (*taṣṭa*) by the confirmed divisor, the remainder is the multiplier. So, in whatever division in which there is no use for the "quotient" (*phala*) there is a use just for the remainder, the symbolic word "taṣṭa" is employed.

So, considering that this (rule is applied) in obtaining the multiplier and the quotient, he speaks of a special property in the occurrence of that (rule) in a task (with the verse that begins): "In this way."

> <50c-51b. Thus (should one proceed) in this at the time when these quotients are even (in number). If (they are) odd, then the quotient and multiplier as they are obtained are to be subtracted from their divisor (*takṣaṇa*); but they are measured by the remainders.>

Thus is (the procedure) at the time when in this pulverizer these quotients are even. The rule was made by (its) maker considering that the described procedure for the task (is to be followed) just at that time. This is the meaning. In accordance with what was said (previously), whatever are the quotients in the mutual division of the dividend and the divisor by the sūtra that begins: "One should divide these two, the confirmed dividend and the confirmed divisor, mutually," if they are even—(i.e.), are of an even number—then the quotient and the multiplier are to be computed by the described procedure. Now if these quotients are odd—i.e., of an odd number—then "the quotient and the multiplier as they are obtained are to be subtracted from their divisor; but"—i.e., again—"they"—i.e., the quotient and the multiplier—"are measured by the remainders." In this rule of the divided (taṣṭa) the divisor is called "takṣaṇa." The sense is: "When a pair of quantities as previously described has been produced, (by the rule) "the one higher (in the chain) is divided (taṣṭa) by the confirmed dividend," the confirmed dividend and the confirmed divisor become the divisors (takṣaṇas) of the multiplier and the quotient; and so in accordance with what was said previously the multiplier and the quotient in a case where the quotients are odd (in number), when they have been subtracted from their divisors (takṣaṇas), become the multiplier and the quotient."

The demonstration in this case is (as follows). Here the quotient and the multiplier are computed by the pulverizer to such an extent. There the order of instructions is that this dividend becomes remainderless when it is multiplied by a certain (number), combined with the additive, and divided by its divisor. Thus that (number) by which it is multiplied, is unknown. So it is proposed that, though the dividend is not remainderless when division of the determined dividend by the divisor occurs, even then mutual division is made in order to learn how great a remainder there may be after (that) division. That is as follows. The dividend divided by the divisor does not become remainderless. In that case, in order to know from what remainder (actually) left over, when it is multiplied how many times, the divisor is again subtracted (with no remainder), the divisor is again divided by the remainder of the dividend. There it is seen that the condition of having the forms of the quotient and the multiplier in order belongs to that which is the chain of results. Thus the inaccuracy (results) from the fact that the chain of results is derived from a dividend that is not combined with an additive. Then the pair of quantities that is produced in the presence of this (chain of results) homogenized by the additive—they are the accurate quotient and multiplier. But, in respect of the production of numbers that are stupifying because of their magnitude, division (takṣaṇa) by the confirmed dividend and the confirmed divisor is carried out in order to diminish them. The teacher, establishing just this in (his) mind, wrote this other procedure (with the words): "joined each with its own divisor multiplied by an arbitrary (number)." This is the meaning. When the pair of quantities is divided by the confirmed dividend and the confirmed divisor, whatever is the remainder—they are the quotient and the multiplier. So the sense is that, assuming that just that which is the quotient is the arbitrary (number), by the operation (described by the words) beginning with "joined each with its own divisor multiplied by an arbitrary (number)," that pair of quantities again becomes the quotient and the multiplier. So, with respect to the two (quantities), the multiplier and the quotient, it is effected by just a pair of quantities. Therefore it is said that the pair of quantities "should be repeated."

Now the inclusion of the quotient in the dividend is seen (from the rule) that "when the dividend is divided by the divisor, a result is obtained." So the inclusion of the multiplier in the divisor is seen (from the rule) that "the quotient multiplied by the divisor is subtracted from the dividend." Therefore it is said: "The one higher (in the chain) is divided (taṣṭa) by the confirmed dividend; (the remainder is) the result (and the other is divided by the confirmed divisor, the remainder is) the multiplier." So it has been demonstrated here that, when the chain of results is being obtained by (the rule) beginning: "One should divide the confirmed dividend and the confirmed divisor mutually," because the first result is obtained from the dividend divided by the divisor, but the second (result is obtained) from the divisor divided by the remainder of the dividend, therefore it is made a rule that "the one higher (is divided) by the dividend."

So something is made clear by us also by means of concise verses (*kārikās*) under the guise of stating a conclusion to the (above) demonstration.

1. Some part of the dividend is called "kṣepa" (the additive). These two, the quotient and the multiplier, are thought to be within (i.e. to depend on) the dividend and the divisor (respectively).

2. (If one wonders:) "If there is a multiplier for the chain of the results of a dividend whose remainder is one, then what is (the multiplier when the dividend) has as its remainder the additive?," (the answer is found) from proportion.

3. The multiplier and quotient are regarded as measured by that which is a pair of quantities. Then in order to diminish them, division (takṣaṇa) is carried out by the sages.

4. The teacher, keeping just this in view, regarded it as most important: the multiplier and the quotient, each added to its own divisor multiplied by an arbitrary (number), become indeed two others (i.e., a multiplier and a quotient).

5. Something is said which causes admiration in ignorant students (who wonder): "(Even) when one of two (quantities) is unknown, yet the second is arrived at."

6. When there is knowledge of the multiplier (but) ignorance of the result (i.e., the quotient), (then) the correct result is obtained (thus): the dividend is multiplied by the multiplier, (the product is) added to the additive, and (the sum) is divided by the divisor.

7. When there is knowledge of the result (but) ignorance of the multiplier, (then) the divisor is multiplied by the result, (the product) is diminished by

the additive, and (the remainder) is divided by the dividend. There is the correct multiplier.

8. Whenever there arises a confirmed dividend which is less than the divisor, there an inversion of the quotient and multiplier is to be carried out by the wise.

9. If, where the quotients are odd (in number and) the additive is subtractive, whatever quotient and multiplier are there are indeed the correct ones.

Now he speaks of a special property when it is impossible to reduce one among the additive, the divisor, and the dividend, which are the previously described causes of the pulverizer (with the verse beginning): "There exists from the method of the pulverizer."

<51c-52b. Or else, there exists from the method of the pulverizer a multiplier of the additive (yuti) and the dividend when they are reduced together. Again, whatever is (the multiplier) of the additive and the divisor that is also (the multiplier) when it is multiplied by the reducer.>

Or else, of the additive and the dividend when they have been reduced together, from the method of the pulverizer a multiplier exists. "Yuti" (means) "additive" (kṣepa). The meaning is: when the reduction of it (the additive) and of the dividend has been made, even if the divisor has not been reduced, yet a multiplier is obtained. Now: again of the additive and the divisor, when they have been reduced together, whatever is the multiplier, that when multiplied by the reducer is the multiplier. The meaning is: when reduction of the additive (yuti) and the divisor (bhājaka)—(i.e.,) of the additive (kṣepa) and the divisor (hāra)—has been made with the exclusion of a reduction of the dividend, whatever multiplier is arrived at, that when it is multiplied by the reduction-number is the multiplier.

The demonstration in this case is (as follows). There, because of the absence of the use of the quotient, even if the additive and dividend have been reduced, even then the multiplier is obtained because the inclusion of the multiplier in the divisor is seen. Thus, because, when the divisor is unaltered, the multiplier also is unaltered, therefore (the verse) beginning: "There exists from the method of the pulverizer" was said. So, when the reduction of the additive and the divisor has been done, (according to the rule): "Whatever is (the multiplier) of the (reduced) additive and divisor," the dividend with respect to the divisor is reduction-number-times greater. Thus, because the divisor is less, the multiplier which is within it also becomes reduction-number-times less (i.e. the new multiplier is $\frac{\text{multiplier}}{\text{reduction - number}}$). It has been demonstrated that, if it is multiplied by the reduction-number, then it is the multiplier.

Having enunciated in this way a collection of sūtras for the perfection of the pulverizer, he now proclaims some rule with his intention being in the derivation of the multiplier and the quotient (with the half-verse that begins): "(In the division) of the multiplier and the quotient, an equal (result) is to be obtained."

<52c. In the division (takṣaṇa) of the multiplier and the quotient, an equal result is to be obtained by an intelligent (person).>

By an intelligent (person) in the division of the multiplier and the quotient an equal result is to be obtained. It has been demonstrated that the meaning is that a rule was made that, after the multiplication of the chain of results by the previously described operation of the pulverizer, as a pair of quantities is achieved by (the rule) here: "In the division of the multiplier and the quotient," when, according to (the verse) which begins:

"The (successively) higher (number) is divided by the confirmed dividend (the remainder) is the result, the other by the divisor, (the remainder) is the multiplier," the pair of quantities is divided in order by the confirmed dividend and the confirmed divisor, then an equal result for them both is to be taken because, as many times as the confirmed dividend is subtracted (from the upper quantity), so many times must (the lower quantity) be diminished by the confirmed divisor.

Now he mentions another peculiarity with the latter half (of the verse that begins): "Arising from addition."

<53b. The multiplier and the quotient, which are arising from addition, when subtracted from (their respective) divisor(s) (takṣaṇa), become (quantities) arising from subtraction.>

Arising from addition the multiplier and the quotient, when they have been subtracted from (their respective) divisor(s), become (quantities) arising from subtraction. The meaning is that whatever multiplier and quotient are obtained "arising from addition" with the meaning: "Having a positive additive," if they are subtracted from their (respective) divisor(s) which are named "confirmed dividend and confirmed divisor," then they become "(quantities) arising from subtraction" with the meaning: "Having a negative additive."

The demonstration in this case is (as follows). It is clear that, if the quotient and multiplier, which are the remainders from division by the confirmed dividend and divisor, are subtracted from their divisors, then they become (quantities) arising from a difference because the additive is made smaller than the dividend.

Now he enunciates a sūtra in (the case when) the divisor is positive or negative (with the verse that begins): "The two produced by a positive dividend."

<54a. The two produced by a positive dividend in the same way become two produced by a negative dividend.>

The two <produced by> a positive dividend in the same way—as previously (described)—become < ... (lacuna) ... (two produced by a negative dividend) ...

<53a. When a positive additive has been divided (taṣṭa) by the divisor, yet the multiplier and the quotient (are found) as previously (described).>

(lacuna) ... > etc. Now, when the divisor is negative, the multiplier and the quotient are (found) as previously (described). In what (circumstance)? When a positive additive has been divided (taṣṭa) by the divisor. The meaning is that if the divisor is negative then, imagining the remainder of the positive additive divided by it (the divisor) to be the (new) additive, the quotient and the multiplier are to be derived by carrying out the method of the pulverizer.

Now again he mentions another peculiarity (with the line that begins): "United with the result produced by division (takṣaṇa) of the additive."

<54b. United with the result (produced by) division (takṣaṇa) of the additive is the quotient; but, if there is subtraction (of the additive because it is negative, the quotient) is diminished.>

Or: the quotient is to be made to be united with the result (produced by) division (takṣaṇa) of the additive. "Kṣepatakṣaṇaṃ" (means) "division of the additive" (and not "by the additive"). The meaning is that, whatever is the result (i.e.) obtained in this (division), with that is the quotient united, (i.e.) joined; (the result) is the (true) quotient. But this (statement) is about a positive additive. So, (when he says): "But, if there is subtraction, (the quotient) is diminished," the meaning is that, when there is a negative additive, the quotient is to be made to be diminished by the result (produced by) division (takṣaṇa) of the additive. The intention is that, when division (takṣaṇa) of the additive is made by the divisor (in accordance) with (the words): "When a positive additive has been divided (taṣṭa) by the divisor," the quotient is joined with whatever is obtained (as the result) if it is a positive additive, and diminished (by it) if it is a negative additive; (the result) is the (true) quotient.

Now again he mentions another peculiarity (with the half-verse that begins): "Or else by the divisor."

> <55a-b. Or else by the divisor of the additive and the dividend divided (taṣṭa), the multiplier (is found) as previously, then the quotient (is found) from the dividend multiplied (by the multiplier) joined (with the additive, and) divided (by the divisor).>

Or else—by another procedure—by the divisor of two divided (taṣṭa), which are the additive and the dividend, as previously the multiplier is to be known—as previously (means) by the method of the pulverizer—the multiplier is to be known because it was said: "There exists from the method of the pulverizer (a multiplier) of the additive and the dividend." When this has been done, the multiplier is obtained, but not the quotient. And so, when the multiplier is known, then the quotient is to be found. The meaning is that the result (i.e. quotient) is obtained when the dividend is multiplied by the multiplier, joined with the additive, and divided by the divisor. Or else the quotient and the multiplier are to be obtained from the dividend increased and divided by the divisor. In this way, all this will be investigated clearly with demonstrations at the time of (giving) examples.

Now he speaks of the possibility of the absence of (a positive or negative value for) a multiplier (with the verse beginning): "The absence of the additive."

<56a-b. Wherever there is the absence of an additive or the additive is reduced (without a remainder) when it is divided by the divisor, in that case, the multiplier is to be known as zero, (and) the additive divided by the divisor is the result.>

Wherever there is the absence of an additive, and likewise wherever the additive divided by the divisor is reduced (without a remainder), in that case—in both (cases)—the multiplier is to be known as zero. Now he speaks of a special property here also: "The additive divided by the divisor is the result." Wherever the additive divided by the divisor is reduced (without a remainder), just in that case the additive divided by the divisor is the result. And likewise wherever by its very form there is an absence of the additive, in that case what is to be divided by the divisor? The meaning is that in that case because of this (absence of the additive) the multiplier and the quotient are just zero.

The demonstration in this case is (as follows). Considering first in that case that there is the absence of an additive, wherever there is the absence of an additive, in that case, in the chain of results which has been obtained from mutual division by (the rule) beginning: "One should divide mutually the confirmed dividend and the confirmed divisor," because of (the rule) beginning: "When (the number) above it has been multiplied by the penultimate," the penultimate is the additive. And that in the case under discussion is measured by zero. In multiplying (the number) above it by it, since (a number) multiplied by zero is zero, it is in all cases zero. Thence it has been demonstrated that zero is the multiplier in that case. Now (with respect to the words): "Wherever the additive is reduced (without a remainder)," in that case the additive, being divided (taṣṭa) by the divisor by the procedure of the sūtra: "When a positive additive has been divided (taṣṭa) by the divisor," becomes remainderless. And so it is correct (to conclude) that in the absence of an additive, the multiplier is zero. So, by this (phrase): "the additive divided by the divisor is the result," the meaning of the sūtra: "United with the result (produced by) division (takṣaṇa) of the additive" has been established. So, (in answer to the question): "How indeed might the multiplier be zero even when the absence of an additive has not been effected?," it is said. If the additive multiplied by one is reduced (without a remainder) when it is divided by the divisor, then, even though multiplied by two, three, and so on, (the additive) will certainly be reduced (without a remainder) when it is divided by the divisor. Thus, when the quantity, which is the lower in a pair of quantities that is obtained from the chain of results when the additive is multiplied, is divided by the divisor, then it (the lower quantity) is certainly remainderless. Thence it is correct (to say) that in that case too the multiplier zero is produced. Now (with respect to the phrase): "the additive divided by the divisor," here the result is obtained when the dividend is multiplied by the multiplier, (the product) increased by the additive, and (the sum) divided by the divisor. And so in the case under discussion the multiplier is zero. When the dividend is multiplied by that, a zero results. Since (the result) is obtained (by the rule): "When this is increased by the additive (the sum) is to be divided by the divisor," consequently the additive is to be divided by the divisor. It has been demonstrated that the result is obtained (in this way) in that case.

Now for the sake of the astonishment of the educable who are lazy in (carrying out) the previously mentioned operations of the pulverizer at the acquisition of several multipliers and quotients, he enunciates a sūtra (with the verse beginning): "(Each) with its own divisor multiplied by an arbitrary (number)."

> <57a-b. (Each) with its own divisor multiplied by an arbitrary (number) when joined, the multiplier and the quotient become (other multipliers and quotients) many times over.>

"Those two, the multiplier and the quotient, (each) with its own divisor multiplied by an arbitrary (number) when joined, many times over become (other multipliers and quotients)." Whatever multiplier and quotient have been established by the method of the pulverizer as described (previously), they are (again) the quotient and the multiplier when they have been joined (each) with its own divisor multiplied by an arbitrary (number). The meaning is this: having multiplied (each of) its own divisor, called the confirmed dividend and divisor, by any arbitrary (number) whatsoever, such as one, two, or three, the previously obtained multiplier and quotient having been increased by them in order become another quotient and another multiplier. The intention is that in this way there are quotients and multipliers many times over—(i.e.,) numerously.

Here the demonstration is (as follows). In that case, whatever pair of quantities has been produced from the multiplication of two additives in the chain of results, the quotient and the multiplier measured by the remainder when division has been carried out by the confirmed dividend and divisor in order are produced from that (pair). Now, it was demonstrated that, if the confirmed dividend and divisor, (each) multiplied by one and becoming its own divisor, are increased by the two remainders which have the nature of a quotient and a multiplier, then they again become the quotient and the multiplier because they are greater (each) by the remainder pertaining to it.

Now he enunciates an example pertaining to the subject of the sūtra on the pulverizer (with the verse that begins): "A pair of hundreds combined with twenty-one."

> <57c-58b. Oh calculator! Tell (me) quickly that multiplier multiplied by which a pair of hundreds combined with twenty-one, joined with sixty-five, and divided by a pair of hundreds diminished by five, arrives at the state of being reduced (without a remainder).>

The (syntactic) connection is: "Oh calculator! That multiplier quickly tell." From reasoning that, from the invariable connection of (the correlative pronouns) "yat" and "tat," the word "tat" anticipates the word "yat," (in answer to the question): "What (is meant by) "tam"? ", he says: "Yadguṇaṃ" ("multiplied by which"). "Multiplied by which a pair of hundreds combined with twenty-one, joined with sixty-five and divided by a pair of hundreds diminished by five arrives at the state of being reduced (without a remainder), that (multiplier)." Thus in this case, from the definition: "That dividend is divided" (given) with the sūtra which begins: "The dividend, the divisor, and the additive are to be reduced (by a common measure)," a pair of hundreds combined with twenty-one becomes the dividend, sixty-five (becomes) the additive, and a pair of hundreds diminished by five (becomes) the divisor. Thus the setting out of these in order is:

Dividend	221	Additive	65
Divisor	195.		

Now it was said in the first verse that in order to make them smaller reduction (by a common measure) is to be performed. And also (it was said) by the procedure of the second sūtra: 'Whatever is the (last non-zero) remainder of two (quantities) which are mutually divided, that is their reducer.' In this case the (last non-zero) remainder of the mutually divided dividend and divisor, the reduction-number, is obtained (as) 13. The dividend, divisor, and additive reduced by this (number) become known as "confirmed":

Dividend	17	Additive	5
Divisor	15.		

Now, whatever are the results (quotients) in the mutual division of the confirmed dividend and divisor until the remainder is one (rūpa) in accordance with (the rule) which begins: "One should divide the confirmed dividend and divisor mutually," placing them one below the other, the additive below them, and placing zero at the end, the chain of results is produced:

1
7
5
0

So, when the operation as described (by the rule) which begins: "When (the number) above it is multiplied by the penultimate" has been carried out, a pair of quantities is produced:

40
35

This (pair), having been divided (taṣṭa) in order by these two (numbers), 17 (and) 15, the confirmed dividend and divisor, (the remainders) become the quotient and the multiplier: 6 (and) 5. In the division (takṣaṇa) of the pair of quantities in this case, an equal quotient, 2, is produced in both places. To compute a quotient and a multiplier numerous times from the quotient and the multiplier obtained in this way, he lays down the sūtra which was previously said (with the words): "Multiplied by an arbitrary (number)." Thus, in this case the basic multiplier and quotient are 5 (and) 6. Their divisors are 15 (and) 17. (By one) multiplying these two by the arbitrary (number) one, they, when increased by the basic quotient and multiplier, become another quotient and another multiplier, 23 (and) 20. (Multiplying) by the arbitrary (number) two (and proceeding) in this way, (they are) 40 (and) 35. (Multiplying) by three (they are) 57 (and) 50. (One should proceed) in this way numerous times.

Having thus enunciated an example of imagining the additive to be positive and of imagining it to be negative, now, having assumed it to be positive or negative, he enunciates an example (which is) pertaining to the subject of this sūtra: "There exists from the method of the pulverizer" (with the verse beginning): "A hundred multiplied by which when joined with ninety."

> <58c-59b. If you are very clever in the pulverizer, tell me correctly that multiplier, by which when one hundred is multiplied, (the product) is added to or diminshed by ninety and (the sum or difference) is divided by sixty-three, there is no remainder.>

The (syntactic) connection is: "Oh mathematician! If in the pulverizer you are very clever, then tell me that multiplier correctly." "Paṭīyān" (is defined as) "excessively clever"; the meaning is "skillful." Now, (in answer to the question): "What is (the pronoun) 'that'? ", he says (the correlative): "by which." (The syntactic connection is): "By which multiplied one hundred increased or decreased by ninety (and) by sixty-three divided is without a remainder."

Thus, here the setting out is:

Dividend	100	Additive	90
Divisor	63.		

Here because of the impossibility of the reduction, just these are the confirmed dividend, divisor, and the additive. So here also, when the method of the pulverizer is being performed as previously, a chain of results is produced:

1
1
1
2
2
1
90
0

In accordance with what was said (previously), the multiplier and the quotient are 18 (and) 30. Now, having imagined the negativity of the additive (which is) ninety, by the procedure of the sūtra: "The multiplier and the quotient, which are arising from addition, when subtracted from (their respective) divisor(s) (takṣaṇa), become (quantities) arising from subtraction," the multiplier and the quotient that have been obtained, when they have been subtracted from their (respective) divisors (takṣaṇa), again become the multiplier and the quotient, 45 (and) 70.

Now, again, the setting out of the dividend, the divisor, and the additive is:

Dividend	100	Additive	90
Divisor	63.		

In this case, in order to display the working of the sūtra: "There exists from the method of the pulverizer," when one has reduced the dividend and the additive by ten, the setting out is:

Dividend	10	Additive	9
Divisor	63.		

Here from mutual division a chain of results (is produced):

0
6
3
9
0

In accordance with what was said (previously), the quotient and the multiplier are 7 (and) 45. Now the quotients in the chain of results are odd (in number). Therefore, proceeding (in consideration of the fact) that "the quotient and the multiplier as they are obtained are to be subtracted from their (respective) divisor(s) (takṣaṇa)," the multiplier is obtained as 18. There is no use for the quotient because the multiplier alone is indicated in the sūtra: "Or else the multiplier of the two (i.e., the additive and the dividend) when they are reduced together."

Now, in order to show working of this (in the sūtra): "Again, whatever is (the multiplier) of the additive and the divisor," when one has reduced the divisor and the additive by nine, the setting out is:

Dividend	100	Additive	10
Divisor	7.		

In accordance with what was said (previously), a chain of results (is produced) in this case:

14
3
10
0

And so the quotient and the multiplier are 30 (and) 2. Here this 2 is the multiplier. Multiplied by this reduction-number 9, the (true) multiplier is produced; it is 18 because it was said: "That is also (the multiplier) when it is multiplied by the reducer." So the multiplier and the quotient produced by the negative additive ninety are 45 (and) 70. Here by (application of the sūtra) beginning: "(Each) with its own divisor multiplied by an arbitrary (number) when joined," the multiplier and quotient are again 108 (and) 170; 171 (and) 270. In this way (one may proceed) variously.

Now in order to display the workings of the remaining sūtras, he again, assuming the negativity of the dividend, enunciates another example (by the verse beginning): "By which multiplied negative sixty increased."

> <59c-60b. Oh mathematician! Tell me separately the multiplier(s) by which when multiplied negative sixty, if it is either increased or diminished by three, is, when divided by thirteen, without a remainder.>

"Tell me the multiplier(s) separately." The purpose of the verse is just as in the previous (case). And so the setting out is:

Dividend	–60	Additive	3
Divisor	13.		

In this case the chain of results (is):

4

1

1

1

1

3

0

In accordance with what was said (previously), the multiplier and the quotient are 2 (and) 9. When one has operated (in realization of the fact) that the quotients are odd (in number) in this case, the quotient and the multiplier subtracted from their (respective) divisor(s) (takṣaṇa) become 51 (and) 11. Here, when one has assumed the positivity of the additive, when that additive is combined with the negative dividend, then, when one has operated (in realization of the fact) that "the addition of a positive and a negative (quantity) is (their) difference," the multiplier and the quotient become 2 (and) –9. Now, when one has assumed the negativity of the additive, when that additive is combined with a negative dividend, then, when one has operated (in realization of the fact) that 'in the addition of two negative (quantities, their) sum occurs,' the multiplier and the quotient (become) 11 (and) –51. Thus all has been accomplished by just this (verse): "The multiplier and the quotient, which are arising from addition, when subtracted from (their respective) divisor(s) (takṣaṇa), become (quantities) arising from subtraction." But for the sake of teaching the dull, it is enunciated by the teacher (Bhāskara): "The two are produced by a positive dividend in the same way." The remainder, which is clear, is understood from the treatise also.

Now, having assumed the negativity of the divisor, he enunciates another example (with the verse that begins): "Eighteen multiplied by what?"

<60c-61a. Eighteen multiplied by what, (if) increased or diminished by ten and divided by negative eleven, becomes (a number) without a fractional part.>

Here the meaning of the verse is easy to be understood. And so the setting out is:

Dividend	18	Additive	10
Divisor	−11.		

Here, when one has assumed the positivity of the divisor, the chain of results (is produced) by mutual division:

1
1
1
1
10
0

Thus in accordance with what was said (previously), the quotient and the multiplier, which are arising from addition, become 14 (and) 8. So, when one has operated (in realization of the fact that): "The multiplier and the quotient, which are arising from addition, when subtracted from (their respective) divisor(s) (takṣaṇa), become (quantities) arising from subtraction," the multiplier and the quotient arising from subtraction become 3 (and) 4. In this example, the quotient is to be understood to be negative because the divisor is negative; (hence the multiplier and the quotient are) 3 (and) −4. (This is) because it has been said previously: "In the multiplication of two positive and of two negative (quantities, the product is) positive, (but it is) negative in the multiplication of a positive and a negative (quantity). But it is also explained in the same way in the division (of positive and negative quantities)."

Now he enunciates another example pertaining to the subject of this (rule which begins): "When a positive additive has been divided (taṣṭa) by the divisor," (with the verse which begins): "By which are multiplied five."

<61b-62a. What is that multiplier, by which multiplied five increased or decreased by twenty-three (and) divided by three is without a remainder?>

The meaning of the verse is easy to be understood in this case also. Thus here the setting out is:

Dividend	5	Additive	23
Divisor	3.		

Here the chain of results (is):

1
1
23
0

and a pair of quantities is produced:

46
23

Here, when the upper quantity has been divided by its divisor (takṣaṇa), the number five, nine is obtained. So when the lower quantity is divided (taṣṭa) by three, seven is obtained. Thus this (method) is improper because the quotients are unequal, since it was said (previously): "(In the division) of the multiplier and the quotient, an equal (result) is to be obtained." Therefore here the achievement of the purpose is by another sūtra. So, from the rule of the sūtra: "When a positive additive has been divided (taṣṭa) by the divisor, yet the multiplier and the quotient (are found) as previously (described)," in this case, when one imagines the remainder of the additive divided (taṣṭa) by the divisor to be the (new) additive, the setting out is:

Dividend	5	Additive	2
Divisor	3.		

Here, from mutual division, the chain of results (is):

1
1
2
0

From this the multiplier and quotient become 2 (and) 4. When they have been subtracted from their divisors, they become the multiplier and the quotient, 1 (and) 1, arising from subtraction. Now in just this example, the subject of this sūtra: "United with the result (produced by) division (takṣaṇa) of the additive, is the quotient; but, if there is subtraction, (the quotient) is diminished," is seen. So here, when the division (takṣaṇa) of this additive, 23, is made by this divisor, 3, the result is 7. United with this, this quotient (produced by) addition, 4 becomes the (true) quotient, 11, and the multiplier is just the previous one, 2. Now (in view of the rule): 'If there is subtraction, (the quotient) is diminished,' in that case the quotient (produced by) subtraction, 1, when diminished by this result, 7, (which is obtained) from division (takṣaṇa) of the additive, again becomes the (true) quotient, –6, and the multiplier is just the previous one, 1. Or else, when one has multiplied by this result, 7 (produced) by division (takṣaṇa) of the additive, these two, the dividend and the divisor, 5 (and) 3, become divisors (takṣaṇa); when one has subtracted (the products) from this pair of quantities previously obtained, 46 (and) 23, again there result the multiplier and the quotient, 2 (and) 11.

The demonstration in this case is (as follows). When the additive is divided (taṣṭa) by the divisor, yet the additive is small. Then the quotient which is produced from it also is small. That (quotient), however, (is obtained) employing a small additive, but not when the additive is large. So, it has been demonstrated that, if the previous quotient is combined with that (number) by which multiplied the divisor is subtracted from the additive, then the quotient is large, because the additive is to be added.

Now he mentions an example pertaining to the subject of this sūtra: 'When there is the absence of an additive and wherever the additive is reduced (without a remainder) when (it is) divided by the divisor,' (with the verse that begins): "By which (when) five are multiplied."

<62b-63b. Oh mathematician! Tell me quickly that multiplier by which (when) five are multiplied and zero is added to or sixty-five is combined with (the product, the two sums) are without a surplus when they are divided by thirteen.>

Here the meaning of the verse is certainly easy. And so the setting out is:

Dividend	5	Additive	0
Divisor	13.		

In this case when there is the absence of an additive, the multiplier and the quotient are 0 (and) 0. Or else, on account of (the verse) beginning: "(Each) with its own divisor multiplied by an arbitrary (number)," when that which is to be added is their own divisor multiplied by one, the multiplier and quotient are 13 and 5 (respectively).

In the second example, the setting out is:

Dividend	5	Additive	65
Divisor	13.		

Here considering that 'the additive reduces (without a remainder) when it is divided by the divisor,' the multiplier becomes 0. And on account of (the rule) beginning: 'The additive divided by the divisor becomes the result,' the result is 5.

<B. *The Constant Pulverizer*>

Having investigated the pulverizer generally in this way, now for the sake of effecting the constant pulverizer which is useful in the computation of (the longitudes of) the planets, he recites a sūtra (with the verse that begins): "If one imagines the additive rūpa to be subtractive."

> <63c-64b. If one imagines the additive rūpa to be subtractive, whatever are the multiplier and the quotient of those two separately, when they have been multiplied by an arbitrary additive or subtractive (and) divided (taṣṭa) by their divisors (takṣaṇas), become their (multiplier and quotient separately).>

The logical order is: "If one imagines the additive rūpa to be subtractive, whatever are the multiplier and the quotient of those two separately, (when) they have been multiplied by an arbitrary additive or subtractive and divided by their divisors, they become the multiplier and quotient." The meaning is that the subtractive (viśuddhi) is the additive which has become negative; if one assumes that rūpa to be the numeral one. The remainder is clear.

Thus, here in the first example, if one assumes rūpa to be the additive, the setting out is:

Dividend	17	Additive	1
Divisor	15.		

So the series of results (is):

1
7
1
0

The multiplier and the quotient are 7 (and) 8. When these two are subtracted from their (respective) divisors (takṣaṇas) because the additive is negative, the multiplier and the quotient are 8 (and) 9. Because of (the rule) beginning: "Multiplied by an arbitrary additive or subtractive," here the arbitrary previous additive is this 5. When the multiplier and the quotient are multiplied by this, (there are produced) 40 (and) 45. And when they are divided (taṣṭa) by their (respective) divisors, there are produced the quotient and the multiplier (arising from the subtractive), 11 (and) 10.

The demonstration in this case is (as follows). There, when the additive rūpa has been assumed to be subtractive, when the multiplication in the chain has been performed by (the sūtra) beginning: "When (the number) above it (i.e. the antepenultimate) has been multiplied by the penultimate," because it has been stated: "If it is multiplied by one, it remains the same," that chain of results remained as it was. In that case, in accordance with what was said (previously), whatever are the (new) quotient and multiplier, those two multiplied by the additive become less with respect to the former quotient and multiplier. If, when one has multiplied those two by the additive, the two (products) are divided by their own divisors, then they become the former quotient and multiplier. Here, whatever two (numbers) are produced when they are multiplied by an arbitrary additive, are the quotient and the multiplier. But in order to diminish them it is said: "Divided by their (respective) divisors." Thus another procedure has been described; otherwise the purpose was accomplished by this (rule beginning): "Those arising from addition, when subtracted from (their respective) divisor(s)." And also here proportion is observed. (If it is asked): "If these two are the multiplier and the quotient by means of a negative additive (which is) measured by one, then what is the use of an arbitrary (additive)?," here the imagining of the negativity of the additive rūpa is for the sake of illustrating the variety of the procedures. Thereby it has been demonstrated that even if one does not assume this, the quotient and multiplier remain the same.

Now, indicating the usefulness of (his) endeavour in describing the pulverizer, in order to compute (the position of) a planet by that, he enunciates a sūtra with a verse and a half (which begin): "Now, the subtractive is to be assumed to be the remainder of the seconds."

<64c-d. Now, the subtractive is to be assumed to be the remainder of the seconds, the dividend to be sixty and the divisor to be the civil days.>

<65a-d. The result produced by them is the seconds, but the multiplier is the surplus of the minutes, and from this (are derived) the minutes and the surplus of the degrees. In this way (one proceeds) higher than that. And, from the surpluses of the intercalary months and the omitted tithis, days of the Sun and the Moon (are to be found).>

So the subtractive is to be assumed to be the remainder of the seconds, and the dividend is to be assumed to be sixty and the divisor to be the civil days. In this way, with the dividend, the divisor and the additive, the pulverizer is to be accomplished. Here it has been said previously that the subtractive is the additive which has become negative. Then the result produced by them is the seconds. But the multiplier is the surplus of the minutes. The meaning is that whatever quotient and multiplier have come from the pulverizer, between those two the quotient is the seconds, but the multiplier is the remainder of the minutes. Now, if one assumes the remainder of the minutes to be the subtractive, sixty is the dividend and the civil days the divisor. In that case too whatever multiplier and quotient (are obtained) with the method of the pulverizer, between those two the quotient is the minutes, but the multiplier is the surplus of the degrees. Now, if one assumes the surplus of the degrees to be the subtractive, thirty is the dividend and the civil days the divisor. Again, whatever multiplier and quotient (are obtained) with the method of the pulverizer, between those two, the quotient is the degrees, but the multiplier is the surplus of the signs (of the zodiac). Now with respect to the signs, twelve is the dividend, the surplus of the signs the subtraction of the additive, and the civil days the divisor. In this case also, in accordance with what was said (previously), whatever multiplier and quotient (are found), between those two the quotient is the signs, but the multiplier is the surplus of the revolutions. In this way, revolutions, intercalary months, omitted tithis, (civil) days, and days of the Sun and the Moon, and so on, are to be derived.

Thus, the demonstration in this case is (as follows). There in the sūtra for computing the mean (longitudes of the) planets mentioned in the *Siddhānta(śiromaṇi)*: "The sum of the (elapsed) days is multiplied by the revolutions of a planet and divided by the civil days; the result in revolutions and so on is (the longitude of) the planet," there is just proportion. (If it is asked): "If the revolutions (of a planet) in a Kalpa are obtained by means of the civil days in a Kalpa, then what (is obtained) by means of the days in an arbitrary ahargaṇa?," the civil days (in a Kalpa) are seen to be the divisor because they are the criterion, and the ahargaṇa to be the multiplier because it is arbitrary. Now, as soon as (the longitude of) the planet is computed by the method of this sūtra, the revolutions of the planet are obtained first because revolutions are the result. Then with respect to the signs, if, when one has multiplied the remainder of the revolutions by twelve, (the product) is divided by the civil days, then signs are obtained. Now, when one has multiplied the remainder of the signs by thirty, as soon as (the product) is divided by the civil days, the degrees are obtained. Then, when one has multiplied the remainder of the degrees by sixty, as soon as (the product) is divided by the civil days, minutes are obtained. Now, when one has multiplied the remainder of the minutes by sixty, as soon as (the product) is divided by the civil days, seconds are obtained. Thus, the remainder of the seconds remains. And so, from the remainder of the revolutions the result is the signs; from the remainder of the signs the result is the degrees; from the remainder of the degrees the result is the minutes; from the remainder of the minutes the result is the seconds. In general the idea is that, in this way, when one has put the revolution first, with respect to each preceding one, the result is seen to be each succeeding one. So, when one has put the remainder of the revolutions first according to that procedure, their own dividends measured by one, twelve, thirty, sixty, and sixty (respectively) are multiplied by as many remainders as have been produced, for the sake of computing the results. Consequently, since it has been seen with respect to each succeeding remainder that each preceding (one) is the multiplier, this sūtra has been composed by the ācārya.

Now, on the other hand, precisely by inversion of the operation in the sūtra for computing the mean (position of) a planet, the computation of (the position of) a planet from the remainder of the seconds, has been told. In that, the remainder of the minutes multiplied by sixty was divided by the civil days; what was obtained was seconds. There the remainder that was produced was the remainder of the seconds. Now the remainder of the seconds was more than (i.e.) in excess of this, because it was a remainder. Now, because, when this has been subtracted from the dividend, the dividend in the division will be remainderless, therefore it has been stated: "The subtractive is to be assumed to be the remainder of the seconds."

Now by whatever remainder of the minutes sixty was previously multiplied, that is described as unknown. For the sake of knowing it the sixty which was previously the multiplicand now is imagined to be the dividend by inversion. In that case, however, since the state of being a divisor pertains to the civil days, therefore it has been stated: "Sixty is the dividend, the civil days the divisor." In this way, when the dividend, the divisor, and the additive have been determined, whatever multiplier is produced by the method of the pulverizer is the remainder of the minutes; because previously sixty was multiplied by the remainder of the minutes. Now here the quotient which is produced is seconds because, considering that, when previously sixty multiplied by the remainder of the minutes was divided by the civil days what was obtained were seconds, therefore it was said: "The result produced by them are the seconds, but the multiplier is the surplus of the minutes."

Now it has been demonstrated that it should be applied in the same way in the case of the surplus.

Now this (procedure) is clearly demonstrated for the sake of teaching students by there being an example. In that, first, for the sake of knowing the remainder of the seconds, (the position of) the planet is determined by (the rule) beginning: "The sum of the (elapsed) days is multiplied by the revolutions of the planet." Thus here the revolutions of the planet are imagined to be 3, the civil days to be 11, the number of (elapsed) days to be 3. Now by the method of the sūtra there is produced (the position of) the planet beginning with revolutions (as follows):

0
9
24
32
43

Here the remainder of the seconds is 7. When one has assumed this to be the subtractive, the setting out for the sake of the pulverizer is:

Dividend	60	Additive	−7
Divisor	11.		

Here the chain of results is produced:

5
2
7
0

The quotient and the multiplier are 17 (and) 3. These two, which are arising from addition, when subtracted from (their respective) divisor(s), are arising from subtraction: 43 (and) 8. Here this quotient 43 is produced as seconds.

Now for the sake of deriving the minutes, the divisor is 11, but the multiplier, (which is) the remainder of the minutes, became 8. When one assumes this to be the subtractive, by (the rule) beginning: "The dividend is sixty (and) the divisor the civil days," again the setting out for the sake of the pulverizer is:

Dividend	60	Additive	–8
Divisor	11.		

Here as previously there are produced the quotient and the multiplier: 32 (and) 6. Here also, the quotient became minutes, the multiplier is certainly the remainder of the degrees. When one imagines this to be the subtractive, (and) imagines (a quantity) measured by thirty to be the dividend, again the setting out is:

Dividend	30	Additive	–6
Divisor	11.		

The multiplier and the quotient are produced as before: 9 (and) 24. Here the quotient became degrees, but the multiplier is the remainder of the signs: 9. When one assumes this to be the subtractive, (and) twelve the dividend, again the setting out is:

Dividend	12	Additive	–9
Divisor	11.		

Here the quotient and the multiplier are 9 (and) 9. Here the quotients are odd and the additive is negative. Therefore, the quotient and the multiplier are just as they were: 9 (and) 9. Moreover, it was said (previously):

> "If, where the quotients are odd (and) the additive is subtractive, whatever quotient and multiplier are there are indeed the correct ones."

In this way here this quotient 9 is produced as (zodiacal) signs, but the multiplier is the remainder of the revolutions, 9. When one imagines this to be the subtractive, the imagined revolutions to be the one dividend, and imagines the civil days to be the divisor, the setting out is:

Dividend	3	Additive	–9
Divisor	11.		

The chain of results (is):

0
3
1
9
0

The multiplier and the quotient are 3 (and) 0. Here this quotient 0 became revolutions; this multiplier became the number of (elapsed) days: 3. In this example, when one proceeds so that (the position of) the planet is derived by an act of imagination, the dividend is imagined to be, the imagined revolutions for the sake of deriving the revolutions. Otherwise, the dividend is to be assumed to be the revolutions in a Kalpa. And so it is in this case, but the multiplier is the remainder of the intercalary months. Considering that intercalary months, omitted tithis, and so on further and further, are to be computed in this way as was (previously) said, there is no need for excessive details.

<C. *The Conjunct Pulverizer*>

Now for the sake of the accomplishment of the "conjunct" pulverizer he proclaims a sūtra (with the verse that begins): "If there is one divisor, (but) two different multipliers."

> <66a-d. If there is one divisor, (but) two different multipliers, then, when one assumes the sum of the (two) multipliers to be the dividend, (and) the sum of the surpluses is made the surplus, in accordance with what was said (previously), the accurate pulverizer is the one which is called "conjunct.">

When there is one divisor and two different multipliers, then the dividend is to be assumed to be the sum of the two different multipliers. Likewise, one should assume the sum of the surpluses—i.e., the sum of the remainders—to be the surplus—i.e., the additive. That additive, even when it is not said, is to be understood as negative. In this way, when the dividend, the divisor and the additive have been determined, in accordance with what was said (previously) that accurate pulverizer which is called "conjunct" was devised by the teacher. "Saṃśleṣa" means "combination" (saṃyoga), "non-disjunction" (aviśleṣa). The "conjunct" pulverizer (saṃśliṣṭakuṭṭaka) (is a compound in which the word "kuṭṭaka") is preceded by that (saṃśleṣa). The meaning is that it is accomplished by the sum of the multipliers and the remainders. The intent is that when one has established the dividend and the additive from the two multipliers and the two remainders, one should compute the multiplier by the method of the pulverizer, because it has been said previously: "But the additive is the remainder from the dividend."

<Here he enunciates an example (with the verse beginning): "What is (the quantity which), when multiplied by five?" >

<67a-d. What is (the quantity which), when multiplied by five and divided by sixty-three, has a remainder of seven? Now, the same quantity, when multiplied by ten and divided by sixty-three, the surplus is fourteen. Tell (me) this quantity.>

<It is clear.>

So here the setting out is:

Multiplier	5	Remainder	7	Multiplier	10	Remainder	14
Divisor	63			Divisor	63.		

Here by the method of the sūtra (beginning): "If there is one divisor," when one has assumed the sum of the multipliers to be the dividend and the sum of the remainders to be the additive, the setting out is:

Dividend	15	Additive	−21
Divisor	63.		

Now, when one has reduced the dividend and so on by three, again the setting out is:

Dividend	5	Additive	–7
Divisor	21.		

In this case, in accordance with what was said (previously), the multiplier and the quotient are produced by the method of the pulverizer: 14 (and) 3. In this way in many cases.

> In the excellent commentary on the *Bīja(gaṇita)*, the *Sūryaprakāśa,* which is distinguished by all the adornments of the virtuous and the wise, which is capable of destroying the darkness in the hearts of unintelligent students, and which has been enunciated by the poet, calculator, and teacher named Sūrya, the son of the astrologer Jñānarāja, a certain auspicious pulverizer having all the (necessary) demonstrations has been perfectly produced, one which enjoys the results of many excellent qualities (*or* which shares in many multipliers and quotients).

Thus, in the commentary on Bhāskara's *Bīja(gaṇita)*, which is called the *Sūryaprakāśa*, written by the astrologer (and) scholar Sūrya, the chapter on the pulverizer has reached its end.

CHAPTER VI

MATHEMATICAL AND HISTORICAL COMMENTARY

ON THE

TEXT ALPHA

1. *Preliminary Remarks.*

The present chapter is a mathematical and historical commentary on the *Text Alpha*, which is a portion of Sūryadāsa's commentary, namely, the *Sūryaprakāśa*. It was written in 1538 A.D. and belongs to the late medieval period (ca. 1200 A.D. – 1700 A.D.).

To facilitate the reading, the verses from Bhāskara's mūla have also been commented on. When a verse contains an example, usually a solution has been provided.

Following Sūrya's example, a set order has been maintained for each commentary on each verse. The grammatical indicators in our commentary are:

(i). Textual problems.

(ii). Mathematical meaning of a verse using modern mathematical language.

(iii). A "setting out," for the examples or problems, when necessary.

(iv). Comments (mathematical and historical) including comparisons of Bhāskara and Sūrya with other ancient and medieval authors.

The above grammatical indicators do not apply to the first chapter of the *Sūryaprakāśa*.

Note that the chapter numbers, headings and sub-headings in our commentary generally correspond to those in our translation of the *Text Alpha*. Nonetheless, a few new sub-sections and hence a few new sub-headings have been introduced, wherever necessary.

2. <*Text Alpha*, First Chapter>

<*Preface*>

In the opening line of the *Text Alpha*, the religious tributes which are for the deities Gaṇeśa and Sarasvatī and for the elders, are probably made by the author Sūryadāsa himself though possibly by some of the scribes of the various manuscripts. These tributes are paid for the success of a particular undertaking or work of composition and form a part of common practice in India.

Verses 1-6 are the verses of the maṅgalācaraṇa (auspicious introduction). They contain tributes to the gods Śiva and Gaṇapati and to Kṛṣṇa as algebra, to Sūryadāsa's father, Jñānarāja, to the author Bhāskara, and to his work, the *Bījagaṇita*. A distinctive feature of Sanskrit poetic writing is the extensive use of paronomasia. The double meanings contained in some of the above-mentioned verses of Sūryadāsa are both poetical and mathematical, as will be shown.

In the following, note the extreme importance attached by Sūrya to algebra, especially in verses 2, 3, 5 and 6.

1. The first verse consists of an invocatory prayer to Gaṇapati which refers to Śiva on the one hand and to his son Gaṇeśa on the other. Śiva is usually portrayed as having the moon (which is a symbol of brilliance) for his crest and snakes around his neck, with Gaṇeśa seated at his feet, and his wife Śrī (i.e. Pārvatī) clinging to his neck. Sūrya compares the radiance from the jewels on the hoods of the snakes to that from the jewel of the day (i.e. the Sun), one of whose names, Bhāskara, was borne by the author of the *Bījagaṇita*.

On the other hand, Gaṇeśa and Gaṇapati mean the same thing since Gaṇeśa is considered to be the superintendent of the gaṇas or the troops of the demigods who are Śiva's attendants. Gaṇeśa's elephant-head is usually decorated with flowers which attract bees.

2. The second verse, replete with paronomasia, contains homage to Kṛṣṇa, the dark-blue one, a word which here refers to algebra (i.e. the mathematics of the unmanifest) on the one hand, and to the unmanifest supreme deity, on the other.

The meaning is that (the unmanifest) algebra or the mathematics of the unknown (as a quantity or number) employs the technical term yāvattāvat (literally 'as much as,' 'so much,' meaning an arbitrary quantity) and the colours black, blue, yellow, white, red etc. to indicate the different unknowns. It (the mathematics of the unknown) involves operations such as division, and uses equal subtractions in the solution of equations. It is understood by the intellect by means of a kind of addition or exercise of the mind.

On the other hand, the unmanifest Supreme deity is in the form of Kṛṣṇa, who is one of the most popular deities and the most celebrated hero of the Indian mythology in the Dvāpara yuga. This supreme deity wears a garment splendid with the colours described in the verse in addition to a necklace, and has laughter on his lips. One can unite with this deity by means of deep, abstract meditation.

The striking fact here is that this same verse can be read either as a purely religious obeisance to a deity or as an appreciation of the importance of algebra.

3. In the third verse, the commentator Sūryadāsa adores his father and teacher Jñānarāja. The commentary *Sūryaprakāśa* includes some citations from the *Bījagaṇitādhyāya* section of the *Siddhāntasundara* of Jñānarāja (see e.g. the method of the approximate square-root following verse 44b-45a of our *Text Alpha*).

Note that in the present verse, Sūryadāsa mentions kuṭṭaka in addition to Pāṭī (arithmetic) and Bīja (algebra). This reveals the importance attached to this topic by Sūryadāsa and the fact that he used this term to denote a branch of Bījagaṇita; because generally kuṭṭaka (i.e. the method and the subject which deals with the solutions of the indeterminate equations of the first degree) was either a part of Pāṭīgaṇita (the section of Gaṇita which then dealt with arithmetic including geometry and mensuration) or of Bījagaṇita (which deals with algebra).

4. This verse is in praise of the rising of the Sun (or of Bhāskara, the author of the *Bījagaṇita*) which is a symbol of inspiration and knowledge. The meaning is that the sunrise of understanding destroys the confusion caused by the darkness (of ignorance), by bringing about knowledge of lunar eclipses, while the real sunrise destroys the union of two geese (a mythological event). Furthermore, the sunrise of understanding allows the poet (Bhāskara) to distinguish between (and write about) the two branches of Gaṇita. The actual sunrise fills the East which is reddened, while the sunrise of understanding fills the learned (e.g., Bhāskara) who are devoted to the poetic sentiments (rasas). This Sun, then, which is victorious, is both the divine awakener of nature's beauty and the poet-author of works on arithmetic, algebra, and mathematical astronomy, namely Bhāskara.

5. In verse five, Sūrya tells why and for whom he wants to write a commentary. He says that his commentary is like a boat which can serve two analogous purposes. On the one hand, those who are bewildered and dull, and desire to learn the tedious methods of algebra (which is the source of arithmetic), can do so through this commentary and thus cross the ocean of algebra. Analogously, the paronomastic implication is that those who cannot think at all, that is, those who are dead and whose souls seek emancipation can do so by merging in the supreme deity, which is the source of the entire visible world, by crossing the worldly ocean on the "boat of liberation."

Recall that Bījagaṇita is also known as avyaktagaṇita (i.e. the mathematics of the unmanifest). Since the various techniques of the avyaktagaṇita are resorted to in the solutions of problems of the vyaktagaṇita (i.e. the mathematics of the manifest, which is Pāṭīgaṇita), the former is considered to be the source of the latter just as the supreme deity is considered to be the source of the entire manifest universe.

6. In this verse, Sūrya compares certain features of algebra to aspects of the religious mantras used in rituals.

He means that Bījagaṇita uses the symbolic single syllables (*yā*, *kā*, *nī* etc. which stand, respectively, for yāvattāvat, kālaka, nīlaka etc.). These symbols are as hard to conceive of and interpret as is the first syllable of a mantra. This indicates that the study of Bījagaṇita requires keen intelligence, concentration of mind and imagination. Also, Bījagaṇita involves demonstrations of rules which need deep thinking, confidence and the exercise of ingenuity. So Sūrya intends to clarify Bījagaṇita and its source.

After the first six maṅgalācaraṇa-verses, Sūryadāsa pays homage in prose to Brahman which is *the unmanifest all-pervading spirit of the universe* (Apte, 1978, p. 705). Sūrya relates that Brahman assumed a body in the form of Brahmā as a favour to the entire creation. It is Brahmā who created Jyotiḥśāstra (the science of mathematics, astronomy, and astrology) along with its scholars such as Bhāskara, Sūryadāsa etc., to lift the world up (through their teachings) when it had been nearly destroyed by the power of the Kaliyuga. Alternatively, Brahmā created the Sun, the radiance of the rays of which brightens the world whenever it plunges into darkness by the power of the wide-spreading night.

A few of the terms e.g. Brahmāṇḍa (the golden egg in which the Universal Spirit itself was born as Brahmā), loka (world), yuga (age), used by Sūrya in his invocation to Brahman require further description in relation to the Indian theology. In this regard, the reader is referred to any scholarly treatment of Indian cosmology, for example, by Kirfel (1920).

Sūrya now begins his commentary on the *Bījagaṇita*. As was customary in India, in order to complete the treatise without any obstacles, Bhāskara begins by paying homage to his chosen deity—Gaṇapati i.e. Gaṇeśa—in his very first verse which is, in fact, the *Bījagaṇita*–maṅgalācaraṇa–verse.

Verse 1. This verse indicates that in Bhāskara's view, the vyakta (manifest) is developed as the product of the avyakta (unmanifest). This means, on the one hand, that all the phenomena of the material world have developed from the unmanifest Supreme Being. On the other hand, Bījagaṇita (algebra) is the source of Pāṭīgaṇita (arithmetic).

A similar verse given by Nārāyaṇa Paṇḍita (ca. 1356 A.D.) is *Bījagaṇitāvataṃsa*, 1, p. 45 (see Shukla, 1970):

यस्मादेतत्सकलं विश्वमनंतं प्रजायते व्यक्तम्।
अव्यक्तादपि बीजाच्छिवं च गणितं च तं नौमि ॥१॥

Nārāyaṇa is saying that as from the unmanifest, the entire manifest and endless universe is produced, so from the unmanifest bīja (algebra) the entire arithmetic (with its various rules) is produced.

The entire verse 1 of Bhāskara is understood in three ways. It praises (i) Gaṇeśa (the son of Śiva) as lord of intellect and remover of obstacles, and Gaṇeśa as his (Gaṇeśa's) father, Śiva, the source of all; (ii) Bhāskara's own father and teacher, Maheśvara (also a name of Śiva); and (iii) algebra. In the commentary which follows this verse, Sūrya is elaborating mainly on these three aspects.

(i) Firstly, Sūryadāsa explains that Gaṇeśa (i.e. Gaṇādhīśa or Gaṇādhipati) is the lord of intellect because our traditional doctrines and the teachings (i.e. the āgamas and śāstras) acknowledge him to be the lord of wisdom and remover of obstacles. Gaṇeśa is asked by us for assistance because the study of the science of the unmanifest is uniquely possible through the intellect. In order to justify this (last) claim with reference to the mathematics of the unmanifest (i.e. Bījagaṇita or algebra), Sūrya quotes the verse (which has no number in our edition) in which Bhāskara describes algebra as "thought accompanied by various colours." Bhāskara includes this verse in his *Bījagaṇita* in the section on varieties of quadratics, i.e. अनेकवर्णमध्यमाहरणभेदाः (see *BG*, 162, p. 122).

In the commentary following the unnumbered verse, Sūrya seems to be elaborating on Gaṇeśa as Śiva, the source or generator of all. Sūrya argues that the words "the generator" have been used by Bhāskara to answer questions such as: who is this chosen deity Gaṇeśa i.e. Śiva? What is his form? Of what sort is he? Sūrya explains that he is the maker of all that is manifest (or of all arithmetic). As an object in the form of a pot implies a potter, so an object in the form of the entire world implies a maker—who is the highest lord (Pareśa)—who has the attributes of Vighneśa (i.e. one who is capable of removing an obstacle i.e. Gaṇeśa). This follows also from Nyāyaśāstra (science of logic), according to which the existence of an object is in itself the testimony of the existence of a creator. Furthermore, the Sāṅkhyas declare that the unmanifest (i.e. formless sky, time and so on) is imbued by the existing Puruṣa. This indicates that here the maker is the highest lord. Therefore, this omnipresence (i.e. all-pervasiveness) is eternal. For further details the reader may look into any scholarly work on Sāṅkhya philosophy, e.g. the book by Sinha (1979).

It is to be noted that the word 'kavayaḥ' (*Text Alpha*, p. 4, 9) has the archaic (Vedic) sense of 'wise men' and not the classical sense of 'poets'. The same applies to the word Sāṅkhyāḥ in Bhāskara's verse 1 (*Text Alpha*, p. 3, 18).

As far as the derivation of the term Sāṅkhyāḥ is concerned, Sūrya explains that there is the suffix aṇ in the first syllable. This explanation is based on the two rules, numbers 4, 1, 83 (प्राग्दीव्यतोऽण्) and 4, 2, 59 (तदधीते तद्वेद), which were formulated by the celebrated Sanskrit grammarian Pāṇini (see e.g. *Pāṇini's Grammatik von Otto Böhtlingk*, 1977, pp. 159 and 176 respectively). These rules state that if a word consists of a preposition followed by a verb, then you add the suffix aṇ after the preposition in some situations, one of which is: studying and knowing something.

Sūrya refers to Pāṇini's rules again in connection with 'sāṅkhya.' Rules 3, 1, 136 and 3, 3, 106 of Pāṇini state आतश्चोपसर्गे, which means that the suffix—ka is applied after a root ending in—ā in a word which has a prefix (see e.g. *La Grammaire de Pāṇini par Louis Renou, Vol. 1*, 1966, pp. 176 and 241 respectively). Thus saṅkhyā (= sam + khyā), by the application of these two rules, becomes sāṅkhya.

Sūrya's explanation about "the numbered one" (i.e. अगणो गणः संजात इति गणितः) is somewhat similar to Nārāyaṇa's explanation about mathematics or computation (i.e. गणितमिति नाम लोके ख्यातमभूदगणितस्य शास्त्रस्य, see *BGV*, 3a, p. 1) which means: the science which was uncounted in the world came to be known as "gaṇitam" (numbered). Furthermore, the pun on the term gaṇaḥ (गण:) is that on the one hand, it refers to a number; and on the other hand, it refers to a multitude, particularly to a troop of demigods considered as Śiva's attendants and under the superintendence of Gaṇeśa who is a demigod of this troop (Apte, 1978, p. 395).

Also, note that the use of "ekabīja" for the highest lord Gaṇapati indicates Sūrya's analogy between Gaṇapati and the mathematics of the unmanifest. This implies that algebra is the most important mathematical subject, so that neglect of its study would mean neglect of the Supreme Lord.

(ii) Having explained Gaṇeśa as Śiva, the source of all and the numbered one, Sūrya explains that Bhāskara next pays homage to his father and teacher, guru Maheśvara. The etymological point here is that 'guru' also means 'father'. Maheśvara taught his son Bhāskara the science of the unmanifest. In fact, Maheśvara was one of several prominent scholars in Bhāskara's lineage (Pingree, 1981b). Bhāskara's ancestors as well as his children had close connections with local political powers. For example, Bhāskara's great-great-great-grandfather (also called Bhāskara) was honoured by the Paramāra king Bhojarāja. Bhāskara's son Lakṣmīdhara and grandson Caṅgadeva were astrologers at the court of the Yādava rulers (Table 8, p. 124).

In connection with the word 'father,' जनितृ, referred to in the preceding paragraph, nearly all the manuscripts of the *Sūryaprakāśa* support the wrong readings जने° instead of जनि° in two places (see *Text Alpha*, p. 4, 24 and p. 5, 1 and the *Apparatus Criticus*). We conclude that Sūryadāsa himself wrote the word incorrectly.

The next verse, beginning with आसीन्महेश्वर and which has no number in our edition, is a tribute by Bhāskara to the wisdom of his father Maheśvara. This verse has, in fact, been stated by Bhāskara in the conclusion to his *Bījagaṇita* (see e.g. *BG*, 207, p. 162). Sūrya has quoted this verse here in order to indicate that the bowing done to his father by Bhāskara is not inappropriate since Maheśvara is the lord (i.e. controller) of the intellect because of his knowledge. This is clear since Bhāskara has proclaimed that Maheśvara earned the epithet "best of the ācāryas (ācāryavarya)."

Now to justify Bhāskara's obeisance to his father at the time of speaking about algebra, Sūrya explains Bhāskara's use of the word Sāṅkhyas as meaning the followers of Jyotiṣa. These followers declare that the existing Puruṣa is the imbuer of the calculation of the unmanifest called "Bīja." This Bīja is the unique source of Pāṭīgaṇita which is composed by the wise; Bhāskara's father being the "best of the ācāryas of the wise."

(iii) As was mentioned before, Bhāskara's verse 1 is also in praise of that mathematics which is called avyakta (algebra) or Bīja and is the unique source of vyakta or Pāṭī. The calculators say that Bīja is the generator of intellect and is imbued by the existing Puruṣa. It is also īśa (the lord). A semi-etymological explanation of the word īśa as given by Sūrya is: "It is the one in whom (any) desire is unopposed."

In addition to the above three points, Sūrya states that Bhāskara's knowledge of Sāṅkhya philosophy is also indicated in verse 1, because in this verse Bhāskara salutes his chosen deity who is also the deity of the intellect and is called "the unmanifest." Another synonym of the unmanifest is Prakṛti (the first being, Puruṣa) which is the cause of the disturbance of the (three) guṇas (sattva, rajas and tamas), and is the generator of the intellect. In this context, the etymological point made by Sūrya is that the Sāṅkhyas are so-called because they teach the science called Sāṅkhya. The Sāṅkhya philosophy treats of twenty-four tattvas (of which one is intellect). The Sāṅkhyas study them, know them and teach them. That is why the suffix 'aṇ' is applied to the word "saṅkhyā" to produce the word "sāṅkhya". Furthermore, in order to elucidate Bhāskara's high esteem for Sāṅkhya philosophy, Sūrya quotes a verse from the *Golādhyāya* section of the *Siddhāntaśiromaṇi* which was composed by Bhāskara (see Āpaṭe, 1943, *GD I* Bhuvanakośapraśna, 1, *ASS 122*, p. 21). In this verse, Bhāskara's description of the creation of the universe has a striking similarity to that of the Sāṅkhya philosophers. The complete verse is the following:

यस्मात्क्षुब्धप्रकृतिपुरुषाभ्यां महानस्य गर्भे-
ऽहंकारोऽभूत्खशिखिजलोर्व्यस्ततः संहतेश्च ।
ब्रह्माण्डं यज्जठरगमहीपृष्ठनिष्ठाद्विरिञ्चे-
र्विश्वं शश्वज्जयति परमं ब्रह्म तत्तत्त्वमाद्यम् ॥१॥

It means: "That first tattva, the highest Brahma, is always victorious from which came into being the great (intellect), from Prakṛti and Puruṣa when they were agitated, (and) in its interior self-awareness, (and) from that the sky, fire, water, and earth, and from (their) combination the whole Brahmāṇḍa (arose) from Brahmā who stood on the surface of the earth within it."

Sūrya's next verse, which begins: "प्रकृतिपुरुषयोगाद्बुद्धितत्त्वमित्यादिना चेति" is verse 9a in the Bhuvanakośa section of the *Golādhyāya* in the *Siddhāntasundara* of Sūrya's father, Jñānarāja. Sūrya quotes it here because its contents are identical with those of Bhāskara's verse, for it means: "The tattva that is intellect (comes) from the union of Prakṛti and Puruṣa."

Thus, having paid obeisance to his chosen deity with the first verse, the teacher Bhāskara, now speaks of the utility of Bīja with the next verse.

Verse 2. Here, by the phrase "previously mentioned," Bhāskara is referring to the manifest (arithmetic) which he explained in the *Līlāvatī*. Bhāskara feels the need to write a treatise on algebra because some mathematical questions are so difficult that they cannot be very well understood by the dull-witted without resorting to algebra.

A similar verse has been given by Brahmagupta in his *Brāhmasphuṭasiddhānta* Kuṭṭakādhyāya XVIII, 1 (see Dvivedin, 1902):

प्रायेण यतः प्रश्नाः कुट्टाकारादृते न शक्यन्ते।
ज्ञातुं वक्ष्यामि ततः कुट्टाकारं सह प्रश्नैः ॥१॥

Brahmagupta says that questions can scarcely be solved without kuṭṭaka (algebra), so he is going to describe kuṭṭaka and provide the rules with problems. This reveals one instance of the influence of Brahmagupta on Bhāskara.

On the other hand, Nārāyaṇa Paṇḍita seems to have been influenced by Bhāskara; for he says (*BGV*, 5-6, p. 1):

यो यो यं यं प्रश्नं पृच्छति सम्यक्करणं न तस्यास्ति।
व्यक्तेऽथाव्यक्ते तु प्रायस्तत्करणमस्त्येव ॥५॥

व्यक्तक्रियया ज्ञातुं प्रश्ना न खिलीभिवन्ति [*sic*] नाल्पधियः।
बीजक्रियां च तस्माद् वच्मि व्यक्तां सुबोधां च ॥६॥

Nārāyaṇa essentially means: "There are questions the solutions of which do not exist in vyaktagaṇita but they (solutions) are generally found in avyaktagaṇita. Since the less intelligent are not able to solve questions by the methods of vyaktagaṇita, therefore I am going to describe the clear and easily intelligible algebraic operations."

3. <*Text Alpha*, Second Chapter>

<The Chapter Concerning the Six-Fold (Operation)>

A. *<The Six-Fold (Operation) of Positive and Negative (Quantities)>—Textual Commentary (Verses 3a-8d).*

Verse 3a-b. Textual problems: Part of Sūrya's text pertaining to the demonstration of this verse was missing in the manuscripts of class A, but present in those of class β (see our Apparatus Criticus). This part of the *Sūryaprakāśa* had to be supplied from the manuscripts of class β in order to maintain continuity and completeness of the demonstration.

Mathematical meaning: Bhāskara is enunciating, though incompletely, the formulas:

For $a, b > 0$, $(-a) + (-b) = -(a + b)$; $a + b = a + b$; $a + (-b) = a - b$;

$(-a) + b = b - a$.

Comments: Sūrya's demonstration of the principle underlying this verse of Bhāskara involves the idea of the computation (of the longitudes) of the planets. Here Sūrya seems to be referring to Bhāskara's *Siddhāntaśiromaṇi GG I* Spaṣṭādhikāra, 64 (see Āpaṭe, 1939, *ASS 110*, p. 125):

चेत् स्वोदयैः स्फुटरवेरसवः कृतास्ते
विश्लेषिताश्च यदि मध्यरवेः कलाभिः।
बाह्वन्तराख्यमुदयान्तरकं चराख्यं
कर्मत्रयं विहितमौदयिके तदा स्यात् ॥६४॥

Unnumbered verse following 3a-b. Textual Problems: The source of this verse is not known. Part of the *Sūryaprakāśa* was also missing in the manuscripts of class A at this point. It was missed due to homoeoteleuton. It had to be supplied from the manuscripts of class β.

Mathematical meaning: For $a, b > 0$, $a + b > 0$; $(-a) + (-b) < 0$. Furthermore, if $0 < b < a$, then $a + (-b) = + (a - b)$; and if $0 < a < b$, then $a + (-b) = - (b - a)$.

Comments: Obviously, this sūtra is a refinement of the sūtra in 3a-b giving due consideration to the sign of the result of the addition.

In the demonstration of this verse, Sūrya seems to be referring to verse 13 of the Grahacchāyādhikāra of Bhāskara's *Siddhāntaśiromaṇi GG II* (see Āpaṭe, 1941, *ASS 110*, p. 86):

स्पष्टा क्रान्तिः स्फुटशरयुतोनैकभिन्नाशभावे
    तज्ज्या स्पष्टोऽपमगुण इतो द्युज्यकाद्यं ग्रहस्य।
कृत्वा साध्या तदुदितघटीभिः प्रभा भानुभाव-
    च्चन्द्रादीनां नलकसुषिरे दर्शनायापि भानाम् ॥१३॥

Verse 3c-4b. Mathematical meaning: Perform the additions:

$(-3) + (-4)$; $3 + 4$; $3 + (-4)$; $(-3) + 4$.

Setting out: -3 and -4; 3 and 4; 3 and -4; -3 and 4.

We provide the solutions as follows:

$(-3) + (-4) = -7$; $3 + 4 = 7$;

$3 + (-4) = - (4 - 3) = -1$;

$(-3) + 4 = 4 - 3 = 1$.

Comments: A similar verse given by Nārāyaṇa is his *Bījagaṇitāvataṃsa*, 1, p. 2 (see Shukla, 1970):

रूपत्रयञ्च रूपकपञ्चकमस्वं धनात्मकं वाऽपि।
वद सहितं झटिति सखे स्वर्णमृणं स्वं च यदि वेत्सि ॥१॥

The only difference in this verse is that it uses the number 5 instead of (Bhāskara's) 4.

Sūrya is referring to the concept of dots over negative numbers described in Bhāskara's *Bījagaṇita*, 3c-4b, p. 1 (see Vidyāsāgara, 1878). Nārāyaṇa's *Bījagaṇitāvataṃsa*, 7, p. 2 has a similar description:

रूपाणामव्यक्तानां नामाद्यक्षराणि लेख्यानि।
उपलक्षणाय तेषामृणगानामूर्ध्वबिन्दूनि ॥७॥

Miśra (1947) comments (on p. 88) that Śrīpati does not state such a definition in his *Siddhāntaśekhara* (see under *SSE* XIV, 3, p. 87) because it is a famous old definition. But another plausible argument is that Śrīpati did not feel the need to provide such a definition because he did not state the (above) problem involving negative numbers.

Sūrya borrows the words 'nyāsaḥ' and 'yoge jātam' from Bhāskara's text; Bhāskara in turn borrows from his predecessors. These terms are, in fact, traditional, though it cannot be stated how old the tradition is.

Verse 4c-d. Textual problems: The explanation of the second half of verse 4d was skipped by the manuscripts of class A. It has been supplied from the manuscripts of class β, which is the counterpart of class A, on the observation that Sūrya generally provides complete explanations for those verses of the mūla which involve sūtras. On the other hand, in the demonstration part, the manuscripts of the β-recension contained some additional text. The demonstration given in this text seemed to be alternative to that given in the (common) text belonging to both A and β-recensions. As the additional text did not exist in the manuscripts of the A-recension, it has been placed by us in the Appendix #1.

Mathematical meaning: For $b > 0$, $a - b = a + (-b)$; $a - (-b) = a + b$.

Comments: In a subtraction, this is the modern rule "change the sign and add."

Kṛṣṇa (ca. 1600 A.D.), who is one of the commentators on Bhāskara's *Bījagaṇita* and *Līlāvatī*, demonstrates the sūtra contained in the present verse in terms of direction, time and wealth in his *Bījapallava*, 3, pp. 13-14 (see Radhakrishna Sastri, 1958, *Madras GOS 67)*.

Verse 5a-b. Mathematical meaning: Perform the subtractions:

$3 - 2$; $(-3) - (-2)$; $3 - (-2)$; $(-3) - 2$.

Setting out: 3 and 2; –3 and –2; 3 and –2; –3 and 2.

We provide the solutions as does Bhāskara (in his *BG*, p. 2):

$3 - 2 = 3 + (-2) = 1$; $(-3) - (-2) = (-3) + 2 = -1$;

$3 - (-2) = 3 + 2 = 5$; $(-3) - 2 = (-3) + (-2) = -5$.

Comments: The corresponding verse stated by Nārāyaṇa is *BGV*, 2, p. 2:

रूपाष्टकं रूपकपञ्चकेन
क्षयं क्षयेनापि धनं धनेन।
धनं क्षयेण क्षयगं धनेन
व्यस्तं च संशोध्य वदाशु शेषम् ॥२॥

Nārāyaṇa's verse contains integers 8 and 5 instead of (Bhāskara's) 3 and 2, however, Nārāyaṇa's line 2d is literally identical with Bhāskara's line 5b.

Sūrya thinks that the verse 5a-b is self-explanatory but Kṛṣṇa furnishes two kinds of explanations: the first involves numbers, the second involves travelling in Eastern and Western lands (see Kṛṣṇa's *BP*, without number, pp. 14-15 in Radhakrishna Sastri, 1958).

Verse 5c-d. Textual problems: In the artha part, the text from β had to be utilized because the writer of manuscript A seems to have omitted it due to homoeoteleuton. The copyist of manuscript ε had to follow this omission.

Mathematical meaning: With $a, b > 0$, $a * b = a * b$; $(-a) * (-b) = a * b$; $(-a) * b = -(a * b)$; $a * (-b) = -(a * b)$; where $*$ stands for $\cdot$ or $\div$.

Comments: Śrīpati's equivalent of Bhāskara's 5c (which contains the rule for multiplication) is *Siddhāntaśekhara* XIV, 4a-b:

वधे धनं स्यादृणयोः स्वयोश्च
धनर्णयोः संगुणने क्षयश्च ||४a-b||

The corresponding equivalent of Brahmagupta is *Brāhmasphuṭasiddhānta* XVIII, 33a (see Dvivedin, 1902, p. 310):

ऋणमृणधनयोर्घातो धनमृणयोर्धनवधो धनं भवति ||३३a||

Note that the first half of Bhāskara's 5c is a modified form of Śrīpati's 4a, which in turn is a modified form of the second half of Brahmagupta's 33a. Also the second half of Bhāskara's 5c and line 4b of Śrīpati are modified forms of the first half of Brahmagupta's 33a.

Nārāyaṇa's equivalent of Bhāskara's 5c is *BGV*, 9, p. 3:

ऋणयोर्धनयोर्घाते स्वं स्यादृणधनहतावस्वम् ||९||

The first half of this line contains, in almost the same order, the synonyms of the words which are contained in the first half of Bhāskara's 5c. The second half seems to be based on the first half of Brahmagupta's 33a.

A comparison of Bhāskara's 5d (which contains the sūtra for division) with the equivalents of Śrīpati (*SSE* XIV, 4c-d):

क्षये क्षयेणाथ धने धनेन
विभाजिते स्याद्धनमन्यथर्णम् ||४c-d||

and Brahmagupta (*BSS* XVIII, 34a-b):

धनभक्तं धनमृणहृतमृणं धनं भवति खं खभक्तं खम्।
भक्तमृणेन धनमृणं धनेन हृतमृणमृणं भवति ।।३४a-b।।

and Nārāyaṇa (*BGV*, without number, p. 3):

ऋणधनगुणने यच्चोपलक्षणं तच्च भागहरणेऽपि।

and Mahāvīra (*GSS*, 50, p. 6, see Raṅgācārya, 1912):

ऋणयोर्धनयोर्घाते भजने च फलं धनम्।
ऋणं धनर्णयोस्तु स्यात्स्वर्णयोर्विवरं युतौ ।।५०।।

reveals that Śrīpati's is a condensed version of that of Brahmagupta, while Bhāskara provides only a short hint. Nārāyaṇa's hint is more explicit than that of Bhāskara. But Mahāvīra explains both the rules (for multiplication and division) together, using very concise but clear language.

Sūrya's commentary on Bhāskara's 5c-d reveals a distinctive feature of his expositions, their specific logical sequence: 'iti sambandhaḥ' (इति संबंधः), 'ityarthaḥ' (इत्यर्थः) and 'ityupapannam' (इत्युपपन्नम्).

In the demonstration part of 5c, Sūrya tries to explain that if the divisor is negative and the dividend is positive, then the quotient will be negative. So the product of the divisor and the quotient will be positive, for if it were negative, then in the division process, in order to get zero as the remainder, we will have to *add* (instead of subtract) the negative product to the positive dividend; but this (last operation) will contradict the principle that division is (repeated) subtraction.

In the demonstration of 5d, the sūtra referred to by Sūrya i.e., "यद्गुणो हारो भाज्याच्छुद्ध्यति तत्फलम्", is Bhāskara's *Līlāvatī*, 18a, p. 18 (see Āpaṭe, 1937, *L I*, *ASS 107*).

The commentator Kṛṣṇa does not seem to follow Sūrya, so far as the sequence of verses is concerned. For example, (in terms of Vidyāsāgara's numbering) Kṛṣṇa comments on 5d after (instead of before) 6a-b because the example contained in 6a-b involves an application of the rule given in 5c and not in 5d. (See Kṛṣṇa's *BP*, pp. 15-18.)

Verse 6a-b. Mathematical meaning: Perform the multiplications:

$2 \cdot 3$; $(-2) \cdot (-3)$; $(-2) \cdot 3$; $2 \cdot (-3)$.

Comments: The corresponding equivalent of Nārāyaṇa is *BGV*, 3, p. 3:

रूपद्वयं रूपकपञ्चकेन
धनं धनेन क्षयगं क्षयेण।
धनं क्षयेण क्षयगं धनेन
निघ्नं पृथक् किं गुणने फलं स्यात् ॥३॥

Verse 6c-7b. Mathematical meaning: Perform the four divisions:

$8 \div 4$; $(-8) \div (-4)$; $(-8) \div 4$; $8 \div (-4)$.

Comments: A similar verse given by Nārāyaṇa is *BGV*, 4, p. 4:

द्विनिघ्नरूपत्रितयं द्विकेन
धनं धनेनर्णमृणेन भक्तम्।
ऋणं धनेन स्वमृणेन वापि
सखे वदाश्वत्र हृतौ फलं मे ॥४॥

Note the striking similarity in the language between Bhāskara's 7a and Nārāyaṇa's 4c. The latter states: ऋणं धनेन स्वमृणेन वापि, while Bhāskara has: ऋणं धनेन स्वमृणेन किं स्याद् ।

Kṛṣṇa provides solutions to problems in 6a-7b, whereas Sūrya leaves the solutions to the reader as exercises. Perhaps Sūrya does not feel the need for solutions because Bhāskara's text has them. Or Sūrya thinks them to be too trivial.

Verse 7c-d. Mathematical meaning: With $a, b > 0$, $a^2 = a^2$; $(-a)^2 = a^2$; $\sqrt{a^2} = a$ and $-a$; and $\sqrt{-b}$ does not exist (since $-b$ cannot equal a square).

Comments: In the explanation to 7d, Sūrya refers to the definition of a square contained in the first half of Bhāskara's *L*, 19a, p. 19 (see Āpaṭe, 1937, *L I*, *ASS 107*):

समद्विघातः कृतिरुच्यतेऽथ स्थाप्योऽन्त्यवर्गो द्विगुणान्त्यनिघ्नाः ॥१९a॥

Note the similarities between this definition, Śrīpati's in *SSE* XIII, 4a:

वर्गोऽभिघातः सदृशद्विराश्योः ॥४a॥

and Śrīdhara's in the first half of *PG*, 24a, p. 16:

सदृशद्विराशिघातो रूपादिद्विचयपदसमासो (वा) ॥२४a॥

Verse 8a-d. Mathematical meaning: Perform the squares and square-roots: $3^2; (-3)^2; \sqrt{9}; \sqrt{-9}$.

Comments: Kṛṣṇa solves these problems as does Bhāskara. But in Sūrya's view, they are straightforward.

B. *<The Six-Fold (Operation) of Zero>—Textual Commentary (Verses 9a-11d).*

Verse 9a-b. Textual problems: In the commentary pertaining to this verse, the manuscripts of class β have some additional text which seems to be a repetition of the text already contained in both A and β-recensions. It is omitted by manuscripts of class A and so also by us. This complete β-text goes to the Appendix #2.

Mathematical meaning: $\pm a + 0 = \pm a$; $\pm a - 0 = \pm a$; $0 - a = -a$; $0 - (-a) = a$.

Comments: The equivalents of this verse given by some of the other mathematicians are as follows: Brahmagupta's *BSS* XVIII, 32a

शून्यविहीनमृणमृणं धनं धनं भवति शून्यमाकाशम् ॥३२a॥

Mahāvīra's *GSS*, 49, p. 6

ताडितः खेन राशिः खं सोऽविकारी हृतो युतः।
हीनोऽपि खवधादिः खं योगे खं योज्यरूपकम् ॥४९॥

Śrīpati's *SSE* XIV, 6

विकारमायान्ति धनर्णकानि
न शून्यसंयोगवियोगतस्तु।
शून्याद्विशुद्धं स्वमृणं क्षयं स्वं
वधादिना खं खहरं विभक्तम् ॥६॥

and Nārāyaṇa's *BGV*, 11, p. 5.

स्वर्णं शून्येन युतं विवर्जितं वा तथैव तद् भवति।
शून्यादपनीतं तत् स्वर्णं व्यत्यासमुपयाति ॥११॥

Verse 9c-d. Mathematical meaning: What is 0 + (± 3), 0 + 0, 0 – (± 3), 0 – 0?

Verse 10a-b. Mathematical meaning: $0 \cdot a = 0, 0 \div a = 0, a \cdot 0 = 0$, but $a \div 0$ is just a (mysterious) quantity "khahara."

Comments: According to Sūrya, "and so on" may also include $0^2 = 0, \sqrt{0} = 0$. Furthermore, in his commentary, *Gaṇitāmṛtakūpikā* (written in 1541 A.D.), on the *Līlāvatī* of Bhāskara, in the section on śūnya, Sūrya refers to the following from the *Sūryaprakāśa* (see ms. *Wai, PrM 9762*, f. 21v., 5): शून्यस्य स्वातंत्र्येण संख्याविषयत्वाभावादिति भावः।

Kṛṣṇa's demonstration of Bhāskara's 10a-b is remarkable in the sense that he considers zero to be an infinitesimal. For 10a, Kṛṣṇa (*BP*, 5, p. 27) constructs examples and then concludes: as the multiplicand decreases (the multiplier being fixed), so does the product. If the multiplicand decreases to the utmost (the multiplier remaining fixed), so does the product. And, in reduction to the utmost, śūnya results. Similar is the situation when the multiplicand stays fixed but the multiplier varies. Also, Kṛṣṇa explains 10b along similar lines.

Some of the equivalents of Bhāskara's 10a-b are the following:

(i) Brahmagupta's *BSS* XVIII, 33b

शून्यर्णयोः सधनयोः सशून्ययोर्वा वधः शून्यम् ॥३३b॥

and Brahmagupta's *BSS* XVIII, 35a;

सोद्धृतमृणं धनं वा तच्छेदं समृणधनविभक्तं वा ॥३५a॥

(ii) Mahāvīra's *GSS*, 49, p. 6 (quoted before); (iii) Śrīpati's *SSE* XIV, 6d (quoted before); and (iv) Nārāyaṇa's *BGV*, 12, pp. 5-6 (quoted below).

खं राशिना विगुणितं खं स्याद्राशिः खगुणश्च खं भवति।
खं राशिना विभक्तं खं स्याद्राशिः खभाजितः खहरः ॥१२॥

In addition, Brahmagupta (*BSS* XVIII, 34a) states that zero divided by zero is zero:

धनभक्तं धनमृणहृतमृणं धनं भवति खं खभक्तं खम् ॥३४a॥

The first mathematician who spoke of division by zero seems to be Brahmagupta. In his verse 35a, he says: "A positive or negative (quantity) divided by zero is taccheda (i.e. having that as divisor)." In verse 34a, he says: "Zero divided by zero is zero."

Of course, in the modern sense, $\frac{0}{0}$ is meaningless outside the use of limits.

Mahāvīra's verse 49 contains: "A quantity multiplied by zero is zero. It remains unchanged when it is divided by, combined with (or) diminished by zero."

Śrīdhara (ca. eighth century A.D.) does not mention division by zero (see *TS*, 8, p. 4 or *PG*, 21, p. 14). Similarly, Āryabhaṭa II ignores division by zero (Datta, 1927, *BCMS 18*, p. 169).

Śrīpati's *SSE* XIV, 6 includes: "(When a quantity is) divided by zero, it is (called) khahara (i.e. having zero as its divisor)."

Bhāskara calls such a quantity "khahara" or "khahāra" (see *Text Alpha*, verses 10b and 11a).

Verse without number. Mathematical meaning: The verse says $a \cdot 0 = 0$; and then attempts to explain that $a \div 0$, a being a finite quantity, is in some sense infinite.

Comments: Sūrya has quoted this verse from Nārāyaṇa's algebra (*BGV*, 14, p. 6). Nārāyaṇa is describing a property of the "khahara" quantity in this verse. This is clear in view of his verse 15, p. 6 (which is exactly Bhāskara's 11a-d, see *Text Alpha*). Nārāyaṇa's preceding two verses are also in the context of a khahara quantity (*BGV*, 12-13, pp. 5-6):

खं राशिना विगुणितं खं स्याद्राशिः खगुणश्च खं भवति।
खं राशिना विभक्तं खं स्याद्राशिः खभाजितः खहरः ॥१२॥

शेषविधौ सति खगुणश्चिन्त्यः शून्ये गुणे खहारश्चेत्।
पुनरेव तदाविकृतो राशिर्ज्ञेयोऽत्र मतिमद्भिः ॥१३॥

These two verses include, among other things, the following: $a \cdot 0 = 0$ but if some operation is remaining, then do not replace $a \cdot 0$ by 0; because in that case $\frac{a \cdot 0}{0} = a$. Furthermore, Nārāyaṇa's explanation to his *BGV*, 8, p. 7 includes that if the multiplier and divisor are both zero, then multiplication and division by zero should not be performed.

It is interesting to note that Nārāyaṇa's *BGV*, 12-14, p. 6 are based on Bhāskara's *L*, 45-46, p. 39 (see Āpaṭe, 1937, *L I*, *ASS 107*):

योगे खं क्षेपसमं, वर्गादौ खं, खभाजितो राशिः।
खहरः स्यात्, खगुणः खं, खगुणश्चिन्त्यश्च शेषविधौ ॥४५॥

शून्ये गुणके जाते, खं हारश्चेत्पुनस्तदा राशिः।
अविकृत एव ज्ञेयस्तथैव खेनोनितश्च युतः ॥४६॥

Furthermore, in his *Līlāvatī* (*L I*, 47, *ASS 107*, pp. 40-42), Bhāskara solves a problem using the sūtra $\frac{a \cdot 0}{0} = a$. In the modern symbolism, the problem can be written as $\frac{(x \cdot 0 + \frac{x \cdot 0}{2}) \cdot 3}{0} = 63$. Bhāskara gives its solution as 14 and remarks: "There is extensive use of this calculation in the computation of (the longitudes of) the planets."

Recall that the calculations of planets usually involve quadratic functions and their derivatives. Bhāskara's computation corresponds very well with the modern limiting operation. For example, let $f(x) = x^2$. Then $f'(x) = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon) - f(x)}{\varepsilon}$

$$= \lim_{\varepsilon \to 0} \frac{x^2 + 2x\varepsilon + \varepsilon^2 - x^2}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{(2x+\varepsilon)\varepsilon}{\varepsilon} = 2x.$$

In a more rough and ready calculus course, as occasionally found in engineering schools, this computation might be taught heuristically as follows:

$$\begin{aligned} f'(x) = \frac{f(x+0) - f(x)}{0} &= \frac{x^2 + 2\cdot x \cdot 0 + 0 \cdot 0 - x^2}{0} \\ &= \frac{2x \cdot 0}{0} + \frac{0 \cdot 0}{0} \\ &= 2x + 0 \qquad \text{(Using } \frac{a \cdot 0}{0} = a\text{)} \\ &= 2x. \end{aligned}$$

Finally, Nārāyaṇa's *BGV*, 14, p. 6 has been cited by Gaṇeśa (b. 1507 A.D.) in his commentary entitled *Buddhivilāsinī* (written in 1545 A.D.) on Bhāskara's *Līlāvatī*, 45-46 (see e.g. Āpaṭe, 1937, *L I*, *ASS 107*, p. 40).

Verse 10c-d. Mathematical meaning: Calculate $0 \cdot 2, 0 \div ?, 3 \div 0, 0^2, \sqrt{0}$.

Comments: Kṛṣṇa solves this problem as does Bhāskara, while Sūrya does not. For the quotient of the division of 3 by 0, Bhāskara and Kṛṣṇa write rū $\frac{3}{0}$ and khahara 3 respectively.

After verse 10c-d, Bhāskara gives the value of the khahara quantity as infinite ("ananta"). The striking fact in this regard is that while Brahmagupta and Śrīpati gave only special technical names ("taccheda" i.e. kha cheda and "khahara" respectively), Bhāskara gave the true value of a khahara quantity as well. In addition, Bhāskara described the khahara quantity by comparing it with God Viṣṇu (see verse 11 below).

Verse 11a-d. Mathematical meaning: $\frac{a}{0} \pm b = \frac{a}{0}$ i.e. infinity remains unaffected by the addition of or subtraction of any finite quantity however large.

Comments: In his commentary, Sūrya is explaining how the infinite quantity 'khahara' remains unchanged by the addition or subtraction of a number of (finite) quantities. Perhaps Sūrya was thinking along the following lines:

$$\frac{a}{0} \pm \frac{b}{c} = \frac{a \cdot c}{0 \cdot c} \pm \frac{b \cdot 0}{c \cdot 0} = \frac{ac}{0} \pm \frac{0}{0} = \frac{ac \pm 0}{0} = \frac{ac}{0}.$$

Here $\frac{ac}{0}$ is again infinite, so both $\frac{a}{0}$ and $\frac{ac}{0}$ are equivalent. They possess the essential property that their denominator is zero (khahara).

Bhāskara's 11a has been referred to by Jñānarāja in his *SSU Bījādhyāya* as follows (see ms. *Berlin 833*, f. 1v., 7-9): अथ भास्करीयव्यक्ताव्यक्ते यदुक्तं खहरे राशौ विकारो नेति भिन्नांके व्यभिचरति। Also 11a-b has been quoted by Sūrya in his commentary *GMK* on the *Līlāvatī* as follows (see ms. *Wai, PPM 9762*, f. 21v., 8-9): तदुक्तं बीजगणिते।

अस्मिन्विकारः खहरे न राशा-
वपि प्रविष्टेष्वपि निःसृतेष्विति।

As mentioned before, Bhāskara's lines 11a-b appear verbatim as 15a in Nārāyaṇa's *BGV*, while the next line (15b) which is the same as Bhāskara's 11c-d is missing from the manuscript but has been supplied by the editor.

Kṛṣṇa explains the infinity of a khahara quantity in the commentary to his *BP*, 5, p. 28 as follows: "As there is reduction of the divisor, so there is rise in the quotient. So when there is the greatest reduction in the divisor, there should be the greatest rise in the quotient."

A mathematical infinity has been mentioned much earlier. In the *Kalpa Sūtra* and *Nava Tattva* (ca. 300 B.C.), infinity was described as (Datta, 1927, *BCMS 18*, p. 175): "A number as great as the number of grains of sand on the brink of all the rivers on the earth or the drops of water in the oceans."

Verse without number. Comments: The last verse of this sub-section, which has no number, has been cited by Sūrya from the Anuśāsanaparvan of the *Mahābhārata*, where it is 135,11 (see Dandekar, 1966, *Vol. 17, Part II*, p. 705). There had been in existence a tradition according to which the Anuśāsanaparvan was regarded as a part or a sub-parvan of the Śāntiparvan, a tradition which Sūrya clearly follows.

The discussion about zero contained in this sub-section B. can be summarized as follows (using the modern notation):

According to Brahmagupta, $\frac{0}{0} = 0$; $\pm\frac{a}{0} =$ kha cheda. According to Śrīpati, $\frac{a}{0} =$ khahara. According to Bhāskara, $\frac{a}{0}$ is khahara and is infinite in its value. Also Bhāskara's *Līlāvatī* has $\frac{a \cdot 0}{0} = a$, while $a \cdot 0 = 0$. Datta (1927) states that Bhāskara's *Līlāvatī* has an instance of the kind $(\frac{a}{0})0 = a$. In the modern sense, the last two expressions are of the type $\frac{0}{0}$ and $\infty \cdot 0$ respectively, which are indeterminate and not finite as stated by Bhāskara (*BCMS 18*, p. 171).

One cannot help thinking however, that the above-mentioned authors were giving correct or logical answers in the particular contexts which they had in mind. The important one is $\frac{0}{0}$, which is 0 according to Brahmagupta and a finite quantity according to Bhāskara (since $\frac{a \cdot 0}{0} = a$ means $\frac{0}{0} = a$). The answers are correct in case of limiting processes where zero is considered as an infinitesimal quantity ('tuchha' or 'kṣudra'). But in general, the answers are wrong. For

$$\lim_{\varepsilon \to 0} \frac{\varepsilon^2}{\varepsilon} = \lim_{\varepsilon \to 0} \varepsilon = 0 \text{ and } \lim_{\varepsilon \to 0} \frac{a\varepsilon}{\varepsilon} = a;$$

but in the general case, using the definition of division, we have: $\frac{0}{0}$ is that (unique) number x such that $x \cdot 0 = 0$. Now since any number multiplied by 0 gives 0, therefore x is indeterminate. This implies that $\frac{0}{0}$ is indeterminate.

Mahāvīra's *GSS*, 49, p. 6 which contains "A quantity multiplied by zero is zero. It remains unchanged when it is divided by, combined with (or) diminished by zero," is no doubt ambiguous. We suggest that the verse is a memory device for the four equations $a \cdot 0 = 0, \frac{a \cdot 0}{0} = a, a + 0 = a, a - 0 = a$. The second of these is logical since $a \cdot 0$ had just been mentioned and is immediately followed by division by 0. Perhaps Bhāskara's statement, $\frac{a \cdot 0}{0} = a$, is not entirely original.

C. *<The Six-Fold (Operation) of One and More Than One Colours>—(Verses 12a-23b).*

(a). *Introduction to Textual Commentary (Verses 12a-23b)—Detailed Treatment by Bhāskara*—This topic has not been treated in detail by the mathematicians earlier than Bhāskara. For example, Brahmagupta states only two verses (*BSS* XVIII, 41-42) which contain rules for the addition, subtraction, multiplication and division of unknown quantities. Śrīpati gives only one verse (*SSE* XIV, 2) which introduces the names of the colours as the measures of the unknown quantities. But Bhāskara's treatment includes rules as well as examples (i.e. problems). Nārāyaṇa treats this topic (see *BGV*, 17-14, pp. 7-13) along lines parallel to those of Bhāsakra. In fact, the sequence of Nārāyaṇa's sūtras and illustrations is almost identical to that of Bhāskara.

(b). *Textual Commentary (Verses 12a-23b).*

Verse 12a-d. Mathematical meaning: The names of the colours correspond to the modern use of the letters, x, y, z etc., as variables or quantities to be determined.

Comments: The measures of the unknown quantities in terms of colours, which are given by Bhāskara in this verse, are similar to those given by Śrīpati (compare Bhāskara's 12a-c to Śrīpati's *SSE* XIV, 2a-b which is):

यावत्तावत् कालको नीलकाद्या
वर्णाः कल्प्या नूनमव्यक्तमाने ।।२a-b।।

Since Brahmagupta did not describe the measures of unknown quantities, although he employs the designation 'varṇa' (which means both colour and letter of the alphabet) for an unknown quantity (see *BSS* XVIII, 42), it cannot be ascertained whether or not any one previous to Śrīpati listed the names in this way.

The source of Amara's saying cited by Sūrya cannot be found. It does not occur in the *Nāmalingānuśāna* (*Amarakośa*) of Amarasiṃha (see Kṣīrasvāmin, 1913). This definition has also been cited by Sūrya's father in his *SSU Bījādhyāya* as follows (see ms. *Berlin 833*, f. 2r., 10-11): यावत्तावच्च साकल्येऽवधौ मानेऽवधारणे इति उक्तवत् यावत्तावदव्ययं मानावधारणे ग्राह्यं ।

Verse 13a-b. Mathematical meaning: It is clear.

Comments: x's can be added together or subtracted, but not x's and y's. For example, $2x + 3x = 5x$, but $2x + 3y$ must be kept as it is (separated). Likewise in case of subtraction. Sūrya explains that the terms, e.g. of $2x + 5$ and of $2x^2 + 3x + 1$, must be kept separate; because the unknowns may not be combined with numbers (rūpas), and similarly the squares of unknowns may not be combined with unknowns to the first power or with numbers. Thus, the idea is that different powers of variables (unknowns) are put separated, and the variable and numbers are not to be combined.

Bhāskara's present verse is similar to Brahmagupta's *BSS* XVIII, 41:

अव्यक्तवर्गघनवर्गवर्गपञ्चगतषड्गतादीनाम् ।
तुल्यानां सङ्कलितव्यवकलिते पृथगतुल्यानाम् ॥४१॥

Verse 13c-14b. Mathematical meaning: What is the sum of $x + 1$ and $2x - 8$? What is the sum when the positive and negative signs of these expressions are reversed?

Setting out: $\pm (x + 1)$ and $\pm (2x - 8)$.

We provide the solutions as follows:

$$(x+1)+(2x-8)=3x-7;$$
$$-(x+1)+(2x-8)=x-9;$$
$$(x+1)-(2x-8)=-x+9;$$
$$-(x+1)-(2x-8)=-3x+7.$$

Verse 14c-d. Mathematical meaning: What is the sum of $3x^2+3$ and $2(-x)$?

Setting out: $3x^2+3$ and $2\,(-x)$.

Our solution is: $3x^2+3+2\,(-x)=3x^2-2x+3$.

Verse 15a-b. Mathematical meaning: From $2x$, subtract $6\,(-x)+8$.

Setting out: $2x$ and $6\,(-x)+8$.

Our solution is: $2x-(6\,(-x)+8)=8x-8$.

Verse 15c-16b. Mathematical meaning: $a \cdot x$ is an unknown. But $x \cdot x$, $x \cdot x \cdot x$, and so on give x^2, x^3, and so on. Also $x \cdot y = xy$.

Comments: In the demonstration part, the sūtra for division quoted by Sūrya is *Text Alpha*, verse 19c. Sūrya demonstrates, among other things, that multiplication of unlike unknowns, e.g. *kā* by *yā*, results in their product, which is written *yākābhā*, so that the multiplier *yā* is first and *bhā* (which stands for bhāvita, meaning product) is last.

Verse 16c-16d. Textual problems: The manuscripts of the β-recension have some (extra) text which is a repetition (with omissions and additions) of the text which already exists under the previous verse 15c-16b. So this (extra) text (which does not exist in the manuscripts of the A-recension) has been placed by us in the Appendix #3.

Mathematical meaning: It is vague.

Comments: By mentioning "Pāṭīgaṇita," Sūrya refers to the *Līlāvatī* (of Bhāskara).

It is to be noted that Bhāskara's lines 15d-16d are parallel to Brahmagupta's *BSS* XVIII, 42:

सदृशद्विवधो वर्गस्त्र्यादिवधस्तद्गतोऽन्यजातिवधः ।
अन्योऽन्यवर्णघातो भावितकः पूर्ववच्छेषम् ॥४२॥

Verse 17a-b. Mathematical meaning: Write the given multiplicand in as many places as there are parts of the multiplier. Then multiply the multiplicand by each part of the multiplier, and add the resulting products.

Comments: This is the distributive law which, in fact, covers examples as difficult as the following:

$$\begin{aligned} &(ax+b)(cx^2+dx+e) \\ &= ax(cx^2+dx+e)+b(cx^2+dx+e) \\ &= acx^3+adx^2+aex+bcx^2+bdx+be \\ &= acx^3+(ad+bc)x^2+(ae+bd)x+be. \end{aligned}$$

But Sūrya gives only the simple case:

$$\begin{aligned} 144 &= (12)(12) \\ &= (1+1+\cdots+1)(12) \\ &= 12+12+\cdots+12 = 144. \end{aligned}$$

In the demonstration, Sūrya also enunciates the basic facts about the operations of multiplication and division: multiplication is the (repeated) addition of the multiplicand as many times as the measure of the multiplier; on the other hand, division is the (repeated) subtraction or (repeated) removal of the divisor from the dividend until the remainder is zero (in case the division is exact; otherwise the remainder is less than the divisor). These explanations have been referred to by Sūrya in his ṭīkā on the *Līlāvatī* (see *GMK*, *Wai*, *PPM 9762*, f. 9r., 10–f. 9v., 1; f. 10v., 8-9) in the following manner:

तदुक्तमस्माभिर्बीजभाष्ये । गुणनं नाम गुणकस्य गुण्यांकप्रमितावृत्तिपूर्वको योगविशेष इति। (f. 9r., 10 – f. 9v., 1)

तदुक्तं बीजभाष्ये । भाज्याद्भाजकस्य यत्प्रमिता आवृत्तयः शुद्ध्यंति तत्पूर्वकांतरविशेष इति। (f. 10v., 8-9)

Verse 17c-d. Mathematical meaning: It is clear.

Comments: The sūtra of the *Pāṭīgaṇita* cited by Sūrya is Bhāskara's *Līlāvatī*, 14b, p. 14 (see Āpaṭe, 1937, *L I*). The method of multiplication by parts referred to, which uses both left and right distributive laws of multiplication, is essentially the following:

$$12 \times 13 = 12\,(8 + 5) = 12 \times 8 + 12 \times 5 = 96 + 60 = 156$$

or $12 \times 13 = (9 + 3)\,13 = 9 \times 13 + 3 \times 13 = 117 + 39 = 156.$

The parts (8 and 5) should be written in a column and not horizontally as we have done.

Verse 18a-d. Mathematical meaning: Multiply $5x - 1$ by $3x + 2$. Do the same after changing the signs of the multiplicand and multiplier.

Setting out: Multiplicand = $5x - 1$, multiplier = $3x + 2$.

In this case, the solution is: The product $= (5x - 1)(3x + 2) = 15x^2 + 7x - 2$. There are three other cases to consider.

Comments: A detailed solution of this problem, using the karaṇasūtra of verse 17a-b, is given by Kṛṣṇa (*BP*, 9, pp. 37-39). Like Bhāskara, Kṛṣṇa discusses four cases; but Sūrya discusses only one.

Verse 19a-d. Textual problems: The meaning and demonstration parts pertaining to this verse are taken from the β-recension. The A-recension had only a one-line hint that the demonstration is to be understood by the reversal of the sūtra for multiplication. This text from the A-recension has been placed in the Appendix #4.

Mathematical meaning: The verse explains the quotient in the division method: $\frac{A}{B} = C$, provided $BC = A$; i.e. $\frac{A}{B}$ is that expression C which is needed to obtain A when it is multiplied by B.

Comments: In the solution of the example following the demonstration, Sūrya alludes to the Indian method of (long) division which involves removing (i.e. moving or casting out) the divisor to the right after each partial quotient. This method (when both dividend and divisor involve numbers only,) is given by most of the Indian mathematicians in their works on Pāṭīgaṇita. Bhāskara describes the process briefly in his *Līlāvatī*, 18, p. 18 (see Āpaṭe, 1937, *L I, ASS 107*):

भाज्याद्धरः शुध्यति यद्गुणः स्यादन्त्यात्फलं तत्खलु भागहारे।
समेन केनाप्यपवर्त्य हारभाज्यौ भजेद्वा सति संभवे तु ॥१८॥

and Gaṇeśa (b. 1507 A.D.) explains it in his commentary *Buddhivilāsinī* (see Āpaṭe, 1937, *L I, ASS 107*, p. 18). Also, verse 16 (see Dvivedi, 1936, *PWSBT 57 I*, p. 5) of Nārāyaṇa's *Gaṇitakaumudī Part I* corresponds to *Līlāvatī*, 18, p. 18:

भाज्यादन्याद् हारः
    शुध्यति येनाहतः फलं तत् स्यात्।
अपवर्त्त्य भाज्यहारौ
    केनापि समेन वा विभजेत् ॥१६॥

Note that in Sūrya's solution of the present example, when two algebraic expressions are separated by a daṇḍa, they are dividend and divisor respectively.

Sūrya discusses only one case pertaining to the present example (which is, in fact, the reverse of that given in Bhāskara's *BG*, 18a-d, pp. 7-8). In this case Sūrya explains that $\frac{15x^2+7x-2}{3x+2}=5x-1$ by a method which is essentially "long division." He leaves the remaining three cases to the reader. Kṛṣṇa on the other hand, discusses all of them very briefly (see *BP*, 10, pp. 41-43) as does Bhāskara (see *BG*, pp. 8-9).

Verse 20a-b. Mathematical meaning: Find $(4x-6)^2$.

Comments: The verse quoted by Sūrya is 17c-d (of Bhāskara's *Bījagaṇita)*.

Verse 20c-21b. Mathematical meaning: It is clear.

Comments: The rule appeals to a knowledge of the formula: $(a+b)^2=a^2+2ab+b^2$ where a and b are knowns or unknowns.

Sūrya's demonstration involves essentially:

$(x+a)^2=x\cdot x+x\cdot a+x\cdot a+a\cdot a=x^2+2xa+a^2$.

The line 20d has been referred to by Sūrya in his ṭīkā on Bhāskara's *Līlāvatī*, 20 in the explanation of the sūtra about squaring, in the following manner (see *GMK*, *Wai*, *PPM 9762*, f. 12v., 3): अत एवोक्तं द्वयोर्द्वयोश्चाभिहतिं ते द्विनिघ्नीमित्यादि ।

Sūrya makes no reference to the problem solved briefly by Bhāskara after verse 20b. Kṛṣṇa discusses it at length.

Verse 21c-22b. Mathematical meaning: Add and subtract the following: $3x + 5y + 7z$ and $-2x-3y-z$.

Comments: The sūtra cited by Sūrya is Bhāskara's *BG*, 13a-b, p. 6.

Verse 22c-23b. Mathematical meaning: What is the product of $-3x-2y+z+1$ and $-6x-4y+2z+2$? What is this product divided by the multiplicand? Find $(\text{multiplicand})^2$ and square-root of this square.

Comments: The sūtras referred to by Sūrya are 17a and 15d-16b, in order, from Bhāskara's *BG*, p. 7. The solutions to the given problems involve straight-forward applications of these sūtras.

This concludes our textual commentary up to and including the six-fold operation of one and more than one colours (i.e. up to verse 23b).

It is worth remarking here that in view of those sections (of the *Text Alpha*) which have been commented on so far, the following is evident:

Sūrya's explanations are concise, while those of Kṛṣṇa are generally more detailed. Sūrya skips some solutions if they exist in the mūla or if they seem to be self-evident. But Kṛṣṇa seems to believe in providing a solution to each problem of the mūla.

D. *<The Six-Fold (Operation) of the Karaṇī (Surd)>—(Verses 23c-46a).*

(a). *Introduction to Textual Commentary (Verses 23c-46a).*

(i). *The Original and Derived Meanings of the Term Karaṇī.* The word karaṇī seems to have been derived from the word "karaṇa" which means "making," "effecting," "producing," "doing"; and hence karaṇī is "the one that makes."

Thibaut (see Chattopadhyaya, 1875/1984) explains that originally karaṇī meant "the cord (i.e. string) used for the measuring of a square" and not "the side of a square" as it meant in the *Śulvasūtras*. Later, possibly it meant the square-root of any number. More specifically:

> The same word which expressed in later times the highly abstract idea of the surd number, originally denoted a cord made of reeds which the adhvaryu stretched out between two wooden poles when he wanted to please the immortals by the perfectly symmetrical shape of their altar. (pp. 65-66)

Datta and Singh (1962) remark that karaṇī denotes square-root in *Śulvasūtras* and Prākṛta literature. In geometry, it denotes a side. "In later times the term is, however, reserved for a surd, i.e. a square-root which cannot be evaluated, but which may be represented by a line" (*Part I*, p. 170).

Similar views are expressed by Chakrabarti (1934) who restricts the use of karaṇī to "the square-root of a number whose root cannot be obtained exactly" (*JDL/UC 24*, p. 36).

The remarks of the above authors seem to be based on the narrow definition of karaṇī given by some of the earlier mathematicians. For example, Śrīpati's definition in his *SSE* XIV, 7a-b is:

ग्राह्यं न मूलं खलु यस्य राशे-
स्तस्य प्रदिष्टं करणीति नाम ॥७a-b॥

which means "of whatever quantity the square-root cannot be obtained, the name 'karaṇī' is fixed for that."

On the other hand, according to Nārāyaṇa's *BGV*, 25a, p. 13:

मूलं ग्राह्यं राशेर्यस्य तु करणीति नाम तस्य स्यात् ॥२५a॥

"Of whatever quantity the square-root is to be obtained, the name of that (quantity) is 'karaṇī'."

A modern author Hayashi (1977) has attempted to clarify the confusion surrounding the meaning of this term karaṇī and its faulty translation by the word "surd." In his study, he concludes that a number K is karaṇī if it satisfies the following two conditions:

Cond 1. K is a number whose root is to be taken.

Cond 2. K is a square of a certain number. (*JSHS 16*, pp. 52, 55)

However, this author does not clarify what he means by 'a certain number' in Condition 2. Nor does he explain anything about the etymology or original meaning of the word 'surd,' which is so essential to the history of the subject.

Quite recently, Professor Shukla (1993b) has revised the exposition of Datta and Singh concerning surds in Hindu mathematics. In this (revised) article, Professor Shukla records the following with respect to the origin and use of the term karaṇī:

> It seems to have been originally employed to denote the cord used for measuring (the side of) a square. It then meant the side of any square and was so called because it made a square *(caturaśra-karaṇī)*. Hence, it came to denote the square-root of any number. As late as the second century of

the Christian era, Umāsvāti (c.150) treated the terms mūla ("root") and karaṇī as synonymous. (*IJHS 28*, p. 253)

The reader is cautioned that the use of the word "karaṇī" and our translation "surd" of this word are not restricted to signifying only the square-root of a non-square rational number. In fact, the authors being considered here, namely Bhāskara and Sūrya, had evolved to a level of sophistication which places the concept of karaṇī very close to an equivalent of our modern concept of square-root. For instance, Bhāskara allows ka 25 (see Bhāskara's commentary following his *BG*, 26c-27c, p. 13). However, the reader should note that, conceptually, in medieval India, ka 5, for example, was not viewed as the "real number" $\sqrt{5}$, but rather "5 is a karaṇī," that is, "we are (for the moment) interested in the square-root of 5." The proof of this is that the rule for "sum" of karaṇīs is something quite different from the sum of two real numbers, although equivalent to it (see (iii) below).

(ii). *Bhāskara's Treatment of Karaṇī in Relation to That of Others.* The mathematicians who treated the topic of karaṇī include, among others, Brahmagupta, Śrīpati, Bhāskara and Nārāyaṇa. The treatment may be found in their works as follows: Brahmagupta's *BSS* XVIII, 38-40; Śrīpati's *SSE* XIV, 7-12; Bhāskara's *BG*, 23c-46a, pp. 11-26; Nārāyaṇa's *BGV*, 25-52, pp. 13-28.

As far as the treatment by Brahmagupta is concerned, the edition of the *Brāhmasphuṭasiddhānta* by Dvivedin (1902) contains only rules but no examples. On the other hand, Colebrooke's (1817) translation of Brahmagupta's *BSS* XVIII, Kuṭṭakādhyāya, contains a few examples and their solutions (see pp. 341-343), in addition to the rules. Most probably, these examples were in what, according to Colebrooke (1817), was a detached copy of a commentary on the eighteenth chapter (see Dissertation, p. xxxii). Professor Pingree informs that Colebrooke did not, in fact, translate the eighteenth chapter of the *Brāhmasphuṭasiddhānta*, but rather its commented and rearranged version which is now in the India Office Library, *IO 2771 (596 A)*. This manuscript belonged to Colebrooke (see Pingree, 1981, *CESS A 4*, p. 255b). Furthermore, according to Professor Pingree, the commentary in this manuscript is anonymous and is certainly not by Caturveda Pṛthūdakasvāmin (who wrote commentaries on Brahmagupta's *Brāhmasphuṭasiddhānta* and *Khaṇḍakhādyaka*, see *CESS A 4*, pp. 221b-222a), which is why it is not listed in *CESS A 4*. Incidentally, all of these examples in Colebrooke's translation have also been treated by Bhāskara in his *Bījagaṇita* in the following verses, in order: 25c-d, p. 12; 26c-27b, p. 13; the first two examples following 30c-31b, pp. 15-16; 32a-b, d, pp. 17-19.

Śrīpati states only rules in his treatment of karaṇī.

As regards the similarities and differences in the treatments of Bhāskara and Nārāyaṇa, both mathematicians a rigorous treatment of the subject. They provide several examples in addition to the rules. Nārāyaṇa gives almost all of the rules given by Bhāskara. A few of Bhāskara's examples appear in Nārāyaṇa's algebra in the same order and have essentially the same solutions as those of Bhāskara. Though their wordings differ, they use the same karaṇīs. Their numberings correspond as follows:

Bhāskara's *Bījagaṇita*	Nārāyaṇa's *Bījagaṇitāvataṃsa*
Verse 41b-42a, p. 23	Verse 22, p. 26
Verse 42b-43a, p. 23	Verse 23, p. 27
Verse 43b-44a, p. 24	Verse 24, p. 27

On the other hand, there are some rules which appear only in Nārāyaṇa's algebra, for example, *BGV*, 46-50, p. 22.

Finally, as a comparison between the treatments of Bhāskara and Nārāyaṇa on the one hand and those of Brahmagupta and Śrīpati on the other, the first two mathematicians give a detailed treatment of the method of extraction of the square-root of a karaṇī-expression, stating rules and examples which the other two mathematicians don't. In fact, Bhāskara complains (see Colebrooke, 1817) that his predecessors did not discuss this subject at length (p. 152).

(iii). *The Rules for Addition and Subtraction of Karaṇīs Formulated by Indian Mathematicians*. These rules may be written as follows (Singh, 1936): The sum or difference of karaṇīs a and b is any of the following expressions:

$$\left(\sqrt{\frac{a}{c}} \pm \sqrt{\frac{b}{c}}\right)^2 \cdot c \tag{1}$$

$$\left(\sqrt{ac} \pm \sqrt{bc}\right)^2 \cdot \frac{1}{c} \tag{2}$$

$$a + b \pm 2\sqrt{ab} \tag{3}$$

$$\left(\sqrt{\frac{a}{b}} \pm 1\right)^2 \cdot b \tag{4}$$

$$\left(\sqrt{aa} \pm \sqrt{ba}\right)^2 \cdot \frac{1}{a} \tag{5}$$

where c is a suitably chosen number so that the quotients or products of a and b by c become perfect squares (*M 12*, p. 104).

Note that each of these expressions equals $(\sqrt{a} \pm \sqrt{b})^2$, so that the "sum" or "difference" of the karaṇīs are karaṇīs (i.e. in the form of squares).

Rule (1) for sum only has been quoted by Bhāskara I in his commentary on the *Āryabhaṭīya* II, 10, though the actual author of this rule is not known (Singh, 1936, *M 12*, p. 104). Nārāyaṇa too states this rule for sum only, but for several surds, in his *BGV*, 30, p. 14:

करणीनां तु बह्वनां योगे केनापि राशिना छित्वा ।
तन्मूलयुतिः स्वघ्ना छेदगुणा स्याद्युतिस्तासाम् ॥३०॥

Brahmagupta in his *BSS* XVIII, 38a and Mahāvīra in his *GSS* VII, 88 state this rule for both sum and difference. The commentator Dvivedin (1902, p. 311) writes that a part of Brahmagupta's verse 37b goes with his verse 38a:

अधिको द्विहतो बाहुः संक्षेप्यो यद्वधो वर्गः ॥३७b॥

इष्टोद्धृतकरणीपदयुतिकृतिरिष्टगुणिताऽन्तरकृतिर्वा ॥३८a॥

This part means the sum or difference of those surds is to be taken, the product of which is a square. However, Mahāvīra seems to be the only mathematician who correctly states the rule in terms of the *square-root* of the whole expression (Singh, 1936):

> After reducing (the surd quantities) by an optional divisor, the square of the sum or difference of the square-roots of the quotients is multiplied by the optional divisor; the *square-root* (of the product) is the sum or difference of the square-root quantities. (*M 12*, p. 104)

As an application of rule (1), let $a = 27, b = 3$. Then $c = 3$ is *one* suitably chosen number.

Rule (2) has been stated by Śrīpati in his *SSE* XIV, 8:

योगे वियोगे करणीं स्वबुद्ध्या
सन्ताडयेत्तेन यथा कृतिः स्यात्।
तन्मूलसंयोगवियोगवर्गौ
विभाजयेदिष्टगुणेन तेन ॥८॥

He clearly mentions that one should multiply the (given) surds by a suitably chosen number so that the products become *squares*. But in his statement in *BSS* XVIII, 38a, Brahmagupta does not mention that the quotients must become squares. Śrīpati's *SSE* XIV, 9c-d

> किन्तूक्तवत् तत्करणीसमास-
> स्तयोर्युतिर्यन्निहतिः कृतिः स्यात् ॥९c-d॥

also states that the composition or sum of only those surds is possible, the product of which is a square.

For an application of rule (2), let $a = 8$, $b = 2$. Then $c = 2, 8, 18$ etc. give perfect squares.

Rules (3) and (4) have been stated by Bhāskara; rule (3) in *BG*, 23c-24b, pp. 11-12 and rule (4) in *BG*, 24c-25b, p. 12:

> योगं करण्योर्महतीं प्रकल्प्य
> वधस्य मूलं द्विगुणं लघुं च।
> योगान्तरे रूपवदेतयोः स्तो
> वर्गेण वर्गं गुणयेद्भजेच्च ॥२३c-२४b॥

> लघ्व्या हृतायास्तु पदं महत्याः
> सैकं निरेकं स्वहतं लघुघ्नम्।
> योगान्तरे स्तः क्रमशस्तयोर्वा
> पृथक्स्थितिः स्याद्यदि नास्ति मूलम् ॥२४c-२५b॥

Rule (3) seems to be an original contribution of Bhāskara. Rule (4) seems to be a derivation from rule (1), because c has been replaced by b, where Bhāskara demands that b is the smaller of the two surds.

Nārāyaṇa states rule (3) in *BGV*, 29, p. 14 and rule (4) in *BGV*, 28, p. 14:

रूपवदपि च करण्योर्घातपदेन द्विसंगुणेन युतिः।
युक्तोना युतिवियुती पृथक्स्थितिः स्यान्न घातपदम् ॥२९॥

अथवा लघ्व्या महती भक्तैतन्मूलमेकयुक्तोनम्।
स्वघ्नं लघ्व्या गुणितं युतिवियुती स्तो महत्यैवम् ॥२८॥

The enunciation, 'if the square-root (of the quotient or product) does not exist, then the surds are put down separately,' has been made both by Bhāskara (*BG*, 25b, p. 12) and Nārāyaṇa (*BGV*, 29b, p. 14). Nārāyaṇa (*BGV*, 28b, p. 14) also provides an alternative to rule (4): "(continue) in this way (by dividing) with the greater surd." Bhāskara does not make this provision. So this is perhaps Nārāyaṇa's own contribution. Nārāyaṇa (*BGV*, 26, p. 13) states another rule which is, in fact, a rewording of his rule (4):

लघ्व्या वापि महत्या पृथक् करण्यौ हृते च तत्पदयोः।
युतिवियुतिकृती च तया गुणिते योगान्तरे भवतः ॥२६॥

It means: "when the two (given) surds are divided separately (either) by the small(er) or by the great(er surd), and when the squares of the sum and difference of the square-roots (of the quotients) are multiplied by that (divisor surd), the sum and difference are produced."

Rule (5) is stated by Nārāyaṇa (*BGV*, 27, p. 14):

गुणिते वापि करण्यावनल्पया वाऽल्पया च तत्पदयोः।
युतिवियुतिकृती भक्ते ह्यभीष्टया योगविवरे स्तः ॥२७॥

It is only a special case of Śrīpati's rule (2). Nārāyaṇa replaces c by the larger surd a or the smaller surd b. For example, when $a = 8$, $b = 2$, then either $c = a = 8$ or $c = b = 2$; and these are the *only* choices for c according to Nārāyaṇa.

We now comment exclusively on the text of the *Sūryaprakāśa* (which refers to verses 23c-46a).

(b). *Textual Commentary (Verses 23c-46a).*

Verse 23c-24b. Mathematical meaning: This verse enunciates the expression in rule (3) above (see sub-section D.(a)(iii)).

Comments: In this rule the great(er surd) (i.e. mahatī) and small(er surd) (i.e. laghu, in fact, laghvī) represent the *names* of the sum and twice the square-root of the product of two given karaṇīs respectively. Obviously, for $a \geq b > 0$,

$$(\sqrt{a} - \sqrt{b})^2 \geq 0 \Rightarrow a + b \geq 2\sqrt{ab}$$

$$\Rightarrow \text{mahatī} \geq \text{laghu}.$$

Further, the sum of the surds a and b is the surd mahatī + laghu, and likewise their difference is the surd mahatī minus laghu. In addition,

$$(\sqrt{a})\, b = (\sqrt{a})\sqrt{b^2} = \sqrt{ab^2}$$

and

$$\frac{\sqrt{a}}{b} = \sqrt{\frac{a}{b^2}}.$$

In his explanation, Sūrya is carefully explaining the definition of karaṇī that it is a number considered to be in the state of being a square (i.e. it has the form of a square). The verse which is cited by Sūrya from Nārāyaṇa's algebra is *BGV*, 25a, p. 13. Nārāyaṇa's definition of karaṇī is evidently an emendation of Śrīpati's narrow definition which is *SSE* XIV, 7a-b, as mentioned before; because Nārāyaṇa's definition implies that the quantity (of which the square-root is to be taken) can be square or non-square, so that its square-root may or may not exist as a rational number. But Śrīpati's definition is restricted to a non-square quantity.

Verse 24c-25b. Textual problems: In the demonstration following Sūrya's examples for the "slow-witted" students, part of the *Sūryaprakāśa* had to be utilized from manuscripts of class β because manuscript A (and hence ε) seemed to have omitted it due to homoeoteleuton. The demonstration remains incomplete without this text.

Mathematical meaning: This verse enunciates the rule given by expression (4) above (see sub-section D.(a)(iii)).

Comments: Unlike their connotations in the previous verse, in this verse mahatī stands for the greater (i.e. for quantity a), and laghu for the smaller (i.e. for quantity b), of the two *given* karaṇīs a and b.

After the artha (meaning) part, in his demonstration Sūrya is using the approximate square-roots of 8 and 2 (in the *sexagesimal* system) so as to demonstrate the validity of Bhāskara's rule (3) in verse 23c-24b for the sum or difference of two given karaṇīs. Therefore, approximate sum of the karaṇīs (modern notation) =

$$(\sqrt{2}+\sqrt{8})^2 = (1;25+2;51)^2 = (4;16)^2 = 18;12,16 \cong 18;12.$$

On the other hand, by Bhāskara's rule (3), the sum of the karaṇīs = greater karaṇī + smaller karaṇī =

$$(2+8)+2\sqrt{2\cdot 8} = 10+8 = 18.$$

Thus the approximate sum 18; 12 is very close to the exact sum 18. The karaṇī of the sum is ka 18.

Similarly, the approximate difference of the karaṇīs =

$$(\sqrt{8}-\sqrt{2})^2 = (1;26)^2 = 2;3,16 \cong 2;3.$$

By Bhāskara's rule (3), the difference = greater karaṇī − smaller karaṇī = 10 − 8 = 2 and the karaṇī of the difference is ka 2. The approximate difference 2; 3 is very close to the exact difference 2.

In the discussion following the above demonstration, Sūrya explicitly mentions that "mahatī" $= a + b = (\sqrt{a})^2 + (\sqrt{b})^2$, and reiterates that the *square* of the sum (difference) of the square-roots of two (given) karaṇīs is the "sum" ("difference") of those two karaṇīs (by definition) or, equivalently, it is the sum (difference) of "mahatī" and "laghu".

Also, Sūrya mentions here that the "product" of karaṇīs a and b is $a \cdot b = (\sqrt{a}\sqrt{b})^2$, the "quotient" of karaṇīs a and b is $\frac{a}{b} = \left(\frac{\sqrt{a}}{\sqrt{b}}\right)^2$, and he justifies Bhāskara's rule in verse 24b: "One should multiply and divide a square by a square."

Furthermore, Sūrya constructs two examples for the "slow-witted" students. The solutions of these examples may be written as follows. The sum or difference of karaṇīs 9 and 4 = greater karaṇī ± smaller karaṇī $= (9 + 4) \pm 2\sqrt{9 \cdot 4} = 25,\ 1 = (\sqrt{9} \pm \sqrt{4})^2$. Here the product of karaṇīs 9 and 4 $= 9 \cdot 4 = 36 = (\sqrt{9} \cdot \sqrt{4})^2$. Now to avoid a fractional quotient in division, Sūrya constructs the second example where the karaṇīs are 16 and 4. Their quotient =

$$\frac{16}{4} = 4 = \left(\frac{\sqrt{16}}{\sqrt{4}}\right)^2.$$

Then using Bhāskara's rule (4), the sum or difference of karaṇīs 16 and 4 is =

$$\left(\frac{\sqrt{16}}{\sqrt{4}} \pm 1\right)^2 (\sqrt{4})^2 = (2 \pm 1)^2 \cdot 4 = 36, 4.$$

Or else, the sum or difference =

$$\left(\frac{\sqrt{16}}{\sqrt{4}} \pm 1\right)^2 \cdot 4 = (2 \pm 1)^2 \cdot 4 = 36, 4.$$

Verse 25c-26b. Textual problems: Some parts of the *Sūryaprakāśa* pertaining to the solution of the first problem (contained in the present verse) were missing from the manuscripts of class A. They have been taken from the manuscripts of class β for otherwise the solution is too brief. Presumably Sūrya came back at some point and revised his text. The author of manuscript β seems to have had access to Sūrya's revised text.

Mathematical meaning: Find $\sqrt{8} \pm \sqrt{2}$; $\sqrt{27} \pm \sqrt{3}$; $\sqrt{7} \pm \sqrt{3}$.

Setting out: Karaṇīs 8 and 2; karaṇīs 27 and 3; karaṇīs 7 and 3.

The solutions are as follows: Using rule (3) or (4), $\sqrt{8} \pm \sqrt{2} = \sqrt{18}, \sqrt{2}$. Likewise, $\sqrt{27} \pm \sqrt{3} = \sqrt{48}, \sqrt{12}$. In the third problem, since the square-root of the product of 7 and 3 is not possible, the sum and difference of karaṇīs 7 and 3 are respectively ka 7 ka 3 and ka 7 ka –3. The intention is that, in such a case, the two given karaṇīs are to be put down separately.

Comments: The anonymous commentator of Brahmagupta's *BSS* XVIII, Kuṭṭakādhyāya, employs rule (1) to solve the first two problems (see Colebrooke, 1817, p. 341). Bhāskara gives only answers (see Bhāskara's commentary following his *BG*, 25c-26b, p. 12).

Verse 26c-27b. Textual problems: The part of the *Sūryaprakāśa* which defines the multiplicand in the second problem was missing from the manuscripts of class A. Our copy of the manuscript A misses folio 10; its two descendants N and R skip this text. So the text from the manuscripts of class β had to be utilized at this point. Perhaps the missing text was in the margin of manuscript A and was overlooked by ε, which is the (hypothetical) ancestor of N and R.

Mathematical meaning: The language is ambiguous. Bhāskara's own commentary (*BG*, 26c-27b, p. 13) suggests that we are to find $(\sqrt{2} + \sqrt{3} + \sqrt{8})\ (\sqrt{3} + 5)$ and $(\sqrt{3} + \sqrt{12} - 5)\ (\sqrt{3} + 5)$.

Setting out: For the first problem, multiplier = $\sqrt{2} + \sqrt{3} + \sqrt{8}$ and multiplicand = $\sqrt{3} + 5$. For the second problem, multiplier = $\sqrt{3} + \sqrt{12} - 5$ and multiplicand = $\sqrt{3} + 5$.

From multiplication, the respective products are: $3 + \sqrt{450} + \sqrt{75} + \sqrt{54}$ and $-\sqrt{625} - \sqrt{75} + \sqrt{675} + \sqrt{81}$. The latter product equals $\sqrt{300} - 16$.

Comments: Both of these problems are also solved by the anonymous commentator of Brahmagupta's *BSS* XVIII (see Colebrooke, 1817, p. 341). But Bhāskara's *BG*, 27b, p. 13 has 'guṇe' in place of 'guṇye' while Kṛṣṇa's *BP*, 13d, p. 56

has 'guṇyo' here. Nonetheless, the multipliers and the multiplicands of these three mathematicians consist of exactly the same karaṇīs.

Recall that for multiplication of karaṇīs, Bhāskara's *BG*, 17c-d, p. 7 suggests the method of *multiplication by parts* as does Brahmagupta's *BSS* XVIII, 38b:

गुण्यस्तिर्यगधोऽधो गुणकसमस्तद्गुणः सहितः ॥३८b॥

But Śrīpati's *SSE* XIV, 9a-b suggests the method called *kapāṭa - sandhi*:

संस्थाप्य गुण्यं गुणकं कपाट-
सन्धिक्रमेणोक्तवदेव हन्यात् ॥९a-b॥

In the solution, for the sake of conciseness, Sūrya first takes the sum of the karaṇīs 2 and 8 as does Bhāskara. The contents of Nārāyaṇa's *BGV*, 31, p. 15 are similar to Bhāskara's instruction in the latter's commentary on *BG*, 26c-27b, p. 13 (in particular, see lines 7-9 in this reference about taking the sum of karaṇīs in the multiplicand, the multiplier, the dividend or the divisor, when it is possible). But the anonymous commentator of Brahmagupta's *BSS* XVIII does not take the sum here. Also, before the multiplication is performed, Sūrya writes rū 5 as ka 25. The underlying concept is that karaṇī indicates a number which is in the state of being a square, and so one should multiply and divide a square by a square (*BG*, 24b, p. 12).

Verse 27c-28b. Textual problems: In the solution of the second example for multiplication, the manuscripts of class A skipped some parts of the *Sūryaprakāśa*. The same have been obtained from the manuscripts of class β because Sūrya does suggest the operations like $\sqrt{3} + \sqrt{12} = \sqrt{27}$, before multiplication is performed (see the previous verse). Furthermore, the solution in A is too brief. Presumably Sūrya added details to his solution later on when he made revisions of his *Sūryaprakāśa* and β possessed a copy of this revised version.

Another place where the text from β had to be utilized is the second example on division, when the karaṇīs in *both* the dividend and divisor are added before performing division. The manuscripts of class A omitted the part of the *Sūryaprakāśa* containing the complete solution by this method. This text has been taken from class β considering again the possibility that Sūrya himself made this augmentation. This text corresponds to Bhāskara's commentary (on pp. 14-15) following his *BG*, 27c-28b, p. 13.

Mathematical meaning: The rule given by the present verse cannot easily be expressed in modern mathematical terms since, e.g., $\sqrt{-16} = -\sqrt{16} = -4$ is clearly not valid. The consistency of Bhāskara's rule arises from a consideration of the *processes* involved. For example, $\sqrt{8} - 2 = \sqrt{8} - \sqrt{4}$ and this would be written as ka 8 ka –4.

Comments: This verse describes the rule containing the *relation* between a karaṇī and a rūpa when they are negative. That is, in the case of karaṇīs, the square of a negative rūpa is a negative karaṇī (i.e. rū –5 = ka –25) and conversely, the square-root of a negative karaṇī is a negative rūpa (i.e. ka –25 = rū –5). This rule seems inconsistent with Bhāskara's rule contained in *BG*, 7c-d, p. 4 which says: "The square of a positive and of a negative (quantity) is positive. The square-root of a negative (quantity) does not exist because it is not a square." But the present rule is necessary due to the notational deficiencies of the time.

In the first two paragraphs following the present verse, Sūrya explains that since the square of a negative quantity is positive, due to this reason the squaring of negative rūpas (when achieved for the sake of being a karaṇī,) is negative. And since the square of negative rūpas is negative (when it is achieved as a karaṇī), the square-root of a negative karaṇī is a negative rūpa.

In his demonstration, the sūtra for square-root cited by Sūrya is Bhāskara's *Līlāvatī I*, 22, *ASS 107*, p. 21. Its content is similar to that of Śrīdhara's sūtra *PG*, 25-26, p. 18:

विषमात् पदतस्त्यक्त्वा वर्गं स्थानच्युतेन मूलेन।
द्विगुणेन भजेच्छेषं लब्धं विनिवेशयेत् पङ्क्तौ ॥२५॥

तद्वर्गं संशोध्य द्विगुणं कुर्वीत पूर्ववल्लब्धम्।
उत्सार्य ततो विभजेच्छेषं द्विगुणीकृतं दलयेत् ॥२६॥

In this sūtra, the odd (viṣama) places are the square (varga) places, and the even (sama) places are the non-square (avarga) places. Sūrya uses this sūtra (and the fact that the squaring of negative rūpas produces a negative karaṇī) in an example concerning the square-root of ka –25. The solution of this example involves the difference of equal karaṇīs ka –25 and ka –25. Sūrya says that this difference is not to be computed by Bhāskara's rule (3) because that rule involves finding the greater karaṇī (i.e. sum of two given karaṇīs). Sūrya thinks that this is excessive occurrence (i.e. a step which is really not needed in case of two equal karaṇīs) because it is certain under all circumstances that the difference of two equal karaṇīs is zero (and their sum is four times one karaṇī).

After the demonstration, Sūrya employs the rule given by verse 27c-28b in the second example in which the setting out is: multiplicand = $\sqrt{25} + \sqrt{3}$; multiplier = $\sqrt{3} + \sqrt{12} - 5$. Using verse 27c-28b, the multiplier = $\sqrt{3} + \sqrt{12} - \sqrt{25} = \sqrt{27} - \sqrt{25}$. This step is needed because one should multiply a square by a square. Now the product = $-\sqrt{625} + \sqrt{675} - \sqrt{75} + \sqrt{81} = -\sqrt{256} + \sqrt{675} - \sqrt{75} = -\sqrt{256} + \sqrt{300}$.

Note that in this example, Sūrya (following Bhāskara's commentary on p. 14 after his *BG*, 27c-28b, p. 13) says that the difference of karaṇīs 675 and –75 is karaṇī 300. But neither Bhāskara nor Sūrya gives the solution. This problem is, in fact, a little tricky as is shown below:

$$\sqrt{675} - \sqrt{75} = 15\sqrt{3} - 5\sqrt{3} = 10\sqrt{3} = \sqrt{300}.$$

On the other hand, the same result can be found using Bhāskara's rules (3) and (4), provided one treats the two karaṇīs as *positive* karaṇīs 675 and 75 and finds their *difference*. A discrepancy arises if one treats them as karaṇīs 675 and –75 and finds their *sum* using rule (3) (though rule (4) works). Because by rule (3), the sum of the karaṇīs 675 and –75

= greater karaṇī + smaller karaṇī

$= (675 + (-75)) + 2\sqrt{675\,(-75)}$

$= 600 - 450 = 150$, which is incorrect.

Therefore, to get the right answer here, one has to take the greater karaṇī $= 675 + 75 = 750$, so that greater karaṇī – smaller karaṇī $= 750 - 450 = 300$. Recall that in the modern sense, rule (3) is not valid when the product is negative. This is the cause of the above discrepancy.

After multiplication of karaṇīs, Sūrya discusses two examples on division as does Bhāskara (on pp. 14-15 following *BG*, 27c-28b, p. 13). Both examples have been dealt with also by the anonymous commentator of Brahmagupta's *BSS* XVIII (see Colebrooke, 1817, p. 342). In the second example, the setting out is: dividend $= \sqrt{81} - \sqrt{625} + \sqrt{675} - \sqrt{75}$; divisor $= -5 + \sqrt{3} + \sqrt{12}$. Adding or subtracting the karaṇīs in the dividend and divisor, the setting out is: dividend $= -\sqrt{256} + \sqrt{300}$; divisor $= -\sqrt{25} + \sqrt{27}$. In this context, Sūrya quotes गुण्ये गुणे वा ... गुणनभजने कार्ये which is part of Bhāskara's commentary under *BG*, 26c-27b, p. 13. To carry out the division, Sūrya uses Bhāskara's method (of long division) given in *BG*, 19, p. 8. This method, in turn, uses *multiplication by parts* (*BG*, 17a-b, p. 7) in order to multiply the divisor $-\sqrt{25} + \sqrt{27}$ by $\sqrt{25} + \sqrt{3}$, which may be displayed as:

Multiplier	Multiplicand	Product
$\sqrt{25}$	$-\sqrt{25}+\sqrt{27}$	$-\sqrt{625}+\sqrt{675}$
$\sqrt{3}$	$-\sqrt{25}+\sqrt{27}$	$-\sqrt{75}+\sqrt{81}$

Adding the partial products, the final product = $-\sqrt{625} + \sqrt{675} - \sqrt{75} + \sqrt{81}$ = $-\sqrt{256} + \sqrt{300}$. Since this product can be subtracted without remainder from the dividend, the quotient is = $\sqrt{25} + \sqrt{3} = 5 + \sqrt{3}$.

Verses 28c-30b. Mathematical meaning: The first half (i.e. 28c-29b) suggests the rationalizing of the denominator in a quotient of karaṇī-expressions. For example:

$$\frac{x}{\sqrt{a}+\sqrt{b}} = \frac{x(\sqrt{a}-\sqrt{b})}{a-b} = \frac{x(\sqrt{a}-\sqrt{b})}{\sqrt{(a-b)^2}}.$$

The second half (i.e. 29c-30b) recommends the following:

$$\frac{\sqrt{r}+\sqrt{s}+\sqrt{t}}{\sqrt{a}} = \sqrt{\frac{r}{a}} + \sqrt{\frac{s}{a}} + \sqrt{\frac{t}{a}}$$

and also the *partitioning* of karaṇīs (e.g. $\sqrt{18} = \sqrt{2} + \sqrt{8}$) in the result, under certain circumstances.

Comments: The equivalents of Bhāskara's 28c-29c are Brahmagupta's *BSS* XVIII, 39:

स्वेष्टर्णच्छेदगुणौ भाज्यच्छेदौ पृथक् युजावसकृत्।
छेदैकगतहृतो वा भाज्यो वर्गः समद्विवधः ॥३९॥

and Śrīpati's *SSE* XIV, 10-11:

छेदे करण्याः समभीप्सितायाः
कृत्वा विपर्यासमृणस्वयोश्च।
गुण्यौ पृथक् भाज्यहरौ युतौ तौ
छेदेऽसकृत् स्यात् करणी यथैका ॥१०॥

तया भजेदूर्ध्वगभाज्यराशि-
मेवं करण्याः खलु भागहारः।
समानराश्योरुभयोश्च घाते
कृते करण्याः कृतिमप्युशन्ति ॥११॥

Nārāyaṇa's equivalent of Bhāskara's 28c-30b is *BGV*, 37-38, p. 19:

छेदेऽभीष्टकरण्या ऋणधनताव्यत्ययोऽसकृत्कार्यः।
भाज्यहरौ सङ्गुणयेद्यावच्छेदे करण्येका ॥३७॥

विभजेत्तया करण्या भाज्योद्भूताः करण्यश्च।
लब्धा योगजकरणी चेत् स्याद्विश्लेषणं प्राग्वत् ॥३८॥

Verse 30c-31b. Textual problems: The manuscripts of class β have some text pertaining to the solution of the first example after Sūrya's demonstration of the present verse. This text has been put in the Appendix #5 because it is out of place and most of it is omitted by the manuscripts of class A. Perhaps manuscript α had some text and corrections in the margin which the author of β did not understand. So he tried to add some text of his own to complete the sense.

On the other hand, class A omits the complete solution pertaining to the second example and a part of the solution of the next (third) example. Without this text there will be disorder and confusion, because the introduction of the second example will be followed by the remaining solution of the third example. Therefore, this part of the *Sūryaprakāśa* had to be taken from class β. This text was somehow omitted by class A.

Mathematical meaning: To "separate" $\sqrt{x}$ into a sum of square-roots, do the following: Find a such that $\frac{x}{a^2} = b$ is an integer. Partition $a = c + d$. then $\sqrt{x} = a\sqrt{b} = \sqrt{c^2b} + \sqrt{d^2b}$.

Comments: In the demonstration, Sūrya says that the *separation-sūtra* in the present verse is just the inverse of the *addition-sūtra* (4) in verse 24c-25b. In particular, Sūrya's explanation can be described as follows: For two surds a and b ($a > b$),

$$\frac{\left(\sqrt{\frac{a}{b}} \pm 1\right)^2 b}{\left(\sqrt{\frac{a}{b}} \pm 1\right)^2} = b = \text{smaller surd.}$$

So taking positive sign, $\frac{\text{surd of addition}}{\text{square number}}$ = smaller surd, where the surd of addition is the sum of the two surds a and b =

$$(\sqrt{a} + \sqrt{b})^2 = \left(\sqrt{\frac{a}{b}} + 1\right)^2 b.$$

For example, we may assume the surd of addition to be 18. Then

$\frac{\text{surd of addition}}{\text{square number}} = \frac{18}{3^2} = 2 = b$. Also

$$3 = 2 + 1 = \sqrt{\frac{a}{b}} + 1.$$

So

$$\sqrt{\frac{a}{b}} = 2 \Rightarrow a = 4b = 8.$$

Thus the separate surds a and b are found.

Note that using modern notation, we can show the relationship of the addition-sūtra and separation-sūtra as follows:

Let x be the sum of the surds a and b (i.e. $x = (\sqrt{a} + \sqrt{b})^2$). Then by rule (4),

$$\sqrt{x} = \sqrt{a} + \sqrt{b} = \left(\sqrt{\frac{a}{b}} + 1\right)\sqrt{b}. \qquad \text{(i)}$$

On the other hand, by the separation-sūtra, let $x = y^2 b$. Then

$$\begin{aligned}\sqrt{x} &= y\sqrt{b} = (m+n)\sqrt{b} \\ &= m\sqrt{b} + n\sqrt{b}.\end{aligned} \qquad \text{(ii)}$$

So the components, of the surd $x = y^2 b$, are $m^2 b$ and $n^2 b$ where (comparing (i) and (ii),) we may take $m = \sqrt{\frac{a}{b}}$ and $n = 1$; where m and n are integers i.e. b divides a and $\frac{a}{b}$ is a square. But then

$$m^2 b = \left(\sqrt{\frac{a}{b}}\right)^2 b = a \text{ and } n^2 b = 1 \cdot b = b,$$

which are the two component-surds as in (i).

As an illustration, let $x = 18$. Then $18 = 3^2 \cdot 2$. Therefore $\sqrt{18} = 3\sqrt{2} = (2 + 1)\sqrt{2} = (m + n)\sqrt{b}$. So the component surds are $m^2 b = 2^2 \cdot 2 = 8$ and $n^2 b = 1^2 \cdot 2 = 2$.

The commentator Kṛṣṇa, following Bhāskara's commentary to *BG*, 34, p. 18, adds (see Kṛṣṇa's commentary following his *BP*, 17, p. 61) that if three component surds are needed, then they are given by $1^2 \cdot 2, 1^2 \cdot 2, 1^2 \cdot 2$ or 2, 2, 2 (since 3 = 1 + 1 + 1).

After the demonstration, Sūrya (following Bhāskara) discusses *three problems*, in order to employ the rules of rationalizing the denominator and separation of surds, given by verses 28c-30b and 30c-31b. Their solutions may be written briefly as follows.

First Problem.

$$\begin{aligned}\frac{\text{Dividend}}{\text{Divisor}} &= \frac{\sqrt{9} + \sqrt{450} + \sqrt{75} + \sqrt{54}}{\sqrt{18} + \sqrt{3}} = \frac{(\sqrt{9} + \sqrt{450} + \sqrt{75} + \sqrt{54})(\sqrt{18} - \sqrt{3})}{(\sqrt{18} + \sqrt{3})(\sqrt{18} - \sqrt{3})} \\ &= \frac{\sqrt{8100} - \sqrt{225} + \sqrt{972} - \sqrt{27}}{\sqrt{324} - \sqrt{9}} = \frac{\sqrt{5625} + \sqrt{675}}{\sqrt{225}} = \sqrt{25} + \sqrt{3}.\end{aligned}$$

Second Problem.

$$\frac{\text{Dividend}}{\text{Divisor}} = \frac{\sqrt{81} - \sqrt{625} + \sqrt{675} - \sqrt{75}}{-\sqrt{25} + \sqrt{27}}$$

$$= \frac{-\sqrt{256} + \sqrt{300}}{-\sqrt{25} + \sqrt{27}} = \frac{(-\sqrt{256} + \sqrt{300})(\sqrt{25} + \sqrt{27})}{(-\sqrt{25} + \sqrt{27})(\sqrt{25} + \sqrt{27})}$$

$$= \frac{\sqrt{8100} - \sqrt{6400} + \sqrt{7500} - \sqrt{6912}}{-\sqrt{625} + \sqrt{729}} = \frac{\sqrt{100} + \sqrt{12}}{\sqrt{4}} = \sqrt{25} + \sqrt{3}.$$

Third Problem.

$$\frac{\text{Dividend}}{\text{Divisor}} = \frac{(\sqrt{3} + \sqrt{450} + \sqrt{75} + \sqrt{54})(\sqrt{25} - \sqrt{3})}{(\sqrt{25} + \sqrt{3})(\sqrt{25} - \sqrt{3})}$$

$$= \frac{\sqrt{8712} + \sqrt{1452}}{\sqrt{484}} = \sqrt{18} + \sqrt{3}.$$

Here 18 is the surd of addition. Its components are surds 8 and 2, as shown before. So the quotient is $\sqrt{2} + \sqrt{3} + \sqrt{8}$.

The first two problems have also been dealt with by the anonymous commentator of Brahmagupta's *BSS* XVIII, where he uses the method of rationalizing the denominator (see Colebrooke, 1817, p. 342).

Verses 31c-32d. Textual problems: The part of the *Sūryaprakāśa*, which contains the detailed explanation of the 'sahajā' and 'nimittajā' quantities in the squaring of a surd-expression as well as the solution of the fourth problem in the present verse, is omitted by the A-recension. So the text from the β-recension had to be utilized to fill this gap; for otherwise the solution (i.e. squaring) of only the fourth problem will be missing which is not a logical occurrence. Clearly, this text was supplied by Sūrya in a later copy of the *Sūryaprakāśa* and β copied it. Furthermore, Appendix #6 shows that the writer of manuscript L intelligently ignores the part of the text which is, in fact, a repetition.

Mathematical meaning: Find x^2 where $x = \sqrt{2} + \sqrt{3} + \sqrt{5}$, $\sqrt{3} + \sqrt{2}$, $\sqrt{6} + \sqrt{5} + \sqrt{3} + \sqrt{2}$, $\sqrt{18} + \sqrt{8} + \sqrt{2}$. And given those squares i.e. x^2, find the square-roots i.e. x.

Comments: The squares are in order, $10+\sqrt{24}+\sqrt{40}+\sqrt{60}$, $5+\sqrt{24}$, $16+\sqrt{120}+\sqrt{72}+\sqrt{60}+\sqrt{48}+\sqrt{40}+\sqrt{24}$, 72.

The third problem has also been solved by the anonymous commentator of Brahmagupta's *BSS* XVIII (see Colebrooke, 1817, pp. 342-343). *Pāṭīgaṇita* is another title of the *Līlāvatī*. The sūtra for squaring cited by Sūrya is Bhāskara's *L I*, 19a, *ASS 107*, p. 19.

Note that according to this sūtra, if a number consists of the digits *a b c* then its square involves a manipulation of the following products, in order: $a^2, 2ab, 2ac, b^2, 2bc, c^2$. (For more details see Datta and Singh, 1962, *Part I*, pp. 157-160). In squaring a sum of surds, Sūrya is referring to the same set of products although the multiplier 2 must be replaced by 4 because of Bhāskara's instruction in verse 24b, that one should multiply a square by a square. Sūrya is using the terminology of even and odd places here also. But clearly this has no particular meaning.

Having solved the fourth problem in the verse, Sūrya is referring to Bhāskara's *BG*, 37c-d, p. 22 in the context of the number of surd-terms (parts) in the square of a surd-expression. He is saying that the number of surd-terms in the square (of a surd-expression) is given by the sums of the natural numbers i.e. by triangular numbers, 1, 1 + 2, 1 + 2 + 3, ... according as the number of surds in the given expression is 2, 3, 4, ... Moreover, the sūtra for square-root of a surd-expression quoted by Sūrya is Bhāskara's *BG*, 39a, 22.

Verses 33a-34d. Textual Problems: In the demonstration part, the one-short-sentence text contained in the manuscripts of class A has been put in the Apparatus Criticus and the corresponding part of the *Sūryaprakāśa* from β has been utilized. This text, which is in β, was presumably provided by Sūrya when he revised his *Sūryaprakāśa* and the author of β could somehow gain access to it. It contains the rule $(a+b)^2-4ab=(a-b)^2$.

Mathematical meaning: The rule for extraction of the square-root of a square-surd-expression can be explained as follows:

Suppose that the given square-surd-expression is of the form $a + b + c + \sqrt{4ab} + \sqrt{4bc} + \sqrt{4ca}$. To find its square-root, consider a difference (as follows) so that it gives a perfect square: $(a+b+c)^2 - (4ab+4ca) = (b+c-a)^2$ or $(a-b-c)^2$ which one of these last two is to be chosen judiciously. Then by the *method of concurrence*

$$\frac{(a+b+c)\pm(b+c-a)}{2} = b+c \text{ or } a.$$

Suppose that the surd $b + c$ is greater (bahvī) than the surd a. Then a is a component-surd in the required square-root of the given expression.

Now to find b and c, proceed as follows: $(b+c)^2 - 4bc = (c-b)^2$ or $(b-c)^2$. Again, by the method of concurrence

$$\frac{(b+c)\pm(c-b)}{2} = c, b. \ .$$

Thus the component-surds in the square-root are a, b, c; and the square-root is $\sqrt{a} + \sqrt{b} + \sqrt{c}$.

Comments: Observe that the above procedure is to be repeated until there are no surds remaining in the given square-expression. Also, 4 times the product of the surds produced by concurrence = $4a(b+c) = 4ab + 4ac$ = the number subtracted from $(a+b+c)^2$.

As an application of the above procedure we consider Sūrya's example. So let $10 + \sqrt{24} + \sqrt{40} + \sqrt{60} = (\sqrt{a} + \sqrt{b} + \sqrt{c})^2$.

We need to find a, b, c. Therefore we consider $10^2 - (24+40)$, which equals $(\pm 6)^2$. This operation is equivalent to considering

$$(a+b+c)^2 - \left[(\sqrt{4ab})^2 + (\sqrt{4ca})^2\right] = (b+c-a)^2 \text{ or } (a-b-c)^2.$$

Next we compute $\frac{10\pm 6}{2}$, which gives 8 and 2. This is equivalent to

$$\frac{(a+b+c)\pm(b+c-a)}{2} = b+c, a;$$

where $a = 2$ and $b + c = 8$.

Now to find b and c, we take the greater i.e. 8 as rūpas and compute $8^2 - 60$, which gives 2^2. This operation means computing $(b+c)^2 - 4bc = (c-b)^2$. Then

$$\frac{8 \pm 2}{2} = 5, 3$$

which is similar to

$$\frac{(b+c) \pm (c-b)}{2} = c, b.$$

Thus the square-root is $\sqrt{2} + \sqrt{3} + \sqrt{5}$.

In the above working, we have assumed $6 = b + c - a$. But if we make the second choice i.e. $6 = a - b - c$, then

$$\frac{10 \pm 6}{2} = \frac{(a+b+c) \pm (a-b-c)}{2},$$

so that $a = 8$ and $b + c = 2$. But then $(b+c)^2 - 4bc = 2^2 - 60$, which is not a perfect square. So the second choice is to be discarded.

If, in the above example, one wants to obtain solutions which are alternative to that of Sūrya, one may proceed by computing $10^2 - (24 + 60)$ or $10^2 - (40 + 60)$ instead of $10^2 - (24 + 40)$. For a comparison, see our commentary to verse 36c-37b.

Bhāskara and Sūrya call this method of extracting the square-root of a square-surd-expression as the *method of remainder* (see verse 40b-41a). Also, it is to be noted that in line 34b, the term 'bahvī' is *elliptical*, though Sūrya does not mention this. Usually it refers to the greater, but sometimes to the smaller (alpā or laghvī) of the two surds obtained by the method of concurrence, as remarked by Bhāskara in his commentary to *BG*, 45b-46a, pp. 25-26; as well as in his introduction to this verse. The example contained in this verse has been worked out by Kṛṣṇa using both implications of the term bahvī (see commentary to *BP*, 19-20, pp. 69-70 and *BP*, 21, p. 83). Also see our commentary to verse 39c-40a.

The concurrence-sūtra [i.e. $\frac{(a+b)\pm(a-b)}{2}=a,b$] referred to by Sūrya in the demonstration part is Bhāskara's *L II*, 56, *ASS 107*, p. 54. The complete sūtra is:

योगोऽन्तरेणोनयुतोऽर्धितस्तौ राशी स्मृतौ संक्रमणाख्यमेतत् ॥५६॥

Observe its similarity with Śrīpati's *SSE* XIV, 13a-b:

योगोऽन्तरेणोनयुतो द्विभक्तः कर्मोदितं संक्रमणाख्यमेतत् ॥१३a-b॥

The only difference is that Śrīpati, following Brahmagupta, states it in his chapter on algebra entitled Avyaktagaṇitādhyāya, while Bhāskara mentions it in his *Līlāvatī* (i.e. *Pāṭīgaṇita* which deals with vyaktagaṇita).

Furthermore, the equivalents of Bhāskara's 33a-34d are Brahmagupta's *BSS* XVIII, 40:

इष्टकरणयूनाया रूपकृतेः पदयुतोनरूपार्धे ।
प्रथमं रूपाण्यन्यत्ततो द्वितीयं करण्यसकृत् ॥४०॥

and Śrīpati's *SSE* XIV, 12:

रूपकृतेः करणीरहिताया
मूलयुतोनितरूपगुणार्धे ।
रूपगुणः प्रथमं हि तदन्यत्
स्यात् करणीपदमित्यसकृच्च ॥१२॥

But Brahmagupta's and Śrīpati's treatment of karaṇī ends with these verses. Unlike Bhāskara, these mathematicians do not state any *specifics* or *limitations* of the method of extracting the square-root. Nor do they discuss how to deal with the *negative* karaṇīs in the square-root of a given (square) karaṇī-expression when this given expression contains negative karaṇīs, which Bhāskara does in the next few verses. That is why Bhāskara complains that this method has not been "explained at length by former writers" (see Colebrooke, 1817, p. 152). Nārāyaṇa, following Bhāskara, continues this topic beyond this point.

Verse 35a-d. Textual problems: The text containing Sūrya's demonstration has been supplied from β because the manuscripts of class A have a brief demonstration which is not as clear as that of β. Sūrya seems to have improved upon it in a later copy of the *Sūryaprakāśa* which seems to have been used by the author of β. The discarded text of class A goes to the Appendix #7.

Mathematical meaning: Bhāskara's rule is that a negative surd in a given square-surd-expression is considered to be positive for the purpose of extracting the square-root specifically when it is to be subtracted from the square of the rūpas. Because then its square becomes positive rūpas, according to the principle: "The square of a negative (quantity) is positive" (verse 7c).

Verse 36a-b. Mathematical meaning: Find $(\sqrt{7}-\sqrt{3})^2$ and $(\sqrt{3}-\sqrt{7})^2$. Further, given this square number, produce the square-root.

Setting out: $\sqrt{7}-\sqrt{3}$; $\sqrt{3}-\sqrt{7}$.

Computing the squares, we get the same number $10-\sqrt{84}$; and the square-root is $\sqrt{3}-\sqrt{7}$ or $\sqrt{7}-\sqrt{3}$.

Comments: Both Bhāskara and Sūrya note that the square of both expressions is the same $10-\sqrt{84}$. Now to find the square-root, according to the rule given in verse 35a-d, we take 10^2-84 (taking $10^2-(-84)$ will not produce an accurate answer). To accomplish the solution successfully, we have to subtract positive 84 from 100 (i.e. taking $(-\sqrt{84})^2 = 84$). Then

$$10^2 - 84 = 4^2, \text{ and } \frac{10 \pm 4}{2} = 7, 3.$$

So the square-root is $\sqrt{3} - \sqrt{7}$ or $\sqrt{7} - \sqrt{3}$.

In the above, if we take $10^2 - (-84)$, we do not obtain a square; but even if we obtain a square, by so doing we will not get the right answer. For example, let the square-surd-expression be $74 - \sqrt{3360}$. Then $74^2 - (-3360) = 5476 + 3360 = 94^2$. So

$$\frac{74 \pm 94}{2} = 84, -10;$$

whence the square-root should be $\sqrt{84} - \sqrt{10}$ or $\sqrt{10} - \sqrt{84}$ but the square of these $= 94 - \sqrt{3360}$ and not $74 - \sqrt{3360}$. On the other hand, $74^2 - (3360) = 2116 = 46^2$. So

$$\frac{74 \pm 46}{2} = 60, 14;$$

whence the square-root $= \sqrt{60} - \sqrt{14}$ or $\sqrt{14} - \sqrt{60}$. Squaring these, we get $74 - \sqrt{3360}$, which is the right answer.

Verse 36c-37b. Textual problems: Here the part of the text of β, which pertains to extracting the square-root, had to be discarded and put in the Appendix #8 because this text makes no sense. It seems that β skipped a few lines, which only the scribe of manuscript L has tried to supply (perhaps using Bhāskara's commentary to *BG*, 36c-37b, p. 21). Since only one manuscript has filled the gap, its text cannot be considered as reliable. Consequently, text A had to be chosen.

Mathematical meaning: Find $(\sqrt{2} + \sqrt{3} - \sqrt{5})^2$ and $(-\sqrt{2} - \sqrt{3} + \sqrt{5})^2$. Further, given this square-number, find the square-root.

Setting out: $\sqrt{2} + \sqrt{3} - \sqrt{5}; -\sqrt{2} - \sqrt{3} + \sqrt{5}$.

Squaring these expressions we get $10 + \sqrt{24} - \sqrt{40} - \sqrt{60}$. The square-roots are found below.

Comments: The problems pertaining to this verse involve making intelligent decisions regarding the negative surds in the square-root-expression. The principles followed seem to be that at least one component-surd of a negative (positive) compound-surd is negative (positive). Also the rule to be followed in squaring a given surd-expression is Sūrya's instruction under verse 35a-d: "But in a square is the state of having the nature of a positive."

In order to find the square-root of $10 + \sqrt{24} - \sqrt{40} - \sqrt{60}$, Sūrya proceeds in two ways (though he leaves both solutions incomplete), as follows:

First solution.

$$10^2 - (40+60) = 0^2; \quad \frac{10 \pm 0}{2} = 5, 5.$$

Taking 5 as a surd in the square-root, the other surd is negative five, integers equal to which are to be taken. Now the setting out is $-5 + \sqrt{24}$. To find the remaining surds, consider

$$(-5)^2 - 24 = 1^2 \text{ and } \frac{5 \pm 1}{2} = 3, 2.$$

Here both components of the compound surd (i.e. surd of addition) -5 have to be negative so as to give the surd positive 24. Thus, the square-root is $-\sqrt{2} - \sqrt{3} + \sqrt{5}$. On the other hand, taking negative five as a surd in the square-root and 5 as rūpas, the setting out is $5 + \sqrt{24}$. Now to find the remaining surds, consider

$$5^2 - 24 = 1^2 \text{ and } \frac{5 \pm 1}{2} = 3, 2.$$

Here both components of surd 5 have to be positive to give the surds positive 24, negative 40 and negative 60. So the square-root is $\sqrt{2} + \sqrt{3} - \sqrt{5}$.

Second solution.

$$10^2 - (24+40) = 6^2; \quad \frac{10 \pm 6}{2} = 8, 2.$$

Taking 2 as a root-surd and –8 as rūpas, the setting out is $-8-\sqrt{60}$. Then $(-8)^2-60=2^2$ and

$$\frac{8\pm 2}{2}=5,3.$$

Here the component-surd 5 of surd negative 8 has to be negative in order to give the correct square. Hence the square-root is $\sqrt{2}+\sqrt{3}-\sqrt{5}$. (Note that

$$\frac{-8\pm 2}{2}=-3,-5$$

which does not give the right answer. Intelligent decision is the key here). On the other hand, taking –2 as a root-surd, and the other surd as 8, the rūpas equal to which are to be taken, the setting out becomes $8-\sqrt{60}$. Proceeding as before, the components of surd 8 are surds 5 and 3, but to get the surd positive 24, surd 3 has to be negative (for $(\sqrt{5}-\sqrt{3})^2=8-\sqrt{60}$). Hence the square-root is $-\sqrt{2}-\sqrt{3}+\sqrt{5}$.

In what follows, Bhāskara explains how to *test* whether or not a given multinomial surd-expression has a square-root. As mentioned previously, Bhāskara observes that the previous writers have not explained this matter in detail. In his *BG*, p. 22, 3-4, Bhāskara proclaims: एवं बुद्धिमतानुक्तमपि ज्ञायते इति पूर्वैर्नायमर्थो विस्तीर्योक्तः। बालावबोधार्थं तु मयोच्यते।

Verses 37c-39b. Textual problems: The manuscripts of class β have some text pertaining to the explanation of 37c-38d which is non-existent in the manuscripts of class A. This text has been discarded and included in the Appendix #9 because it seems to have been borrowed by the writer of β from Kṛṣṇa's commentary (*BP*, 21a-22b, p. 77) with some additions. This commentary was written ca. 1600 A.D., that is, at least 55 years after the *Sūryaprakāśa*. On the contrary, the one-sentence-explanation of 39b which is in A has been placed in the Appendix #10 but the corresponding detailed explanation of β has been utilized on the observation that Sūrya generally provides detailed explanations to the verses involving sūtras. In this case, Sūrya seems to have elaborated on his brief text at the time of revision of his *Sūryaprakāśa*, and the writer of β had access to this revision.

Mathematical meaning: $(\sqrt{a}+\sqrt{b})^2=(a+b)+\sqrt{4ab}$ has one surd, $(\sqrt{a}+\sqrt{b}+\sqrt{c})^2=(a+b+c)+\sqrt{4ab}+\sqrt{4ac}+\sqrt{4bc}$ has 3 = 1 + 2 surds, $(\sqrt{a}+\sqrt{b}+\sqrt{c}+\sqrt{d})^2$ will have 6 = 1 + 2 + 3 surds, etc. Furthermore, if the number of surds in a given multinomial square-surd-expression is 1, 3, 6, 10, 15 etc., then for the purpose of extracting the square-root, the rūpas equal to the sum of the rūpas in 1, 2, 3, 4, 5 etc. of the surds are to be subtracted *first* from the square of the rūpas in the given square-expression. Also, Bhāskara claims that any situation other than that described here will *fail* to produce an accurate result.

Comments: After subtracting from the square of the rūpas, the rūpas equal to the number of surds just described, one should follow the method of extracting the square-root given in verses 33a-34d.

The example which Sūrya has referred to is contained in verse 44b-45a. In this example the square-root cannot be found if one subtracts rūpas equal to three, two and one surd, which is the usual order; though an incorrect square-root can be found using the opposite order which is one, two and three surds. It is incorrect because its square is not equal to the given square-surd-expression.

Verse 39c-40a. Mathematical meaning: Bhāskara seems to be *defining* which is the "smaller" surd produced by the method of concurrence, and is used in determining which surds are to be subtracted from the square of the rūpas in the procedure for extracting square-roots. The surds to be subtracted are those which are divisible by four times that "smaller" surd.

Comments: Though Sūrya does not mention anything, it is to be noted here that "smaller" can be replaced by "greater" (also see our commentary to verses 33a-34d). That is, smaller does not mean 'smaller in the numerical sense.' Smaller (alpā) is that which is taken as a surd in the required square-root (-expression); and greater (mahatī or bahvī) is that the square of which is taken. For example, in verse 45b-46a, the setting out of the given square-surd-expression is $17 + \sqrt{40} + \sqrt{80} + \sqrt{200}$.

To extract its square-root, we compute $17^2 - (200+80) = 3^2$. The two surds are obtained as

$$\frac{17 \pm 3}{2} = 10, 7.$$

Taking 7 as the smaller and 10 as the greater surd, $10^2 - 40 = 60$, but 60 is not a square. We observe here that four times the smaller surd = $4 \cdot 7 = 28$, but the surds 200 and 80 are not divisible by 28. Therefore, taking 7 as the *greater* surd, $7^2 - 40 = 3^2$, whence the remaining surds in the square-root are given by

$$\frac{7 \pm 3}{2} = 5, 2.$$

Clearly, here four times "smaller" = $4 \cdot 10 = 40$, which divides into both 200 and 80 exactly. However, numerically 10 is the larger of 7 and 10. Also, here the required square-root is $\sqrt{2} + \sqrt{5} + \sqrt{10}$.

Observe that in this problem, the quotients obtained on division of 200 and 80 by '4 times the "smaller" surd 10' are respectively 5 and 2. These are the other two surds in the square-root (-expression), as is specified in the next verse.

Verse 40b-41a. Textual problems: This verse along with its introduction is missing from the manuscripts of class A, though β has both. The explanation of the verse has more details in β than in A. So these parts of the *Sūryaprakāśa* have been supplied from β (the brief explanation belonging to A at this point goes to the Appendix #11) because presumably Sūrya has supplied the missing parts and details to which the writer of β had access. Also, almost the same brief explanation which was in A appears in Sūrya's commentary on verse 43b-44a; because the example contained in the verse 43b-44a is an application of the present verse (40b-41a). It is this example which is being promised by Sūrya in the present verse.

Mathematical meaning: If, in a square, e.g., $(a + b + c) + \sqrt{4ab} + \sqrt{4ac} + \sqrt{4bc}$, we obtain, say a, by the previous methods given, then clearly

$$c = \frac{4ac}{4a} \text{ and } b = \frac{4ab}{4a}.$$

If these surds, which are obtained as quotients on division, are not the same as those produced by the method of remainder (described in verses 33a-34d), then the square-root is impossible.

Comments: In the artha part, Sūrya is referring to verse 34b. In the demonstration part, he is referring to verses 37c and 39c. Also referred to again is the sūtra for squaring, i.e. Bhāskara's *L I*, 19a, *ASS 107*, p. 19, which was also stated in connection with verse 31c-32d.

Verse 41b-42a. Textual problems: The part of the *Sūryaprakāśa* containing the faulty approach to finding the square-root of the given square-surd-expression is missing from the A-recension. The same has been supplied from the β-recension since this solution also seems to have been supplied by Sūrya at the time of revision of his *Sūryaprakāśa*; and is also given by Bhāskara (see *BG*, 41b-42a, p. 23).

Mathematical meaning: Find the square-root of the surd-expression $10 + \sqrt{32} + \sqrt{24} + \sqrt{8}$.

Setting out: $10 + \sqrt{32} + \sqrt{24} + \sqrt{8}$.

In order to solve this problem, we use the method of verses 38a-39b and 33a-34d. So 100 – (32 + 24) = 44, which is not a square. Likewise 100 – (32 + 8) = 60, not a square. So also 100 – (24 + 8) = 68, which is not a square. Therefore the square-root cannot be found by the prescribed method. Now transgressing the prescribed method, a faulty attempt is as follows: $100 - (32 + 24 + 8) = 36 = 6^2$. Then

$$\frac{10 \pm 6}{2} = 8, 2.$$

Since there are no surds remaining in the given square-expression, the square-root is $\sqrt{2} + \sqrt{8}$. But its square = $10 + \sqrt{64} = 10 + 8 = 18$, which is different from the given square-expression. Thus the square-root cannot be found here.

Comments: The above is an illustration of a faulty problem where the square-root is not possible. Also this illustration proves the validity of the statement in verse 39b that if the square-root is found otherwise, it is incorrect.

Bhāskara gives another faulty approach as well (see *BG*, p. 23, 15-17), which Sūrya does not: Adding surds 32 and 8, the sum

$$= \left(\sqrt{\frac{32}{8}} + 1\right)^2 8 = 3^2 \cdot 8 = 72.$$

Therefore, the given square-expression reduces to $10 + \sqrt{72} + \sqrt{24}$. This means that we don't have a triangular number of surds. So the square-root $2 + \sqrt{6}$, which is obtained by Bhāskara, is wrong. Bhāskara does not show the working here, but we show it as follows. Since $100 - 72 = 28 \neq$ a square and $100 - 24 = 76 \neq$ a square, we compute $100 - (72 + 28) = 4 = 2^2$. Now

$$\frac{10 \pm 2}{2} = 6, 4.$$

So the square-root $= \sqrt{4} + \sqrt{6} = 2 + \sqrt{6}$, which is wrong because its square is $10 + \sqrt{96} \neq 10 + \sqrt{72} + \sqrt{24}$.

Nārāyaṇa's equivalent of Bhāskara's 41b-42a is *BGV*, 22, p. 26:

वसुरसनेत्रप्रमिता यत्र करण्यश्चतुर्गुणा वर्गे ।
युक्ता रूपैर्दशभिस्तत्र पदं ब्रूहि मे गणक ॥२२॥

Verse 42b-43a. Textual problems: The Apparatus Criticus shows that β has some text which does not exist in its counterpart A. This additional text is put in the Apparatus

Criticus because it contains one sentence which is also contained in Kṛṣṇa's commentary (*BP*, 29, p. 80).

Mathematical meaning: What is the square-root of $10+\sqrt{60}+\sqrt{52}+\sqrt{12}$?

Setting out: $10+\sqrt{60}+\sqrt{52}+\sqrt{12}$.

Extracting its square-root,

$$10^2-(52+12)=6^2 \text{ and } \frac{10\pm 6}{2}=8, 2.$$

But the division of 52 and 12 is not possible by 4 times the smaller surd 2, i.e. by 8. So these surds are not to be subtracted. Sūrya and Bhāskara stop here. But if one proceeded further with Sūrya's solution, then

$$8^2-60=2^2 \text{ and } \frac{8\pm 2}{2}=5, 3.$$

So the square-root = $\sqrt{2}+\sqrt{3}+\sqrt{5}$ but this is wrong because its square is $10+\sqrt{24}+\sqrt{40}+\sqrt{60}$, and not the given square.

Comments: This problem reveals the restrictions or limitations of the method given in verses 38a-39b as well as the necessity of the instructions stated in 39c-40a. The surds to be subtracted must comply with them. These instructions do not seem to have been stated by any of the predecessors of Bhāskara.

Nārāyaṇa's equivalent of Bhāskara's 42b-43a is *BGV*, 23, p. 27:

षष्टिर्द्वापञ्चाशद् द्वादश करणीत्रयं कृतौ यत्र।
दशभी रूपैर्युक्तं तत्र सखे किं पदं ब्रूहि ॥२३॥

Verse 43b-44a. Textual problems: In the artha part, the texts in A and β differ. While the text in A states that the surds obtained by the method of remainder are different from 1 and 7, the text in β states, also, that they are 3 and 5. But this latter text contains one sentence from Bhāskara's commentary to *BG*, 43b-44a, p. 24. Hence it has been discarded and placed in the Appendix #12 so as to avoid contamination of the *Sūryaprakāśa* (i.e. *Text Alpha*) with Bhāskara's *Bījagaṇita*.

Mathematical meaning: Extract the square-root of $10 + \sqrt{8} + \sqrt{56} + \sqrt{60}$.

Setting out: $10 + \sqrt{8} + \sqrt{56} + \sqrt{60}$.

Solving the problem by the method of remainder,

$$10^2 - (8 + 56) = 6^2 \text{ and } \frac{10 \pm 6}{2} = 8, 2.$$

Taking 2 as the root-surd, and 8 as the greater surd,

$$8^2 - 60 = 2^2 \text{ and } \frac{8 \pm 2}{2} = 5, 3.$$

Thus the square-root = $\sqrt{2} + \sqrt{3} + \sqrt{5}$. But this is inaccurate because its square is $10 + \sqrt{24} + \sqrt{40} + \sqrt{60}$ and not the given square-expression.

Comments: Notice that taking 2 as the smaller surd, both 8 and 56 are divisible by $4 \cdot 2$ and the respective quotients 1 and 7 are the remaining surds in the square-root. On the other hand, the remaining two root-surds obtained by the method of remainder are 3 and 5. Since they are not the same, in view of 40b-41a, the square-root which is obtained is not correct. So these surds 8 and 56 should not be subtracted from 100. But then $100 - (8 + 60) = 32$ and $100 - (56 + 60) = -16$ are not squares. So the square-root is not possible in this example. Thus this example is an application and verification of the rule in 40b-41a. Sūrya says that in this case the square-root is not exact or is approximate.

Moreover, when Sūrya says: "Here is the manifestation of the sūtra," he is referring to his remarks which he made in his commentary to 40b-41a (see the brief explanation belonging to A which was put in Appendix #11).

Nārāyaṇa's equivalent of Bhāskara's 43b-44a is *BGV*, 24, p. 27:

तिथिमनुनयनकरण्यश्चतुर्गुणा रूपदशकसंयुक्ताः ।
किं मूलं ब्रूहि सखे करणीगणिते श्रमोऽस्ति यदि ॥२४॥

Verse 44b-45a. Textual problems: The Apparatus Criticus indicates that the last two sentences of Sūrya's explanation to the present verse are different in the two recensions A and β. This part of the *Sūryaprakāśa* has been supplied from the A-recension and the corresponding text from the β-recension goes to Appendix #13 because a part of this latter text is from Bhāskara's *BG*, p. 25, 10-12. In addition to this text from the β-recension, the manuscript S contains some text from Kṛṣṇa's *BP*, p. 82, 22; whereas the writer of the manuscript H copies from Bhāskara's *BG*, p. 25, 12-15.

Mathematical meaning: Calculate the square-root of the surd-expression $13+\sqrt{48}+\sqrt{60}+\sqrt{20}+\sqrt{44}+\sqrt{32}+\sqrt{24}$.

Setting out: $13+\sqrt{48}+\sqrt{60}+\sqrt{20}+\sqrt{44}+\sqrt{32}+\sqrt{24}$.

The square-root cannot be found by the usual method described in verses 37c-39b; i.e. by subtracting rūpas equal to three, two and one surd, in order. Although a faulty square-root can be found by reversing the order, its square does not equal the given surd-expression. Making the faulty attempt,

$$13^2-48=11^2 \text{ and } \frac{13\pm 11}{2}=12, 1.$$

Now

$$12^2-(60+20)=8^2 \text{ and } \frac{12\pm 8}{2}=10, 2.$$

Similarly

$$10^2-(44+32+24)=0^2 \text{ and } \frac{10\pm 0}{2}=5, 5.$$

Thus the square-root is $\sqrt{1}+\sqrt{2}+\sqrt{5}+\sqrt{5}$. But its square is $13+\sqrt{8}+\sqrt{20}+\sqrt{20}+\sqrt{40}+\sqrt{40}+\sqrt{100}=23+\sqrt{8}+2\sqrt{20}+2\sqrt{40}=23+\sqrt{8}+\sqrt{80}+\sqrt{160}$.

Comments: Both Bhāskara and Sūrya maintain that in such cases, the approximate square-roots should be computed. More specifically, Bhāskara says (*BG*, p. 25, 11-14): यैरस्य मूलानयनस्य नियमोन कृतस्तेषामिदं दूषणं। एवंविधवर्गे करणीनामासन्नमूलकरणेन मूलान्यानीय रूपेषु प्रक्षिप्य मूलं वाच्यम्। Here Bhāskara is saying: "This is the fault of those (former mathematicians) who made an incomplete rule (i.e. without mentioning the limitations of the rule) for extracting the square-root (of an expression involving karaṇīs). In a square (karaṇī-expression) of this kind, one should (first) compute the approximate square-roots of the (given) karaṇīs, combine them with the (given) rūpas, and (then) tell the square-root (of the given karaṇī-expression)".

Verse Without Number. Textual Problems: The text β places this verse, along with its introduction and commentary, after the introduction, lemma and solution for verse 45b-46a. But we have chosen the order of text A for our edition because it preserves the continuity of the context of the approximate square-root of karaṇī-expressions.

Mathematical meaning: This rule is identical to what we now think of as Newton's iterative method for finding square-roots, namely,

$$x_{n+1} = \frac{\frac{a}{x_n} + x_n}{2}$$

where x_1 is a first approximation to $\sqrt{a}$.

Comments: The rule for approximate square-root of a karaṇī, which is cited by Sūrya from his father's *Siddhāntasundara* is *Bījādhyāya* ms. *Berlin 833*, f. 3v., 10-13. Using this rule, we can present the solution for Sūrya's example as follows:

Let the given (non-square) number be 5. Then the imagined near square-root (i.e. the largest integer whose square is closest to 5 but smaller than 5) is 2. Now

$$\frac{\frac{5}{2} + 2}{2} = \frac{9}{4}$$

is nearer than the imagined square-root 2.

Again,

$$\frac{\frac{5}{(9/4)} + \frac{9}{4}}{2} = \frac{161}{72}$$

is a square-root of 5 and is closer than $\frac{9}{4}$, which was closer than 2. Repeating this process over and over again, the (nearly) true square-root emerges.

Note that the fraction

$$\frac{161}{72} = 2.236\overline{1}.$$

The reading of class A is 2; 14 which stands for

$$2 + \frac{14}{60} = 2.2\overline{3}.$$

On the other hand, the reading of class β is 2; 14, 10 which is exactly $\frac{161}{72}$. So the reading of class β is better than that of class A as it gives a better approximation of $\sqrt{5}$.

Verse 45b-46a. Mathematical meaning: What is the square-root of $17 + \sqrt{40} + \sqrt{80} + \sqrt{200}$?

Setting out: $17 + \sqrt{40} + \sqrt{80} + \sqrt{200}$.

The solution has already been given in our commentary to verse 39c-40a.

Comments: This problem verifies the idea that the "smaller" surd is that which is taken as a root-surd and not as a smaller surd in the numerical sense.

Verse without number. Comments: This verse has been composed by Sūrya. It is the verse of upasaṃhāra (summing up). It marks the end of the second chapter of our *Text Alpha*, that is, the section of the *Sūryaprakāśa* which deals with the six-fold operations of positive and negative quantities, zero, colours and karaṇī.

Colophon. Textual problems: This part of the *Sūryaprakāśa* has been supplied from class A because it is missing from the manuscripts of class β.

Comments: A colophon is an identification at the end of a section, manuscript or book. It contains some of the important information about the author and his work, such as the name of the author and his father, religion, work, place, date, day, time, patronage, ruling king and his kingdom.

(c). *Conclusion to Textual Commentary (Verses 23c-46a).*

(i). *A Summary of Bhāskara's Method of Square-Root of a Karaṇī-Expression.* In light of the previous sub-section, Bhāskara's discussion for the square-root of a given karaṇī-expression can be summarized as follows:

Verses 33-34 describe the general method, called method of remainder, which involves subtracting rūpas equal to the sum of a few of the surd-terms from the square of the rūpas in the given expression. Verse 35 deals with the negative surd-terms in the given expression. Verses 37c-39b inform us of the specifics of the method given in verses 33-34. They tell about the exact number of surd-terms, the rūpas equal to which are to be subtracted. Also they include the warning that if the square-root is found otherwise, it will be wrong. Verse 39c-40a explains the limitation that those surd-terms, (the rūpas equal to) which have been subtracted, must be divisible evenly by 4 times the smaller surd generated by the method of concurrence. (The surds obtained by division will be the remaining surds in the square-root). Verse 40b-41a states that if the surds obtained as quotients by this division are not the same as those obtained by the method of remainder, then the square-root is not correct. The problem in verse 43b-44a tests the implication of the rule given by 40b-41a (in which problem Sūrya suggests approximate square-root).

Later on, Bhāskara remarks (*BG*, 45b-46a, p. 25) that the term "smaller" is metaphorical for sometimes it implies "greater." Regarding the number of surd-terms in the square of a surd-expression involving 2, 3, 4 etc. surd-terms, Bhāskara says (verses 37c-38d) that this number is the sum of the first one, two, three etc. natural numbers (i.e. the triangular numbers). On the other hand, if the surd-expression, of which the square-root is to be found, does not contain the number of surd-terms as described in 37c-38d, the compound (i.e. addition) surds, if any, should be separated first so as to obtain the required (i.e. requisite) number of surds (see Bhāskara's *BG*, p. 23, 1-4).

Finally, if the exact square-root cannot be found following the rules in verses 33-34 and 37c-40a (as in examples 43b-44a and 44b-45a), then one should take the apporoximate square-roots of the surds in the given square-surd-expression (*BG*, p. 25, 11-14).

(ii). *Approximate Square-Root of a Non-Square Number.* Jñānarāja's method can be written as follows:

Let a be the given non-square number. Let r_1 be the imagined near square-root, so that

$$0 < r_1 < \sqrt{a}, \text{ and } \sqrt{a} = r_1$$

is the first approximation. Then averaging $\frac{a}{r_1}$ and r_1, we get

$$r_2 = \frac{\frac{a}{r_1} + r_1}{2}$$

which is a "nearer square-root", i.e. a closer approximation to the actual square-root. Continuing thus we can find a sequence of approximate square-roots which converges to $\sqrt{a}$. This is precisely Newton's (1642 – 1727 A.D.) method.

Nārāyaṇa's (ca. 1356 A.D.) method involves indeterminate equations of the second degree (Datta, 1931a). That is, one has to soive $ax^2 + 1 = y^2$. If $x=\alpha$ and $y=\beta$ be a solution of this equation, then

$$\sqrt{a} = \frac{\beta}{\alpha}$$

approximately (*BCMS 23*, p. 187). Garver (1932) has calculated the limits to the error in Nārāyaṇa's approximation (*BCMS 24*, pp. 99-100).

The method given by Śrīdhara in his *Pāṭīgaṇita*, 118, p. 175 (or English translation p. 91; see Shukla, 1959) and restated in his *Triśatikā*, 46, p. 34 (see Dvivedin, 1899) can be (substantially) written as

$$\sqrt{a} = \frac{\sqrt{a \cdot b^2}}{b} = \frac{\sqrt{n^2 + r}}{b} \cong \frac{n}{b},$$

where b is some large number. As an illustration, one may calculate

$$\sqrt{10}=\frac{\sqrt{10\cdot 1000^2}}{1000}=\frac{\sqrt{3162^2+1756}}{1000}\cong\frac{3162}{1000}=3.162.$$

Bhāskara applied this method of Śrīdhara to find the approximate square-root of fractions in his gloss following his *Līlāvatī*, 140, p. 280, as follows (see Sarma, 1975, *VIS 66*):

$$\sqrt{\frac{169}{8}}=\frac{\sqrt{169\cdot 8}}{8}=\frac{\sqrt{1352}}{8}=\frac{\sqrt{1352\cdot 10000}}{800}$$

$$=\frac{\sqrt{13520000}}{800}=\frac{\sqrt{3677^2-329}}{800}\cong\frac{3677}{800}=4\frac{477}{800}.$$

So the approximate square root of $\frac{169}{8}$ is $4\frac{477}{800}$.

Bag (1979) surmises that Bhāskara, Nārāyaṇa and other Indian mathematicians knew how to find approximate square-roots by the method of continued fractions (*CORS 16*, p. 99).

The formula given in the *Bakhshālī Manuscript* (200 – 400 A.D.) is (Channabasappa, 1976, *IJHS 11*, p. 112):

$$\sqrt{a}=\sqrt{p^2+r}=p+\frac{r}{2p}-\frac{\left(\frac{r}{2p}\right)^2}{2\left(p+\frac{r}{2p}\right)}$$

approximately. As far as the date of composition of the *Bakhshālī Manuscript* is concerned, historians of mathematics are of varying opinions. For example, according to Hayashi (1985), the date of the *Bakhshālī Manuscript* (its writing) is, at the latest, the twelfth century (p. 43), while the text itself is tentatively dated to the seventh century by this same historian (p. 249).

Professor Shukla (1993a) comments that one can find applications of the following formula in the early canonical works of the Jainas which were written during 500 B.C. – 300 B.C. (*IJHS 28*, p. 266):

$$\sqrt{a} = \sqrt{p^2 + r} = p + \frac{r}{2p}.$$

As far as the date of the canonical works of the (Śvetāmbara) Jainas is concerned, Professor Pingree remarks, that they were extensively revised in the early sixth century A.D., so that no material in them can be securely dated before then.

Professor Datta (1932b) states that the *Baudhāyana* (800 B.C.), *Āpastamba* and *Kātyāyana Śulbasūtras* contain a rule which gives

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3 \cdot 4} - \frac{1}{3 \cdot 4 \cdot 34}.$$

In terms of decimal fractions this yields $\sqrt{2} = 1.414215...$ (pp. 188-189), which is accurate to 5 decimal places.

Chakrabarti (1934, *JDL/UC 24*, pp. 29-58), Gāṅguli (1932, *SM 1*, pp. 135-141) and Gurjar (1942, *JUB NS 10*, pp. 6-10) explain how one can obtain the above approximation for the square-root of 2.

This concludes our commentary on the second chapter of our *Text Alpha* which deals with the six-fold operation of positive and negative quantities, zero, colours and karaṇī (verses 3a-46a).

4. <*Text Alpha*, Third Chapter>

<*The Chapter Concerning the Kuṭṭaka (Pulverizer)*>

We begin our commentary on this chapter with a mathematical and historical study (sub-sections A. – R.) which discusses the etymology of the term kuṭṭaka, the importance of the subject of kuṭṭaka, the origin of the Indian indeterminate equations, kinds of problems in connection with kuṭṭaka, description of the method of kuṭṭaka by Āryabhaṭa I (b. 476 A.D.), rationale of the method of kuṭṭaka in modern notation, the modifications or innovations suggested by Indian mathematicians after Āryabhaṭa I, and the similarity between kuṭṭaka and continued fractions. The textual commentary begins with the sub-section S.

A. *Preliminary Remarks.*

Kuṭṭaka (pulverizer) is the name of the method and the subject which deals with the solution of indeterminate equations of the first degree, i.e. equations of the form

$by = ax + c$

where x and y are integer solutions for a, b and c which are given integers. (In most cases the writers are interested in positive integer solutions x and y).

Āryabhaṭa I was the first Indian mathematician who gave a method for finding the general solution in positive integers of the above simple indeterminate equation, where a, b, c are positive (Datta & Singh, 1962). He further suggested how to extend the method to obtain positive integral solutions of several simultaneous indeterminate equations of the first degree. Bhāskara I (ca. 600 A.D.) showed that the same method can be used to solve $by = ax - c$, $(a, b, c > 0)$; and the general solution of $by = ax - c$ would follow from that of $by = ax - 1$, as will be seen later. Brahmagupta (b. 598 A.D.) seems to have adopted the methods of these two mathematicians. (Brahmagupta may have used at least the ***Mahābhāskarīya*** of Bhāskara I. The *Āryabhaṭīya-Bhāṣya* of Bhāskara I was written in 629 A.D., a year after Brahmagupta wrote his *Brāhmasphuṭasiddhānta*.) Āryabhaṭa II (fl.

950 A.D.) suggested abridgements of the operations in some cases. Also, he suggested further innovations concerning the methods for $by = ax \pm c$ (*Part II*, p. 87).

In fact, almost every Indian mathematician touched upon the subject of kuṭṭaka. The other Indian mathematicians who treated this subject include Govindasvāmin (fl. ca. 800 – 850 A.D.), Mahāvīra (fl. ca. 850 A.D.), Pṛthūdakasvāmin (fl. 864 A.D.), Śrīpati (fl. 1039 A.D.), Ācārya Jayadeva (fl. before 1073 A.D.), Bhāskara II (b. 1114 A.D.), Nārāyaṇa Paṇḍita (fl. 1356 A.D.), Devarāja, Jñānarāja (fl. 1503 A.D.) and Sūryadāsa (1507 – 1588 A.D.). In case of Jayadeva, the only reference to his work in existence is to his dealing with indeterminate equations of the second degree (see Chapter I, section 2.G., The Sources Used by Bhāskara).

Some of these mathematicians (e.g. Āryabhaṭa I, Āryabhaṭa II and Śrīpati) have stated only the rules; the others (e.g. Brahmagupta and Mahāvīra) have stated rules and numerical problems, but no solutions. Bhāskara I provides brief solutions to his numerical problems in his commentary *Āryabhaṭīya-Bhāṣya* on the *Āryabhaṭīya*, and states several rules in his *Mahābhāskarīya*. Bhāskara II states rules and provides detailed solutions to almost all of his numerical problems in his *Līlāvatī* and *Bījagaṇita*.

B. *Etymology of the Term Kuṭṭaka.*

Kuṭṭa and its synonyms (e.g. kuṭṭaka, kuṭṭīkāra) are derived from the Sanskrit root "kuṭṭ" which means (Apte, 1978) "to cut," "to divide," "to grind," "to multiply" (p. 360). Thus the noun kuṭṭaka means a grinder, or a pulverizer, or a multiplier.

Various explanations have been offered by mathematicians as to why this subject came to be known by the term kuṭṭaka. The commentators on the works of Bhāskara II (e.g. Sūryadāsa (b. 1507 A.D.), Gaṇeṣa (b. 1507 A.D.), Kṛṣṇa (fl. ca. 1600 – 1625 A.D.), and Raṅganātha (fl. 1630 A.D.)) are of the view that kuṭṭaka stands for the multiplier (which is x in the equation $by = ax + c$) because in Sanskrit, multiplication is called by words signifying "injuring" and "killing" (Datta & Singh, 1962). Mahāvīra states that kuṭṭīkāra is another name for "the operation of 'prakṣepaka'," which means throwing or scattering, and which implies division into parts. Raṅgācārya, who edited and translated the *Gaṇitasārasaṅgraha* of Mahāvīra, describes kuṭṭīkāra as "a special kind of division or distribution," "proportionate division." Bhāskara I once remarked that to get the multiplier, the operation of pulverizing (kuṭṭana) is employed (*Part II*, pp. 90-91).

In essence, the method of solution of the equation $by = ax + c$ involves the process of continued division by means of which new equations, similar to the one which is given, are obtained such that the values of a and b become smaller and smaller in the new equations (as in the Euclidean algorithm). Sūryadeva Yajvan, a commentator of Āryabhaṭa I, explicitly mentioned that the process is to be continued until there is the "smallness" of the divisor b and the dividend a: "भाज्यभाजकशेषयो स्वरूपमन्योन्यभक्तं स्यात्। यावद्धरभाज्ययोरल्पता।" (cited in Sarma, 1976, p. 71, lines 3-4). Datta and Singh (1962) agree that it is this very idea, which suggested this name kuṭṭaka to the ancient Indian mathematicians for the process (*Part II*, p. 91).

Ganguli (1931 – 32) explained in a comprehensive article on India's contribution to this area: As by repeated operation the kuṭṭaka reduces the size of an object, so by repeated operation (inherent in it,) this method of kuṭṭaka reduces the size of the given equation to the extent that it can be easily solved by inspection (*JIMS/NQ 19*, p. 116).

The synonyms of kuṭṭaka are kuṭṭākāra, kuṭṭīkāra and simply kuṭṭa. For example, Bhāskara I's *Mahābhāskarīya* I, 49-50 (see Shukla, 1960, p. 8) contains kuṭṭākāra and kuṭṭa; and his commentary to the *Āryabhaṭīya* contains kuṭṭākāra and kuṭṭaka (see Shukla, 1976, p. 135). Brahmagupta uses (see *BSS* XVIII, 16, 19, 20) kuṭṭaka, kuṭṭākāra and kuṭṭa; while Mahāvīra likes kuṭṭīkāra (see *GSS*, p. 80, 2 in Raṅgācārya, 1912).

C. *The Importance of Kuṭṭaka.*

As mentioned previously, kuṭṭaka was treated by all Indian mathematicians. Āryabhaṭa I (b. 476 A. D.) described the method in verses 32-33 of the Gaṇitapāda section of his *Āryabhaṭīya* (499 A.D.). In order to explain Āryabhaṭa I's method, his commentator Bhāskara I (ca. 600 A.D.) cited six mathematical problems involving remainders and twenty-four astronomical problems (Bag, 1977, *IJHS 12*, p. 8). Also, Bhāskara I gave his own description of the method in his *Mahābhāskarīya* (which is discussed later). He applied the method to solve astronomical problems in his treatises *Mahābhāskarīya* and *Laghubhāskarīya.*

Brahmagupta (b. 598 A.D.) was so fascinated by kuṭṭaka that he called the complete algebraical section of his treatise *Brāhmasphuṭasiddhānta* as the Kuṭṭakādhyāya, even though that section contains many topics not related to kuṭṭaka.

Āryabhaṭa II (fl. 950 A.D.) enumerated kuṭṭaka as a distinct branch of mathematics alongside Pāṭī and Bīja, in the first verse of the first chapter of his treatise *Mahāsiddhānta* (Bag, 1979, *CORS 16*, p. 24). Likewise did Sūryadāsa in his maṅgalācaraṇa verse 3 of his *Sūryaprakāśa.*

Bhāskara II discussed this topic both in his *Līlāvatī* and *Bījagaṇita*. The wordings in the two treatises are almost the same (the few differences will be discussed later in a separate sub-section, R.). Similarly, Nārāyaṇa Paṇḍita dealt with kuṭṭaka in his two treatises—the *Bījagaṇitāvataṃsa (Part I)* and the *Gaṇitakaumudī (Part II)*. Most of the verses and commentaries on them in the two treatments of kuṭṭaka are almost the same, word for word. The *Gaṇitakaumudī* (which was written after the *Bījagaṇitāvataṃsa*) adds 6 sūtras and 6 illustrations (see e.g. Dvivedī, 1942, *PWSBT 57 II*, Verses 32-35, pp. 222-230) and skips 2 sūtras and 1 illustration in comparison with the *Bījagaṇitāvataṃsa* (see e.g. Shukla, 1970, *Part I*, verses 68-69, p. 35).

Devarāja, a commentator on Āryabhaṭa I, wrote his work *Kuṭṭākāraśiromaṇi* exclusively on kuṭṭaka and composed a ṭīkā (commentary), the *Mahālakṣmīmuktāvalī*, on his own work (see e.g. Āpaṭe, 1944, *ASS 125)*. Devarāja mentions Bhāskara II in his ṭīkā (Pingree, 1976, *CESS A 3*, pp. 120b-121a).

In his commentary on Bhāskara II's *Līlāvatī*, 242, p. 251 the commentator Gaṇeśa proclaims (in 1545 A.D.) that the separate mention of kuṭṭaka has been made by Bhāskara II and others in order to explain the pre-eminence of this topic (see Āpaṭe, 1937, *L II, ASS 107*, p. 252).

The comments made by Colebrooke (1817) regarding the achievements of the Indian mathematicians in this field are worth quoting:

> ...general methods for the solution of indeterminate problems both of the first and second degrees, are taught in the *Bījagaṇita*, and those for the first degree repeated in the *Līlāvatī*, which were unknown to the mathematicians of the West until invented anew in the last two centuries by algebraists of France and England. (Dissertation, p. iv)

D. *The Origin of the Indian Indeterminate Equations.*

The construction of the Vedic sacrificial altars gave rise to certain kinds of indeterminate problems, the solutions of which caused the evolution of simultaneous indeterminate equations (Datta, 1931b, *Archeion*, p. 401).

For example, consider the case of the Gārhapatya Vedi (altar) of the square type with breadth one vyāyāma (fathom) (Datta, 1931b). It is to be constructed with 5 layers of square (or rectangular) bricks so that each layer consists of 21 bricks and the rifts of the bricks in two consecutive layers never coincide (perhaps for strength and beauty). The *Śulbasūtras* which deal with the measurement and construction of different altars contain only the solutions pertaining to such problems and not the method of obtaining them. For the present problem, Professor Datta has provided the following algebraical explanation: At least two varieties of square bricks are needed in a layer, since 21 is not a square. Let x and y be the number of bricks of each of these varieties. Let their sides be mth and nth part of a fathom, respectively. Then we have the following simultaneous indeterminate equations.

$$\left.\begin{aligned} \frac{x}{m^2}+\frac{y}{n^2}=1 \\ x+y=21. \end{aligned}\right\} \qquad (1)$$

The author Baudhāyana (800 B.C.) gives the following solutions of (1) in his *Śulbasūtra*: When m and n are taken to be 6 and 4 respectively, then $x = 9$ and $y = 12$; and when m and n are taken to be 3 and 6 respectively, then $x = 5$ and $y = 16$. We do not know how Baudhāyana obtained these solutions. (*Archeion*, pp. 401-402)

Another indeterminate problem arises in connection with the Śyena-cit (Falcon-shaped Fire-altar) (Datta, 1931b). Here the total area has to be $7\frac{1}{2}a^2$, where a = one puruṣa. The instructions to be followed are that the number of layers is 5, each layer consists of 200 bricks, and the rifts of bricks in successive layers must not be identical. Baudhāyana describes two methods of construction: one employing four kinds of square bricks, the other employing rectangular bricks also. Professor Datta represents this problem algebraically as follows: Let the number of bricks of each variety in a layer be x, y, z, u and let the areas of the bricks be $\frac{a^2}{m}, \frac{a^2}{n}, \frac{a^2}{p}$ and $\frac{a^2}{q}$ respectively, where m, n, p, q are squares of integers. Then the problem leads to the following simultaneous indeterminate linear equations:

$$\left.\begin{aligned} \frac{x}{m}+\frac{y}{n}+\frac{z}{p}+\frac{u}{q}=7\tfrac{1}{2} \\ x+y+z+u=200. \end{aligned}\right\} \qquad (2)$$

Baudhāyana gives four solutions of equations (2):

$m = 16, n = 25, p = 36, q = 100,$

$x = 24, y = 120, z = 36, u = 20$

or

$x = 12, y = 125, z = 63, u = 0;$

and

$m = 25, n = 50, p = \frac{50}{3}, q = 100,$

$x = 160, y = 30, z = 8, u = 2;$

or

$x = 165, y = 25, z = 6, u = 4.$

For the same altar, Āpastamba (ca. 500 B.C.) uses 5 different varieties of square bricks. (*Archeion*, pp. 402-403)

E. *Problems in Connection With the Indian Indeterminate Equations of the First Degree.*

Two kinds of problems, which directed the Indians to the investigations of the indeterminate equations of the first degree, are:

(i). To find a number N which when divided by two numbers a, b leaves remainders R_1, R_2 respectively.

Clearly, here $N = ax + R_1 = by + R_2$

$\Rightarrow by - ax = R_1 - R_2 = c, c > 0$; or $by - ax = -(R_2 - R_1) = -c, c > 0$

$\Rightarrow by = ax \pm c$, according as R_1 is greater than or less than R_2 respectively.

(ii). To find a number x such that its product with a given number a when increased or decreased by another given number c, and the result when divided by another given number b, leaves no remainder. That is, to solve

$$\frac{ax \pm c}{b} = y$$

for positive integers x and y.

Here a is called the dividend (भाज्य, विभाज्य), x the multiplier (गुणः, गुणकः, गुणकारः), c the additive or interpolator (क्षेपः, क्षेपकः, प्रक्षेपः, literally that which is thrown into or away from something, i.e., that which is added to or subtracted from something), *b* the divisor (छेदः, भागहारः, भाजकः, हरः, हारः), and *y* the quotient (फलं, लब्धिः).

The rules given by the earlier writers, such as Āryabhaṭa I and Brahmagupta, corresponded to the solutions of the problems of the first kind. Bhāskara I solved (non-astronomical) problems of the first kind and (both non-astronomical and astronomical problems of the) second kind in his commentary on the *Āryabhaṭīya*. In his *Mahābhāskarīya* and *Laghubhāskarīya*, the problems discussed are usually astronomical and of the second kind.

Govindasvāmin's rules from twenty-two stanzas in his *Govindakṛti,* which is now lost, are cited by his pupil Śaṅkaranārāyaṇa (fl. 869 A.D.) in his commentary on the *Laghubhāskarīya* of Bhāskara I. These rules aim at the solutions of astronomical problems of the second kind (see Shukla, 1963, pp. 103-114).

Mahāvīra stated problems of the second kind and problems of the first kind involving simultaneous indeterminate equations. Āryabhaṭa II discussed methods pertaining to the solutions of astronomical problems, particularly of the second kind. Bhāskara II solved problems mainly of the second kind in his *Līlāvatī* and *Bījagaṇita.* The problems in these two treatises are, for the most part, non-astronomical.

Bhāskara I is the first mathematician who classified the mathematical problems involving indeterminate equations of the first degree into two kinds (Shukla, 1976): (a) Residual Pulverizer ('Sāgrakuṭṭākāra'), which refers to (the solution of) a problem of the first kind; (b) Non-residual Pulverizer ('Niragrakuṭṭākāra'), which refers to (the solution of) a problem of the second kind (Introduction, p. lxxxi).

Furthermore, the solution of an astronomical problem based on the indeterminate equation of the first degree is called by Bhāskara I, the Planetary Pulverizer ('Grahakuṭṭākāra') (Shukla, 1976). Bhāskara I illustrates numerous kinds of such problems in his commentary, for example, Week-day Pulverizer ('Vārakuṭṭākāra') and Time Pulverizer ('Velākuṭṭākāra'). (Introduction, p. lxxxi)

Mahāvīra (see e.g. Raṅgācārya, 1912, p. 87, 15) employs special terms in connection with the problems he states, such as 'Suvarṇakuṭṭīkāra' (Gold Pulverizer).

Thus, in view of the above it follows that kuṭṭaka is used in the solution of:

(i). Non-astronomical problems involving division with remainder (e.g. the Residual Pulverizer of Bhāskara I).

(ii). Astronomical or non-astronomical problems involving division without remainder (e.g. the Non-residual Pulverizer of Bhāskara I).

We note that the "distribution problems" of Mahāvīra, which are of the second kind, are sometimes considered to be a third type of problem employing the method of kuṭṭaka (e.g. see problem $117\frac{1}{2}$ and its solution, English translation of Raṅgācārya, 1912, pp. 117-118, 121. This problem will be dealt with later).

F. *The Method of Kuṭṭaka As Given by Āryabhaṭa I.*

It was mentioned before (see the sub-sections C. and E.), that the method of solution described by Āryabhaṭa I in the last two verses, number 32-33, of the second section (which is entitled Gaṇitapāda) of his *Āryabhaṭīya* (written in 499 A.D.), is the method of solution of the problems of the first kind. Nonetheless, this method can be explained to provide a method of solution for a problem of the second kind. Āryabhaṭa's verses are (Shukla & Sarma, 1976, p. 74):

अधिकाग्रभागहारं छिन्द्यादूनाग्रभागहारेण ।
शेषपरस्परभक्तं मतिगुणमग्रान्तरे क्षिप्तम् ॥३२॥

अधउपरिगुणितमन्त्ययुगूनाग्रच्छेदभाजिते शेषम् ।
अधिकाग्रच्छेदगुणं द्विच्छेदाग्रमधिकाग्रयुतम् ॥३३॥

We give a literal translation of these verses as follows:

Verse 32. One should divide the divisor pertaining to the greater remainder by the divisor pertaining to the smaller remainder. The remainder (obtained from this division and the divisor pertaining to the smaller remainder are) mutually divided, (and then the last remainder) is multiplied by mati, (this product) is increased by the difference of the (given) remainders.

Verse 33. The one just below is multiplied by the one above it and (this result) is added to the one immediately following it (i.e. the final or ultimate). When it (i.e. the uppermost number so formed) is divided by the divisor pertaining to the smaller remainder, the remainder (so obtained) is multiplied by the divisor pertaining to the greater remainder. (The product) is combined with the greater remainder. It (i.e. the resulting number) is the remainder (*or* aim i.e. the required number) pertaining to the two (given) divisors.

The following observations may be made concerning Āryabhaṭa's *AB*, 32-33, p. 74 just translated:

(i). The verses are too concise and obscure.

(ii). It seems that either of the given divisors can be greater than the other.

(iii). It is not clear how far the division is to be carried.

(iv). It is not stated whether the number of quotients is even or odd.

(v). The verses do not articulate that the first quotient from the mutual division is to be discarded.

(vi). The verses do not state that the other quotients are to be placed one below the other in a column. (However, statements (v) and (vi) are easily gleaned from the operations given in the verses and the explanations given by the commentators).

(vii). The verses do not state that mati is to be placed in the column of quotients.

As is evident, Āryabhaṭa's verses are difficult to translate (because they do not describe all operations clearly). Due to this reason, more than one interpretation of the above two verses have been given by the translators. The Indian translators who translated these verses into English include Mazumdar (1911 – 12), Sen Gupta (1927), Ganguly (1928) (all cited in Datta, 1932a, *BCMS 24,* pp. 20-21), Datta (1932a), and Shukla and Sarma (1976).

Datta (1932a) discards the translation of Mazumdar for he thinks that it is based on Kaye's (1908) wrong translation.[16] Kaye had himself admitted his translation to be unsatisfactory. Sen Gupta has not made an independent attempt to explain Āryabhaṭa I's verses which contain a truly enigmatical rule. His interpretation of the rule is admittedly based on Brahmagupta's rule. Ganguly's translation is restricted, for his interpretation is based on the working out of a numerical example. It does not represent the true intent of the author Āryabhaṭa I (*BCMS 24,* pp. 20-21).

Datta (1932a) procured three early commentaries on the *Āryabhaṭīya*. He based his translation on the interpretation given in the commentary which was the earliest of these three and was written by the commentator Bhāskara I (in 629 A.D.). The other two commentators, namely Sūryadeva Yajvan (b. 1191 A.D.) and Parameśvara (ca. 1380 – 1460 A.D.) have followed Bhāskara I in many respects (*BCMS 24,* p. 21).

Datta (1932a) writes that as far as Bhāskara I is concerned, he has made augmentations at one or two places in his commentary, which do not seem to follow easily from the text of Āryabhaṭa I's two verses. But to justify himself, Bhāskara I has explicitly

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16. It is worth remarking here that though Datta is of the opinion that Mazumdar's (1911–12) translation is based on Kaye's wrong translation, Mazumdar did differ from Kaye on several points and found several mistakes in Kaye's translation, and hence in Kaye's interpretation of Āryabhaṭa's rule (see e.g. pp. 11 and 16-17 of Mazumdar, N.K. (1911–12); "Aryyabhatta's rule in relation to Indeterminate Equations of the First Degree"; *BCMS 3*, pp. 11-19).

mentioned in such places "संप्रदायाविच्छेदाद्व्याख्यायते"; that is, "his interpretation is that which has been generally accepted in his (Āryabhaṭa I's) school." (See e.g. pp. 23, 35 अग्रान्तरे क्षिप्तम् समेषु क्षिप्तं विषमेषु शोध्यमिति संप्रदायाविच्छेदाद्व्याख्यायते।) Thus his interpretation should likely be considered as that which expresses the intentions of the author Āryabhaṭa I (*BCMS 24*, pp. 21, 23, 35).

Having written the two verses in the prose form, and having discussed the various meanings or implications of the parts of the prose form, Datta (1932a) concludes that two translations are possible. The first one is in accordance with Bhāskara I's interpretation of Āryabhaṭa's rule. Also, Āryabhaṭa's rule is intended for the solution of the problem and its general case, which may be algebraically represented as:

$$N = ax + R_1 = by + R_2, R_1 > R_2 \text{ (say)};$$

and in general

$$N = a_1x_1 + R_1 = a_2x_2 + R_2 = \ldots = a_nx_n + R_n. \qquad (BCMS\ 24, \text{pp. } 19\text{-}20, 33)$$

Now the first translation is *essentially* the following (Datta, 1932a):

Divide the divisor corresponding to the greater remainder by the divisor corresponding to the smaller remainder (so here, assuming $R_1 > R_2$, divide a by b). Let r_1 be the remainder and q the quotient. Then divide the divisor corresponding to the smaller remainder (i.e. b) by r_1 and let q_1 be the quotient. Continue this process of mutual division. Multiply the last *remainder* by an optional integer (called 'mati') t so that the product being increased or diminished by c $(= R_1 - R_2)$, according as the number of quotients (beginning with q_1) in the mutual division is even or odd, will be exactly divisible by the last but one remainder. Let this (new) quotient be Q. Now place the quotients (beginning with q_1) in a column one below the other, then place t. Below that, place the quotient Q just obtained. Then (reduce the numbers in this column as follows): multiply the penultimate by the one just above it and add the number just below it (i.e. the ultimate) to this product. (Replace the number above the penultimate with this result and remove the ultimate number.) Repeat this operation with the other numbers in the (altered) column. Divide the last (i.e.

the uppermost) number obtained by performing this operation (repeatedly) by the divisor corresponding to the smaller remainder (i.e. by b). The remainder will be (the least value of) x. Then calculate $ax + R_1$. The result will be (द्विच्छेदाग्रम् i.e.) a number (N), corresponding to the two divisors a and b (*BCMS 24*, p. 32).

Datta (1932a) also gives another interpretation of the word द्विच्छेदाग्रम् in the last sentence in the above translation which serves for the general case when the number of divisors is more than two: The result will be the residue corresponding to the product of the two divisors (i.e. corresponding to the divisor ab). Bhāskara I and Sūryadeva follow both interpretations but Parameśvara gives only the first (*BCMS 24*, pp. 23, 35).

According to the second translation (Datta, 1932a), r_1 and b should be mutually divided until the remainder *becomes zero*. The last *quotient* should be multiplied by an optional integer t and increased or diminished by c according as the number of quotients (beginning with q_1) in the mutual division is even or odd. Then the quotients (beginning with q_1 but not including the last quotient) of the mutual division should be placed in a column. Underneath them, should be placed the result just obtained (= (last quotient) $t \pm c$) and finally, under that, the value t. The rest of the procedure is the same as that in the first translation. This translation is very similar to the one given by Ganguly. In this translation, we do not have to supply the words "will be exactly divisible by the last but one remainder" (*BCMS 24*, pp. 32-33).

G. *The Rationale of the Method of Kuṭṭaka Using Modern Notation.*

It is clear that in Āryabhaṭa I's method, c is positive because c is the (positive) difference of the remainders R_1 and R_2. So we have to solve $by = ax + c$ if $R_1 > R_2$ or $ax = by + c$ if $R_2 > R_1$, where R_1 and R_2 are remainders corresponding to the divisors a and b respectively (see sub-section E. above). Recall that Āryabhaṭa I's verses 32-33 can be explained in terms of a Residual Pulverizer as well as a Non-residual Pulverizer (see the preceding sub-section F.). In this context, the commentator Bhāskara I comments (cited in

Datta, 1932a, *BCMS 24*, p. 36): "एवं साग्रकुट्टाकारो व्याख्यातः। इदानीं ते एव सूत्रे निरग्रकुट्टकार्थं व्याख्यास्यामः। अधिकाग्रभागहारं छिन्द्यादपवर्त्तितयोरित्यर्थः।" Similarly, Sūryadeva Yajvan comments (cited in Sarma, 1976, p. 74): "एतदेवार्यासूत्रद्वयं निरग्रकुट्टाकारे योज्यते। अधिकाग्रभागहारम् अधिकसंख्यभागहारं भागहारभाज्ययोः अत्र परस्परभाजकत्वात् भागहारशब्देन द्वयोरपि निर्देशः।"

Now in regard to the specifics of the method of solution, Datta (1932a) proclaims that all Indian algebraists posterior to Āryabhaṭa I have observed that the above equations can be solved if and only if the greatest common divisor of a and b divides c. So, if possible, a, b, c must be divided by the gcd (a, b). Thus, as a preliminary operation, a and b must necessarily be made relatively prime. Bhāskara I proclaims that the first line of Āryabhaṭa I's rule (i.e. verse 32a) implies this (preliminary) operation and that this interpretation was prevalent in Āryabhaṭa I's school. This is contained in the ensuing extracts from Bhāskara I's commentary on the *Āryabhaṭīya* (cited in *BCMS 24*, pp. 24, 36): "अधिकाग्रभागहारं छिन्द्यादपवर्त्तितयोरित्यर्थः। ... येन भागहारोऽपवर्त्तितः तेनैव भाज्योऽपवर्त्तनीयः। कथमिदमवगम्यते येनैव भागोऽपवर्त्तितस्तेनैव भाज्योऽपवर्त्तनीय इति। संप्रदायाविच्छेदादथवा न्याय एषः। अपवर्त्तितस्य भागहारस्यापवर्त्तितेनैव भाज्येन भवितव्यमिति। ... अधिकाग्रभागहारमित्यादिना ग्रन्थेनैतत् प्रतिपादयति। अपवर्त्तितयोर्भागहारभाज्ययोः कुट्टाकार इति।"

The commentator Sūryadeva Yajvan provides a complete explanation about the operations to be performed (cited in Sarma, 1976, p. 74): "अधिकाग्रभागहारमधिकसंख्यंभागहारं भागहारभाज्ययोः अत्र परस्परभाजकत्वात् भागहारशब्देन द्वयोरपि निर्देशः। तमधिकसंख्यं भाज्यभाजकात्मकं राशिद्वयम्। ऊनाग्रभागहारेण छिन्द्यादूनसंख्येन भागहारेण सम्भवे सत्यपवर्तयेदित्यर्थः। येन हरभाज्यावपवर्त्येते तेन क्षेपस्याप्यपवर्तनम् अर्थसिद्धमिति न कण्ठोक्तम्। ... उक्तं च-

भाज्यहरप्रक्षेपात् सदृशच्छेदेन सम्भवे छिन्द्यात्।
स्याच्चेद् विभाज्यहरयोः छेदो न क्षेपकस्य खिलम् ॥

इति।"

Similar explanations are contained also in other mathematicians' works, see e.g., Āryabhaṭa II's *MS* XVIII, 1; Śrīpati's *SSE* XIV, 22a-b and 26a-b; Bhāskara's *BG*, 46b-47b, p. 26 which contain respectively:

भाज्यक्षेपच्छेदा यथोदिताः संस्थिताः क-विधिरेषः।
ते च करण्या भक्ता दृढाभिधाना अयं स्व-विधिः ॥१॥

विभाज्यहारं च युतिं निजच्छिदा।
समेन वाऽऽदावपवर्त्य सम्भवे ॥२२a-b॥

विभाज्यहृत्योरपवर्तनं यदा।
भवेद्युतौ नैव खिलं हि तत्तदा ॥२६a-b॥

भाज्यो हारः क्षेपकश्चापवर्त्यः
केनाप्यादौ संभवे कुट्टकार्थम्।
येन छिन्नौ भाज्यहारौ न तेन
क्षेपश्चैतद्दुष्टमुद्दिष्टमेव ॥४६b-४७b॥

Furthermore, for the phrase शेषपरस्परभक्तं, in Āryabhaṭa I's verse 32b above, Bhāskara I's explanation is (cited in Datta, 1932a, *BCMS 24*, p. 35): "शेषपरस्परभक्तं लब्धेन नास्ति प्रयोजनं, शेषेण सह कर्म क्रियते, परस्परेण भक्तं परस्परभक्तं इतरेतरभक्तमित्यर्थः। शेषेण सह परस्परभक्तं शेषपरस्परभक्तम्।" Likewise, Sūryadeva

Yajvan states (cited in Sarma, 1976, p. 71): "शेषपरस्परभक्तं भाज्यभाजकशेषयो स्वरूपमन्योन्यभक्तं स्यात्। यावद्धरभाज्ययोरल्पता। लब्धानि फलान्युपर्यधोभावेन स्थापितानि समानि च भवन्ति तावदन्योन्यं भजेदित्यर्थः।" In this regard, the interpretation of the commentator Paramesvara is very clear (cited in Datta, 1932a, *BCMS 24*, p. 22): "शेषपरस्परभक्तं। अनन्तरं शेषपरस्परहरणं कार्यम्। शेषशब्दोऽत्र हृतशेषस्य तत्समीपस्थितस्योनाग्रहारकस्य च प्रदर्शकः। हृतशेषस्योनाग्रभागहारस्य च परस्परहरणं कार्यमित्यर्थः।"

Thus the following is the rationale of the general solution in the case of two remainders (Datta, 1932a, *BCMS 24*, pp. 25-29):

Let us suppose $R_1 > R_2$, so that $c = R_1 - R_2 > 0$. We have to solve the equation

$$by = ax + c \qquad \text{(I)}$$

for x, y in positive integers; where a, b, c are positive integers and a, b are prime to each other.

Dividing b and a mutually we have:

$$\begin{array}{lllllll}
b) & a & (q & & & & \\
 & \underline{bq} & & & & & \\
 & r_1) & b & (q_1 & & & \\
 & & \underline{r_1q_1} & & & & \\
 & & r_2) & r_1 & (q_2 & & \\
 & & & \underline{r_2q_2} & & & \\
 & & & r_3) & r_2 & (q_3 & \\
 & & \cdots & \cdots & \cdots & \cdots & \cdots \\
 & & & r_{m-1}) & r_{m-2} & (q_{m-1} & \\
 & & & & \underline{r_{m-1}q_{m-1}} & & \\
 & & & & r_m) & r_{m-1} & (q_m \\
 & & & & & \underline{r_mq_m} & \\
 & & & & & r_{m+1} &
\end{array}$$

(If $a < b$, note $q = 0$ and $r_1 = a$.)

The mutual division can be continued either (i) to the finish or (ii) until a certain number of quotients is obtained and then stopped.

This process is equivalent to the Euclidean Algorithm:

$$a = bq + r_1$$

$$b = r_1 q_1 + r_2$$

$$r_1 = r_2 q_2 + r_3$$

$$r_2 = r_3 q_3 + r_4$$

$$\dots \dots \dots$$

$$r_{m-2} = r_{m-1} q_{m-1} + r_m$$

$$r_{m-1} = r_m q_m + r_{m+1}.$$

Here $r_1 > r_2 > \dots > r_{m+1} \geq 0$.

Now substituting the value of a into the given equation

$$by = ax + c \tag{I}$$

we get

$$by = (bq + r_1)\, x + c = bqx + r_1 x + c = b\left(qx + \frac{r_1 x + c}{b}\right).$$

Therefore

$$y = qx + \frac{r_1 x + c}{b}$$

i.e.

$$y = qx + y_1 \tag{1}$$

where

$$by_1 = r_1 x + c\,. \tag{I.1}$$

Equivalently, using $a = bq + r_1$ and (1), the given equation (I) reduces to (I.1). That is,

$by = ax + c$ reduces to $by_1 = r_1 x + c$.

Again, putting $b = r_1 q_1 + r_2$ in (I.1), we get

$$(r_1 q_1 + r_2) y_1 = r_1 x + c$$

$$\Rightarrow r_1 x = r_1 q_1 y_1 + r_2 y_1 - c = r_1\left(q_1 y_1 + \frac{r_2 y_1 - c}{r_1}\right)$$

$$\Rightarrow x = q_1 y_1 + \frac{r_2 y_1 - c}{r_1}$$

$$\Rightarrow x = q_1 y_1 + x_1 \qquad (2)$$

where

$$r_1 x_1 = r_2 y_1 - c. \qquad (I.2)$$

Thus, (I.1) further reduces to (I.2). That is, $by_1 = r_1 x + c$ further reduces to $r_1 x_1 = r_2 y_1 - c$. This procedure is continued.

Writing down the successive values and the corresponding reduced equations separately, we get the following table:

$y = qx + y_1$	(1)	$by_1 = r_1x + c$	(I.1)
$x = q_1y_1 + x_1$	(2)	$r_1x_1 = r_2y_1 - c$	(I.2)
$y_1 = q_2x_1 + y_2$	(3)	$r_2y_2 = r_3x_1 + c$	(I.3)
$x_1 = q_3y_2 + x_2$	(4)	$r_3x_2 = r_4y_2 - c$	(I.4)
$y_2 = q_4x_2 + y_3$	(5)	$r_4y_3 = r_5x_2 + c$	(I.5)
$x_2 = q_5y_3 + x_3$	(6)	$r_5x_3 = r_6y_3 - c$	(I.6)
$y_3 = q_6x_3 + y_4$	(7)	$r_6y_4 = r_7x_3 + c$	(I.7)
....	....	....	....
$y_{n-1} = q_{2n-2}x_{n-1} + y_n$	$(2n-1)$	$r_{2n-2}y_n = r_{2n-1}x_{n-1} + c$	$(\text{I.}(2n-1))$
$x_{n-1} = q_{2n-1}y_n + x_n$	$(2n)$	$r_{2n-1}x_n = r_{2n}y_n - c$	$(\text{I.}(2n))$
$y_n = q_{2n}x_n + y_{n+1}$	$(2n+1)$	$r_{2n}y_{n+1} = r_{2n+1}x_n + c$	$(\text{I.}(2n+1))$

(This table is for $m = 2n$; for $m = 2n-1$ leave off the last line.)

Now whether the mutual division is continued to the finish or stopped at some stage, the number of quotients obtained (neglecting the first quotient q, as does Āryabhaṭa I) may be even or odd. Thus, we have the following cases and sub-cases.

Case I. Let us suppose that the mutual division is continued to the finish, so that the last remainder r_{m+1} is zero. Then the last but one remainder (i.e. r_m) will be unity because a and b are prime to each other.

Sub-case (I.i). Let the number of quotients (excluding the first) be even, i.e. $m = 2n$. Then $r_{2n+1} = 0$, $r_{2n} = 1$, $r_{2n-1} = q_{2n}$. Therefore, from equation (I. (2n+1)), we have

$$1 \cdot y_{n+1} = 0 \cdot x_n + c \Rightarrow y_{n+1} = c.$$

(Note that any $x_n = t$ gives a positive value for y_{n+1}.)

From equation (I. (2n)) we have

$$q_{2n}x_n = 1 \cdot y_n - c \Rightarrow y_n = q_{2n}x_n + c.$$

(Or, we could have put $y_{n+1} = c$ in equation (2n+1) and obtained the same equation $y_n = q_{2n}x_n + c$.)

Putting $x_n = t$ where t is an arbitrary non-negative integer, we get a positive value of y_n, i.e. $y_n = q_{2n}t + c$. Then we can obtain x_{n-1} from equation (2n). Proceeding backwards (i.e. upwards) step by step, we can find x and y finally from equations (2) and (1), in *positive* integers. This x and y form a solution for the given equation (I), but not necessarily the least positive solution.

Sub-case (I.ii). Suppose the number of quotients (excluding the first) is odd, i.e. $m = 2n-1$. Then $r_{2n} = 0$, $r_{2n-1} = 1$, $r_{2n-2} = q_{2n-1}$. In this situation, equations (2n+1) and (I. (2n+1)) will not exist because q_{2n} and r_{2n+1} are absent. Therefore, in view of equation (I. (2n)), we have

$$1 \cdot x_n = 0 \cdot y_n - c \Rightarrow x_n = -c.$$

(Note that any $y_n = t'$ is all right here.)

In view of equation (I. (2n–1)), we have

$$q_{2n-1}y_n = 1 \cdot x_{n-1} + c \Rightarrow x_{n-1} = q_{2n-1}y_n - c.$$

(Or, we can obtain the same equation by putting $x_n = -c$ in equation (2n).)

Putting $y_n = t'$ where t' is a sufficiently large positive integer, we get a positive value of x_{n-1}, i.e. $x_{n-1} = q_{2n-1}t' - c$. Proceeding upwards as before, we can find x and y in *positive* integers, so that a solution (which is not necessarily the least positive solution) for the given equation (I) is found.

Case II. Let us suppose that the mutual division is stopped at some stage ($r_{m+1} > 0$).

Sub-case (II.i). Suppose the number of quotients (excluding the first) is even, i.e. $m = 2n$. Then in equation (I. (2n+1)), $r_{2n+1} \neq 0$ and $r_{2n} \neq 1$. Therefore,

$$y_{n+1} = \frac{r_{2n+1}x_n + c}{r_{2n}}.$$

Putting $x_n = t$, where t is a suitably chosen positive integer (so that the division is exact), we get

$$y_{n+1} = \frac{r_{2n+1}t + c}{r_{2n}} = \text{a positive integer.}$$

Then from equation (2n+1), we find $y_n = q_{2n}t + y_{n+1}$, where y_{n+1} is known. Proceeding upwards as before, we can find x and y in positive integers.

Sub-case (II.ii). Suppose the number of quotients (excluding the first) is odd, i.e. $m = 2n-1$. Here $r_{2n} \neq 0$ and $r_{2n-1} \neq 1$. Also equations (2n+1) and (I. (2n+1)) do not exist. So the reduced form of the original equation (I) will be the equation (I. (2n)), i.e. $r_{2n-1}x_n = r_{2n}y_n - c$, whence

$$x_n = \frac{r_{2n}y_n - c}{r_{2n-1}}.$$

Now choosing $y_n = t'$, where t' is a positive integer such that

$x_n = \dfrac{r_{2n}t' - c}{r_{2n-1}}$ = a positive whole number, we can find a positive integral value of x_{n-1} from equation (2n). That is

$x_{n-1} = q_{2n-1}t' + x_n$, where x_n is known.

Proceeding upwards as before, we can calculate the values of x and y in positive integers.

Thus a solution for the given equation is found in all cases.

H. *Comparison of the Above Rationale With Āryabhaṭa I's Process.*

(i). In the first translation of Āryabhaṭa I's rule, the first sentence "Divide the divisor corresponding to the greater remainder by the divisor corresponding to the smaller remainder," means that Āryabhaṭa aims at solving the problems:

For $R_1 > R_2$ and $c = R_1 - R_2$, find x such that $\dfrac{ax+c}{b}$ is a positive integer.

For $R_2 > R_1$ and $c = R_2 - R_1$, find y such that $\dfrac{by+c}{a}$ is a positive integer.

(ii). In the first translation, the portion "Multiply the last remainder ... exactly divisible by the last but one remainder," refers to calculating the (new integral) quotients $y_{n+1} = c$ and $x_n = -c$ in the sub-cases (I.i) and (I.ii) respectively, and the quotients

$$y_{n+1} = \frac{r_{2n+1}t + c}{r_{2n}} \text{ and } x_n = \frac{r_{2n}t' - c}{r_{2n-1}}$$

in the sub-cases (II.i) and (II.ii) respectively. In both cases, the operations performed are

$$y_{n+1} = \frac{\text{(last remainder) } t + c}{\text{last but one remainder}}$$

and

$$x_n = \frac{\text{(last remainder) } t' - c}{\text{last but one remainder}}, \text{ respectively.}$$

(iii). In the second translation, the portion "The last quotient should be multiplied by an optional integer t ... odd," refers to the operations $y_n = q_{2n}t + c$ and $x_{n-1} = q_{2n-1}t' - c$ of the sub-cases (I.i) and (I.ii) respectively, when the mutual division is continued to the finish.

Note that this rule does not apply to case II. Because in sub-case (II.i), $y_n = q_{2n}t + y_{n+1}$ where y_{n+1} is a positive integer not necessarily equal to c; and in sub-case (II.ii), $x_{n-1} = q_{2n-1}t' + x_n$ where x_n is a *positive* whole number, not equal to $-c$.

(iv). In the rationale, case I corresponds to the second translation and case II, to the first translation.

(v). Āryabhaṭa I's operation of writing the quotients in a column and then reducing them, is equivalent to writing the expressions for $(y), x, y_1, x_1, \dots, y_{n-1}, x_{n-1}$ etc., as is evident from the tables on the following page.

Sub-case (I.i)

A = Āryabhaṭa I's unreduced column (Datta's second translation).
B = Calculated values from the table.

	A	B	
		$qx+y_1=y$	
↓	q_1	$q_1y_1+x_1=x$	
↓	q_2	$q_2x_1+y_2=y_1$	
↓	q_3	$q_3y_2+x_2=x_1$	
	⋮	⋮	
↓			↑
	⋮	⋮	
↓	q_{2n-2}	$q_{2n-2}x_{n-1}+y_n=y_{n-1}$	↑
	q_{2n-1}	$q_{2n-1}y_n+x_n=x_{n-1}$	↑
	$q_{2n}t+c$	$q_{2n}x_n+y_{n+1}=y_n$	↑
	t	$t=x_n$	↑
		$c=y_{n+1}$	

Sub-case (I.ii)

A = Āryabhaṭa I's unreduced column (Datta's second translation).
B = Calculated values from the table.

	A	B	
		$qx+y_1=y$	
↓	q_1	$q_1y_1+x_1=x$	
↓	q_2	$q_2x_1+y_2=y_1$	
↓	q_3	$q_3y_2+x_2=x_1$	↑
	⋮	⋮	
↓			↑
	⋮	⋮	
↓	q_{2n-2}	$q_{2n-2}x_{n-1}+y_n=y_{n-1}$	↑
	$q_{2n-1}t'-c$	$q_{2n-1}y_n+x_n=x_{n-1}$	↑
	t'	$t'=y_n$	↑
		$-c=x_n$	

Sub-case (II.i)

A = Āryabhaṭa I's unreduced column (Datta's first translation).
B = Calculated values from the table.

	A	B	
		$qx+y_1=y$	
↓	q_1	$q_1y_1+x_1=x$	
↓	q_2	$q_2x_1+y_2=y_1$	
	⋮	⋮	
↓			
	⋮	⋮	
↓	q_{2n-2}	$q_{2n-2}x_{n-1}+y_n=y_{n-1}$	↑
↓	q_{2n-1}	$q_{2n-1}y_n+x_n=x_{n-1}$	↑
	q_{2n}	$q_{2n}x_n+y_{n+1}=y_n$	↑
	t	$t=x_n$	↑
	$Q=\dfrac{r_{2n+1}t+c}{r_{2n}}$	$Q=\dfrac{r_{2n+1}x_n+c}{r_{2n}}=y_{n+1}$	↑

Sub-case (II.ii)

A = Āryabhaṭa I's unreduced column (Datta's first translation).
B = Calculated values from the table.

	A	B	
		$qx+y_1=y$	
↓	q_1	$q_1y_1+x_1=x$	
↓	q_2	$q_2x_1+y_2=y_1$	
	⋮	⋮	
↓			↑
	⋮	⋮	
↓	q_{2n-2}	$q_{2n-2}x_{n-1}+y_n=y_{n-1}$	↑
↓	q_{2n-1}	$q_{2n-1}y_n+x_n=x_{n-1}$	↑
	t'	$t'=y_n$	↑
	$Q=\dfrac{r_{2n}t'-c}{r_{2n-1}}$	$Q=\dfrac{r_{2n}y_n-c}{r_{2n-1}}=x_n$	↑

(vi). Note that by omitting the first quotient q, the value of x only is found, which is sufficient because Āryabhaṭa I wants to find the number $N = ax + R_1$ (Datta, 1932a). So retaining the first quotient q in the column is not necessary. This is also indicated by (Āryabhaṭa I's) statement in the first translation, "Divide the last number obtained ...." Then the remainder will be the least value of x and then the corresponding value of N will be least (*BCMS 24*, pp. 29-30).

We can explain this statement as follows:

Let α, β be non-negative integers such that

$$\left.\begin{array}{l} b\beta = a\alpha + c \\ \alpha \text{ is least.} \end{array}\right\} \qquad (*)$$

We wish to show that α is the remainder after division by b of the calculated value of x in the procedure. Conditions (*) clearly imply that $0 \le \alpha < b$ so that α is a remainder on division by b. Furthermore, the general solution to $by = ax + c$ in positive integers is then $x = bm + \alpha$ and $y = am + \beta$ for $m \ge 0$. The *calculated* value of x ($= q_1 y_1 + x_1$) is one such solution. Thus it is clear that α is the remainder upon dividing the calculated x by b.

Therefore, the minimum value of N will be $a\alpha + R_1$. Similarly, the general values for N in positive integers are $N = ma + (a\alpha + R_1)$, (or $N = mb + (b\beta + R_2)$), where m is any integer ≥ 0.

(vii). In the sub-case (I.ii), i.e. when the number of quotients is odd, then we have the calculated value $q_{2n-1} y_n + x_n = x_{n-1}$, where $y_n = t'$ and $x_n = -c$. This is in accordance with Āryabhaṭa I's instruction: the product of the last quotient and an optional integer (which is here t') is to be diminished by c if the number of quotients (beginning with q_1) in the mutual division is odd. (For another solution to $by = ax + c$ when the number of quotients is odd, see our commentary on Bhāskara II's verse 50c-51b).

(viii). When $R_1 = R_2$, (I) reduces to $by = ax$. So the minimum solution is $x = 0$ and $y = 0$. Thus Āryabhaṭa's method works here, except that here the minimum solution $x = 0$ and $y = 0$ is self-evident and the required number N equals the common remainder.

I. *An Application of the Above Rationale to Āryabhaṭa I's Rule.*

The following example is briefly solved in the *Āryabhaṭīya-Bhāṣya* of Bhāskara I (see Shukla, 1976):

A number yields 5 as the remainder when divided by 12, and the same number is again seen by me to yield 7 as the remainder when divided by 31. What is that number?

Solution. Dividing the divisor 31 (= a) corresponding to the greater remainder by the divisor 12 (= b) corresponding to the smaller remainder, the quotient $q = 2$, and the remainder $r_1 = 7$. The quotient q is to be neglected. Now dividing $r_1 = 7$ and $b = 12$ *mutually* we have,

$$
\begin{array}{llllll}
r_1 = & 7) & 12 & (1 & = q_1 & \\
 & & \underline{7} & & & \\
 & r_2 = & 5) & 7 & (1 & = q_2 \\
 & & & \underline{5} & & \\
 & & r_3 = & 2 & &
\end{array}
$$

Here $c = R_1 - R_2 = 7 - 5 = 2$. Also, using sub-case (II.i) in the rationale, the number of quotients $= 2 \Rightarrow 2n = 2 \Rightarrow n = 1$. Therefore,

$$y_{n+1} = \frac{r_{2n+1}x_n + c}{r_{2n}} \Rightarrow y_2 = \frac{r_3x_1 + 2}{r_2} = \frac{2x_1 + 2}{5}.$$

Putting $x_1 = t = 4$, $y_2 = 2 =$ the new quotient Q. So we have the following columns:

A = Āryabhaṭa I's unreduced column. B = Calculated values from the table.

A	B
	$qx + y_1 = 2 \cdot 10 + 6 = 26 = y$
$q_1 = 1$	$q_1y_1 + x_1 = 1 \cdot 6 + 4 = 10 = x$
$q_2 = 1$	$q_2x_1 + y_2 = 1 \cdot 4 + 2 = 6 = y_1$
$t = 4$	$t = 4 = x_1$
$Q = 2$	$Q = 2 = y_2$

Ignoring our topmost line, the last calculated value is 10. Dividing 10 by $b = 12$, the remainder $x_{\text{least}} = 10$. Therefore, the minimum value of the number $N = ax_{\text{least}} + R_1 = 31 \cdot 10 + 7 = 317$ (pp. 133-134, *or* English translation p. 309).

Note that equivalently, Bhāskara I is using Āryabhaṭa I's rule in solving the problem which corresponds to $N = 12y + 5 = 31x + 7$, which gives rise to the indeterminate equation $12y = 31x + 2$. From the calculated values, a solution is $x = 10$, $y = 26$. (In this case, this solution is also the least solution.)

J. *Bhāskara I's Rules.*

Bhāskara I proclaimed his rules concerning kuṭṭaka in his *Mahābhāskarīya* I, 41-52 (see Shukla, 1960, pp. 7-9 *or* English translation pp. 29-46). He applied these rules to solve the problems contained in verses 13-23 of the eighth chapter of this work (pp. 49-51 *or* English translation pp. 219-224), and in his commentary on the *Āryabhaṭīya* which he wrote in 629 A.D. (see Shukla, 1976, English translation pp. 311-334). Also Bhāskara I's *Laghubhāskarīya* VIII, 17-18 contain two astronomical problems the solutions of which involve the method of kuṭṭaka (see Shukla, 1963, p. 26 *or* English translation pp. 99-102). The order in which these three astronomical works were composed by Bhāskara I is as follows (Shukla, 1960, Introduction, p. I): (i) The *Mahābhāskarīya*, (ii) the commentary on the *Āryabhaṭīya*, and (iii) the *Laghubhāskarīya*.

Bhāskara I describes his method of kuṭṭaka in his *MB* I, 42-44 as follows:

भाज्यं न्यसेदुपरि हारमधश्च तस्य
खण्डयात्परस्परमधो विनिधाय लब्धम् ।
केनाऽऽहतोऽयमपनीय यथाऽस्य शेषं
भागं ददाति परिशुद्धमिति प्रचिन्त्यम् ॥४२॥

आप्तां मतिं तां विनिधाय वल्ल्यां
नित्यं ह्यधोऽधः क्रमशश्च लब्धम्।
मत्या हतं स्यादुपरिस्थितं य-
ल्लब्धेन युक्तं परतश्च तद्वत् ॥४३॥

हारेण भाज्यो विधिनोपरिस्थो
भाज्येन नित्यं तदधःस्थितश्च।
अह्नांगणोऽस्मिन् भगणादयश्च
तद्वा भवेद्यस्य समीहितं यत् ॥४४॥

We give the following translation of these verses based on that of Datta and Singh (1962, *Part II*, p. 100) as well as that of Shukla (1960, English translation, p. 30):

Place the dividend a above and the divisor b below it. (Divide them mutually.) Place the quotients (obtained) from their mutual division one below the other (so that they form a chain). (When an *even* number of quotients has been obtained, discarding the first quotient which is zero,) find out by what (number t called mati) the (last) remainder be multiplied, so that when this (resulting product) is *diminished* by the (given) residue (c = revolutions of a planet), the remainder (i.e. difference) will be exactly divisible (by the divisor corresponding to that remainder. This division will yield a new quotient). Place the mati, (i.e. the number t) which was found, under the chain (of quotients) and the (new) quotient in turn underneath (that t). Then (reduce this column as in Āryabhaṭa I's rule, until only two numbers remain in the altered chain; i.e.) by the mati multiply the number which stands just above it and (to the product) add the (new) quotient (which is below the mati). (Replace the number above the mati by this resulting sum and discard the new quotient which is below the mati). Proceed further (i.e. upward) the same way (until only two numbers remain in the altered chain). (Then) by the (prescribed) method, the upper (number, called the "multiplier") is to be divided by the divisor b, and as usual the one which is the lower (called the "quotient" is to be divided) by the dividend a. In this (operation of division, the remainders) will (respectively) be the (required) ahargaṇas (x) and the (complete) revolutions (y, performed by a planet) etc.; or (the result is) that which is one's desire.

In view of the above verses just described, we can make the following observations:

(i). Bhāskara I is solving a problem of the second kind because two numbers, x and y, are obtained.

(ii). The first quotient (q) in the mutual division of the dividend a and the divisor b has been discarded, because the upper number in the reduced column gives x.

(iii). This first quotient (q, which has been discarded) was, in fact, zero because the number below x (i.e. y_1) gives a value of y (see equation (1) of the rationale).

(iv). The dividend a is less than the divisor b, because the first quotient is zero.

(v). c is positive because c is the residue of revolutions ($c = 0$ is a special case).

(vi). The number of quotients retained, discarding the first, is assumed to be *even* (because c is subtracted i.e. $-c$ is *added* to the product of t and the (last) remainder).

Thus, in this rule Bhāskara I intends to solve $\frac{ax-c}{b}$ = some positive integer, where $a < b$ and $-c < 0$.

As mentioned earlier (see sub-sections A. and E.), Bhāskara I solves astronomical problems which can be expressed mainly by the form

$$\frac{ax-c}{b} = y,$$

and since in such problems a, b, c are usually very large, to simplify the solutions, Bhāskara I has suggested a few procedures as follows:

(i). In his *Mahābhāskarīya* I, 45a-46b, Bhāskara I suggests that alternatively, the pulverizer is employed by subtracting unity. What he means is that to solve

$$\frac{ax - c}{b} = y \tag{1}$$

first solve

$$\frac{ax' - 1}{b} = y' \tag{2}$$

where

$$\frac{x}{c} = x' \text{ and } \frac{y}{c} = y'.$$

Then if $x' = \alpha$ and $y' = \beta$ is a solution of (2), then $x = c\alpha$ and $y = c\beta$ will be the corresponding solution of (1). Divide $c\alpha$ and $c\beta$ by b and a respectively to obtain the minimum solution of (1).

(ii). In *MB* I, 47, Bhāskara I essentially says that if $a > b$, then the largest multiple of b should be subtracted from a first (Shukla, 1960, English translation, pp. 34-35). The pulverizer is to be employed after this operation. That is, let the pulverizer be

$$\frac{ax - c}{b} = y \tag{1}$$

where $a > b$. Let $a = kb + a'$; $k, a' > 0$ and $a' < b$. Then we first solve

$$\frac{a'x - c}{b} = y'. \tag{3}$$

If $x = \alpha$, $y' = \beta$ is a solution of (3), then $x = \alpha$, $y = k\,\alpha + \beta$ is a solution of (1).

(iii). The contents of Bhāskara's *MB* I, 50 may be symbolically expressed as follows (Shukla, 1960, English translation, p. 41): If $x = \alpha$, $y = \beta$ is the *minimum* solution of

$$\frac{ax - c}{b} = y,$$

then the *other* solutions are given by $x = mb + \alpha$, $y = ma + \beta$, where $m = 1, 2, 3, \ldots$.

(iv). In *MB* I, 51, Bhāskara refers to a pulverizer of the form (Shukla, 1960, English translation, p. 41)

$$\frac{ax + c}{b} = y, \tag{4}$$

which is to be solved in the same manner as equation (1) above except that c should be added (i.e. $-c$ *diminished*) in the operations in (1), since

$$\frac{ax-(-c)}{b}=\frac{ax+c}{b}.$$

Alternatively, the solution of (4) may be deduced from the solution of

$$\frac{ax+1}{b}=y. \tag{5}$$

In light of the observations and procedures just mentioned, we can conclude that Bhāskara I's chain for

$$\frac{ax-c}{b}=y$$

($a<b$, $c>0$) will look like:

$$\begin{array}{l} q_1 \\ q_2 \\ \vdots \\ q_{2n-1} \\ q_{2n} \\ t \\ Q=\dfrac{r_{2n+1}t-c}{r_{2n}}=\dfrac{r_{2n+1}t+(-c)}{r_{2n}}, \end{array}$$

where $2n$ is the number of quotients retained (discarding the first quotient q).

Likewise, the chain for

$$\frac{ax+c}{b}=y$$

($a<b$, $c>0$) will look like:

$$\begin{array}{l} q_1 \\ q_2 \\ \vdots \\ q_{2n-1} \\ q_{2n} \\ t \\ Q=\dfrac{r_{2n+1}t+c}{r_{2n}}, \end{array}$$

where $2n$ is the number of quotients retained (discarding the first quotient q).

Compare this latter chain with Āryabhaṭa I's chain (according to Professor Datta's first translation) when the number of quotients retained (discarding the first) is even (= $2n$). The chains are identical.

In section 4.E. of this chapter, it was mentioned that Bhāskara I classified the problems involving indeterminate equations of the first degree into Residual Pulverizer and Non-residual Pulverizer. The following is an illustration of a Residual Pulverizer in which the number of divisors is more than two. It is solved briefly by Bhāskara I in his *Āryabhaṭīya-Bhāṣya* on *AB* II, 32-33 (see Shukla, 1976, English translation, p. 309):

Illustration. Calculate what is that number which is said to yield 5 as the remainder when divided by 8, 4 when divided by 9, and 1 when divided by 7.

Solution. We provide the following solution based on that of Shukla and Sarma (1976, pp. 76-77):

Here the divisors and the corresponding remainders are:

Divisor	8	9	7
Remainder	5	4	1

First apply the pulverizer to the first and second pairs. So let $a = 8$, $b = 9$, $R_1 = 5$, $R_2 = 4$, $c = R_1 - R_2 = 5 - 4 = 1$.

The divisor corresponding to the greater remainder is $8 = a$. Dividing $a = 8$ by $b = 9$, the quotient is $q = 0$. This quotient is to be discarded. Now divide the remainder 8 and the divisor 9 mutually until the number of quotients is *even*, as follows:

$$\begin{array}{lllll} r_1 = & 8) & 9 & (1 & = q_1 \\ & & \underline{8} & & \\ & r_2 = & 1) & 8 & (8 \quad = q_2 \\ & & & \underline{8} & \\ & & r_3 = & 0 & \end{array}$$

Here the final remainder is = 0. Last divisor = 1 = last but one remainder. Let $t = 1$. Then using sub-case (II.i) of our rationale (which corresponds to Datta's first translation), with $n = 1$ and $c = 1$, we have:

$$y_2 = \frac{r_3 t + c}{r_2} = \frac{0\,(1) + 1}{1} = 1 = Q.$$

Discarding $q = 0$, we have the following chain (column) and its reduction:

q_1	=	1	1	10
q_2	=	8	9	9
t	=	1	1	
Q	=	1		

Dividing the upper number 10 by $b = 9$, the remainder $x_{\text{least}} = 1$. Therefore, the minimum value of the number $N = ax_{\text{least}} + R_1 = 8 \cdot 1 + 5 = 13$. Thus $N = 13$ is the (द्विच्छेदाग्रम् i.e. the) number which corresponds to the two divisors 8 and 9. Also $N = 13$ is the (द्विच्छेदाग्रम् i.e. the) remainder which corresponds to the divisor $8 \cdot 9$ (= 72).

Now apply the pulverizer to the pairs:

Divisor	72	7
Remainder	13	1

So let $a = 72, b = 7, R_1 = 13, R_2 = 1, c = R_1 - R_2 = 13 - 1 = 12$.

The divisor corresponding to the greater remainder is $a = 72$. Dividing $a = 72$ by $b = 7$, the quotient is $q = 10$. This quotient is to be discarded. Now divide the remainder 2 and the divisor 7 mutually until the number of quotients is *even*, as follows:

$$
\begin{array}{llllll}
r_1 = & 2) & 7 & (3 & = q_1 & \\
 & & \underline{6} & & & \\
 & r_2 = & 1) & 2 & (2 & = q_2 \\
 & & & \underline{2} & & \\
 & & r_3 = & 0 & &
\end{array}
$$

Here the final remainder is = 0. Last divisor = 1 = last but one remainder. Let $t = 1$. Then proceeding as before, we have:

$$y_2 = \frac{r_3 t + c}{r_2} = \frac{0\,(1) + 12}{1} = 12 = Q.$$

Discarding $q = 10$, we have the following chain (column) and its reduction:

q_1	=	3	3	43
q_2	=	2	14	14
t	=	1	1	
Q	=	12		

Dividing the upper number 43 by $b = 7$, the remainder $x_{\text{least}} = 1$. Therefore, the minimum value of the number $N = ax_{\text{least}} + R_1 = 72 \cdot 1 + 13 = 85$. Thus $N = 85$ is द्विच्छेदाग्रम् i.e. the number which corresponds to the two divisors 72 and 7, and hence 85 is त्रिच्छेदाग्रम् i.e. the number which corresponds to the three divisors 8, 9 and 7.

Thus the least integral solution of the given problem is $N = 85$. The general solution is $N = (8 \cdot 9 \cdot 7)\,\lambda + 85 = 504\,\lambda + 85$, where $\lambda = 0, 1, 2, 3, \ldots$.

Illustration. The following illustration of a Non-residual Pulverizer is discussed in Shukla and Sarma (1976, p. 78):

Solve $\dfrac{16x - 138}{487} = y.$

Solution. Comparing with

$$\frac{ax-c}{b}=y,$$

we have $a = 16, b = 487, c = 138$. Here $a < b$ and $(a, b) = 1$. Dividing a by b, and stopping the division when the number of quotients (discarding the first) is *even*, we get:

$$q = 0, q_1 = 30, q_2 = 2, r_1 = 16, r_2 = 7, r_3 = 2.$$

Letting $t = 76$ and using Bhāskara I's rule, (or using sub-case (II.i) of our rationale, with $n = 1$ and $-c$ in place of c, which gives $y_2 = \frac{r_3x_1 + (-c)}{r_2}$),

we have:

$$y_2 = \frac{r_3t - c}{r_2} = \frac{2\,(76) - 138}{7} = \frac{14}{7} = 2 = Q\,.$$

Discarding $q = 0$, we have the following chain and its reduction, according to Bhāskara I's method:

q_1	=	30	30	4696
q_2	=	2	154	154
t	=	76	76	
Q	=	2		

Dividing 4696 by the divisor $b = 487$, the remainder is 313, which is x_{least}. Similarly, dividing 154 by the dividend $a = 16$, the remainder is 10, which is y.

Thus the least integral solution of the given problem is $x = 313, y = 10$. The general solution is $x = 487\,\lambda + 313, y = 16\,\lambda + 10$ where $\lambda = 0, 1, 2, 3, \ldots$.

K. *Brahmagupta's Rule.*

Brahmagupta deals with kuṭṭaka in chapter eighteen, entitled Kuṭṭakādhyāya, of his *Brāhmasphuṭasiddhānta* (see e.g. Dvivedin, 1902, verses 3-29, pp. 294-308). In verses

3-5, Brahmagupta gives his rule for the solution of a problem of the *first* kind. This rule is the same as that of Āryabhaṭa I as is given by the *first* translation of Professor Datta (1932a). Brahmagupta's verses are the following *BSS* XVIII, 3-5 (see Dvivedin, 1902, p. 294):

अधिकाग्रभागहारादूनाग्रच्छेदभाजिताच्छेषम् ।
यत् तत् परस्परहृतं लब्धमधोऽधः पृथक् स्थाप्यम् ॥३॥

शेषं तथेष्टगुणितं यथाऽग्रयोरन्तरेण संयुक्तम् ।
शुध्यति गुणकः स्थाप्यो लब्धं चान्त्यादुपान्त्यगुणः ॥४॥

स्वोर्ध्वोऽन्त्ययुतोऽग्रान्तो हीनाग्रच्छेदभाजितः शेषम् ।
अधिकाग्रच्छेदहतमधिकाग्रयुतं भवत्यग्रम् ॥५॥

Note that in the remaining verses Brahmagupta states rules for solving astronomical problems. Thus the context of Brahmagupta's treatment of kuṭṭaka is astronomical. Moreover, like Bhāskara I, Brahmagupta also treats the *sthira* (constant) kuṭṭaka where the additive is –1 (see Dvivedin, 1902, verses 9-11, p. 299).

An anonymous commentator of Brahmagupta's *BSS* XVIII, Kuṭṭakādhyāya, indicates that the solution of $by = ax + c$ can also be obtained by changing it into $ax = by - c$, and thus starting with the division of b by a (instead of a by b, where a is the divisor corresponding to the greater remainder). But in this case, the difference of remainders, must be made negative, i.e. $-c$. The commentator solves an example to this effect which may be enunciated as follows (see Colebrooke, 1817, p. 329):

Example. What number divided by seventy-three yields a remainder of eight; and divided by thirteen, a remainder of three?

Solution.

Setting out:	Divisor	73	13
	Remainder	8	3.

The divisor corresponding to the greater remainder is $73 = a$ and the divisor corresponding to the smaller remainder is $13 = b$. Dividing $b = 13$ by $a = 73$, the quotient is $q = 0$. This quotient is to be discarded. Now divide the remainder 13 and the divisor 73 mutually. The quotients are 5, 1, 1, 1, 1 and the last remainder is 1. Here the additive $c = 8 - 3 = 5$ must be made negative, -5, because the process was inverted. Choosing the mati $t' = 1$, we have (using sub-case (II.ii) of our rationale, with $n = 3$):

$$x_3 = \frac{r_6 t' - c}{r_5} = \frac{1(1) - (-5)}{2} = 3 = Q.$$

Discarding $q = 0$, we have the following chain and its reduction:

q_1	=	5	5	5	5	5	79
q_2	=	1	1	1	1	14	14
q_3	=	1	1	1	9	9	
q_4	=	1	1	5	5		
q_5	=	1	4	4			
t'	=	1	1				
Q	=	3					

Dividing the अग्रान्त (the upper number at the end, i.e.) 79 by $a = 73$, the remainder $y_{least} = 6$. Therefore, the minimum value of the number $N = by_{least} + R_2 = 13 \cdot 6 + 3 = 81$. Thus $N = 81$ is the required number.

L. *Mahāvīra's Rules.*

Mahāvīra's rules aim at the solution of

$$\frac{ax \pm c}{b} = y$$

in positive integers. He describes the rules briefly in verses $115\frac{1}{2}$ and $136\frac{1}{2}$ in his *Gaṇitasārasaṅgraha* and calls these pulverizers as 'Vallikākuṭṭīkāra' and 'Sakalakūṭṭīkara' respectively (see e.g., Raṅgācārya, 1912, pp. 80, 83):

छित्वा छेदेन राशिं प्रथमफलमपोह्याप्तमन्योन्यभक्तं
स्थाप्योर्ध्वाधर्यतोऽधो मतिगुणमयुजाल्पेऽवशिष्टे धनर्णम् ।
छित्वाधः स्वोपरिघ्नोपरियुतहरभागोऽधिकाग्रस्य हारं
छित्वा छेदेन साग्रान्तरफलमधिकाग्रान्वितं हारघातम् ॥११५$\frac{१}{२}$॥

भाज्यच्छेदाग्रशेषैः प्रथमहृतिफलं त्याज्यमन्योन्यभक्तं
न्यस्यान्ते साग्रमूर्ध्वैरुपरिगुणयुतं तैस्समानासमाने ।
स्वर्णघ्नं व्याप्तहारौ गुणधनमृणयोश्चाधिकाग्रस्य हारं
हृत्वा हृत्वा तु साग्रान्तरधनमधिकाग्रान्वितं हारघातम् ॥१३६$\frac{१}{२}$॥

Raṅgācārya (1912) treats an example, which is contained in Mahāvīra's *GSS*, $117\frac{1}{2}$, as follows (English translation, pp. 117-118, 121):

(There were) 63 (numerically equal) heaps of plantain fruits put together and combined with 7 (more) of those same fruits; and these were (equally) distributed among 23 travellers so as to leave no remainder. You tell (me now) the (numerical) measure of a heap (of plantains).

(The problem is thus

$$\frac{63x + 7}{23} = y,$$

where x and y are whole numbers.)

Solution

23)	63	(2	$= q$				
	46						
$r_1 =$	17)	23	(1	$= q_1$			
		17					
	$r_2 =$	6)	17	(2	$= q_2$		
			12				
		$r_3 =$	5)	6	(1	$= q_3$	
				5			
			$r_4 =$	1)	5	(4	$= q_4$
					4		
				$r_5 =$	1		

The first quotient 2 is discarded.

Letting $t = 1$(and using sub-case (II.i) of our rationale, with $n = 2$),

$$Q = y_3 = \frac{r_5 t + 7}{r_4} = \frac{1 \cdot 1 + 7}{1} = 8.$$

So we get the following column and its reduction:

q_1	=	1	1	1	1	51
q_2	=	2	2	2	38	38
q_3	=	1	1	13	13	
q_4	=	4	12	12		
t	=	1	1			
Q	=	8				

Dividing 51 by the divisor 23, the remainder $5 = x$ is the least number of fruits in a bunch.

Note that Mahāvīra states only rules and problems. He does not give any solutions. However, Srinivasiengar (1967) writes in connection with this last example: "Mahāvīra has dodged so as to avoid a zero remainder by dividing 5 by 1, so as to get a quotient of 4 and remainder 1" (p. 102). But Aiyar's (1910) explanation, like that of Raṅgācārya (1912, p. 117), is that Mahāvīra's rule does not require the mutual division to be continued until the remainder is unity; instead it requires *the least remainder in the odd position of order*; and the same value of x can be obtained if one stops with remainder 5, which is in the third (i.e. odd) position of order (*JIMS 2*, pp. 218-219).

In the translation of Mahāvīra's verse $136\frac{1}{2}$, Datta and Singh (1962) have emended the (Mahāvīra) text by changing 'sāgra' to 'khāgra,' which seems quite logical. According to their translation, the mutual division is to be continued until the last remainder is 0, the optional multiplier t (mati) is taken to be zero, and the first quotient is to be discarded (*Part II*, p. 103).

Based on the translation of verse $136\frac{1}{2}$ by the above authors, Mahāvīra's chains seem to look like:

$$\begin{array}{l} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_{2n-1} \\ q_{2n} \\ t=0 \\ c \end{array} \qquad \text{or} \qquad \begin{array}{l} q_1 \\ q_2 \\ q_3 \\ \vdots \\ q_{2n-1} \\ t=0 \\ -c \end{array}$$

according as the number of quotients (rejecting the first) is even (= $2n$) or odd (= $2n$–1). Compare these chains with those of Āryabhaṭa I (given by Datta's first translation). The above is a special case of Āryabhaṭa I's chain when the mutual division is carried until the remainder is 0. Also one can compare Mahāvīra's above chains with those of Āryabhaṭa II and Bhāskara II.

Raṅgācārya (1912, English translation, pp. 126-129) and Jain (1963, *JJG 12*, Hindi translation, p. 124) do not emend the text of Mahāvīra's verse $136\frac{1}{2}$. According to their translations, the mutual division is to be carried on until the divisor and the remainder become equal; and so on. We will not go into further details concerning these translations.

In his verses $115\frac{1}{2}$ and $136\frac{1}{2}$, Mahāvīra also includes the rule for the solution of simultaneous indeterminate equations of the first degree.

M. *Āryabhaṭa II's Rules.*

Āryabhaṭa II dealt with kuṭṭaka in chapter eighteen of his *Mahāsiddhānta* (see e.g. Dvivedi, 1910, verses 1-66, pp. 224-245). He made a few innovations, as follows:

(1). To simplify the given equations $by = ax \pm c$, Āryabhaṭa II suggested reductions of a, b, c in this way (*MS* XVIII, 1-2):

भाज्यक्षेपच्छेदा यथोदिताः संस्थिताः क-विधिरेषः।
ते च करण्या भक्ता दृढाभिधाना अयं ख-विधिः ॥१॥

भाज्यक्षेपौ ग-विधिः क्षेपच्छेदौ यदा तदा घ-विधिः।
भाज्यक्षेपौ क्षेपच्छेदौ ङ-विधिर्विभिन्नकरणीभ्याम् ॥२॥

That is, divide by their greatest common factor (i) the dividend, additive (interpolator) and divisor; or (ii) the dividend and additive; or (iii) the additive and divisor. Or (iv) divide the dividend and additive, and then the (reduced) additive and divisor, by their respective greatest common factors.

(2). To solve $by = ax \pm c$, $c \neq 0$, Āryabhaṭa II firstly solves $by = ax \pm 1$ (as did Bhāskara I in his *MB* I, 45a-46b and *MB* I, 51). Then, if $x = \alpha$, $y = \beta$ is a solution of $by = ax \pm 1$, the corresponding solution of $by = ax \pm c$ is given by the residues of $c\alpha$ and

$c\beta$ on division by b and a respectively (in case the quotients on division are *equal*; *MS* XVIII, 7b-8a):

कुट्टौ स्वक्षेपहतावूर्ध्वाधःस्थौ क्रमाद्धक्तौ।
निजभाज्यच्छेदाभ्यां फलगुणकौ शेषकौ भवतः ||७b-८a||

What this *essentially* means is that if $x = \alpha$ and $y = \beta$ is a solution of $by = ax \pm 1$, and if $c\alpha = bq + r_1$ and $c\beta = aq + r_2$ $(q, r_1, r_2 \geq 0)$, then $x = r_1$ and $y = r_2$ is a solution of $by = ax \pm c$. To prove this statement (Āryabhaṭa II does not prove it), one can first observe that $x = c\alpha$ and $y = c\beta$ is a solution of $by = ax \pm c$; i.e. $b\,(c\beta) = a\,(c\alpha) \pm c$. Then

$$b\,(aq + r_2) = a\,(bq + r_1) \pm c$$

$$\Leftrightarrow b\,r_2 = a\,r_1 \pm c$$

$$\Leftrightarrow x = r_1 \text{ and } y = r_2$$

is a solution of $by = ax \pm c$. The minimum solution will be obtained by taking the largest non-negative (common) quotient q.

The innovation made by Āryabhaṭa II is: If the quotients in the (above) division of $c\alpha$ and $c\beta$ by b and a respectively are *not equal*, then only the "multiplier" should be accepted but the "quotient" discarded, when the additive is positive; and vice-versa if the additive is negative (*MS* XVIII, 15-16; see also our commentary under verse 52c):

अन्यत्र प्रश्नोक्तावथ तत्सम्बधजे यदा लब्धी।
न समे गुण एव तदा ग्राह्यो हेयं फलं धनक्षेपे ||१५||

फलमृणसंज्ञे ग्राह्यं हेयो गुणको गुणात् फलोत्पत्तिम्।
वक्ष्ये फलतोऽपि तथा सर्वत्र समां गुणोत्पत्तिम् ||१६||

The earlier writers e.g. Āryabhaṭa I and Bhāskara I etc. seem to assume that equal quotients need to be taken when finding the minimum values of x and y but they make no explicit mention of it.

(3). To find the "quotient" when only the "multiplier" x is accepted (i.e. when c is positive), one should calculate

$$\frac{ax+c}{b}$$

(by substituting for x the accepted value). Similarly, to find the "multiplier" when c is negative and the "quotient" y is accepted, one should calculate

$$\frac{by-c}{a}$$

(by substituting for y the accepted value). (See *MS* XVIII, 17-18 below):

गुणपृच्छाभाज्यवधं पृच्छाक्षेपेण संस्कृतं विभजेत्।
प्रश्नोक्तच्छेदेन स्पष्टं लब्धं फलं भवति ॥१७॥

प्रश्नच्छित्फलघातं व्यस्तात्व्यक्षेपकेण संस्कृत्य।
प्रश्नोदितेन पृच्छाभाज्येन भजेद् गुणो भवेल्लब्धम् ॥१८॥

(4). To solve $by = ax + 1$, where $(a, b) = 1$, Āryabhaṭa II carries the mutual division until the remainder is *unity*. He warns that if the remainder is zero, the questioner does not know the method of kuṭṭaka (*MS* XVIII, 3-4):

ऐषां टा-शेषं स्याद्वल्लीकरणेऽत्र तैः सिद्धिः।
ना-शेषं चेदिह तत् कुट्टाकारं न पृच्छको वेत्ति ॥३॥

भाज्यहारावन्योन्यं विभजेत् टा-शेषकं भवेद्यावत् ।
सा वल्ली तेन हतेऽन्त्येनोर्ध्वे कान्विते स्फुटा वल्ली ॥४॥

(5). While Āryabhaṭa I, Bhāskara I and Mahāvīra reject the first quotient, Āryabhaṭa II uses it. He includes it in the chain of quotients. This is clear in view of his *MS* XVIII, verses 1b, 4a (stated above), and verse 66 (stated below):

का-शेषे नो करणी फलान्यधोऽधः क्रमेण धार्याणि ।
करणीजं नो धार्यं वल्ली सा मध्यमा स्व-विधौ ॥६६॥

In verse 66, Āryabhaṭa II specifically mentions that the (last) *quotient* which is obtained when the divisor is 1 (and the corresponding remainder is 0), is not to be placed in the chain of quotients.

Also Āryabhaṭa II seems to place the additive 1 underneath all the quotients (see *MS* XVIII, 4b). It seems that Āryabhaṭa II takes the mati to be zero as is also stated by Ganguli (1931 – 32, *JIMS/NQ 19*, p. 153). Though Āryabhaṭa II does not mention this fact, it can be easily gleaned from the operation given in his verse 4b. (Also see our rationale under sub-section O., Bhāskara II's Treatment of the Kuṭṭaka.)

(6). Āryabhaṭa II also considers particular cases of $by = ax + c$ as follows (see *MS* XVIII, 19 below):

(i) If $c > 0$ and b divides c, then the (minimum) solution is $x = 0,\ y = \frac{c}{b}$;

(ii) if the additive is negative (i.e. c is subtracted, so that the equation is $by = ax - c$) and b divides c, then $x = 0,\ y = \frac{c}{b}$ is not a solution;

(iii) if $c = 0$, then $x = 0, y = 0$.

That is (*MS* XVIII, 19),

स्वक्षेपे छेदहृते निरग्रके ना गुणः फलं लब्धिः ।
एवमृणक्षेपे नो ना-क्षेपे फलगुणौ नौ स्तः ॥१९॥

Also see our commentary on verse 56a-b.

(7). Considering the chain of Āryabhaṭa II for $by = ax + 1$ and that of Āryabhaṭa I for $by = ax + c$ $(c > 0)$, we have (from the tables in section 4.H.):

Āryabhaṭa II No. of Quotients including the first = $2n$	Āryabhaṭa I No. of Quotients excluding the first = $2n$ (Datta's second translation or Sub-case I.i)
q	
q_1	q_1
.	.
.	.
.	.
q_{2n-1}	q_{2n-1}
1	$q_{2n}t + c$
	t

Putting $t = 0$ and $c = 1$ in Āryabhaṭa I's chain, we essentially get the chain of Āryabhaṭa II. Āryabhaṭa II uses $t = 0$ but does not write it under the 1. Note that the last quotient (q_{2n}) is included *implicitly* in Āryabhaṭa I's chain, but the last quotient (q_{2n-1}) is included *explicitly* in Āryabhaṭa II's chain.

When the number of quotients considered is odd = $2n-1$ (including the first quotient in case of Āryabhaṭa II, and excluding it in case of Āryabhaṭa I, the last quotient q_{2n-1} being included *implicitly* in Āryabhaṭa I's chain, though the last quotient q_{2n-2} is included *explicitly* in Āryabhaṭa II's chain), the corresponding chains for the above indeterminate equations are:

Āryabhaṭa II	Āryabhaṭa I (Datta's second translation or Sub-case I.ii)
q	
q_1	q_1
.	.
.	.
.	.
q_{2n-2}	q_{2n-2}
1	$q_{2n-1}t' - c$
	t'

In the case when the number of quotients is odd, the chain of Āryabhaṭa II gives a solution of $by = ax - 1$. Therefore, in order to get a solution of $by = ax + 1$, he subtracts the values of y and x (which are obtained using the above chain) from their "takṣaṇas" a and b respectively (see *MS* XVIII, 14, below). Āryabhaṭa I's chain does not require such a subtraction.

The term "takṣaṇa" (divisor) is used for a and b, after the complete reduction of the chain of quotients is made resulting in a pair of numbers, x and y. In order to get smaller values for x and y, the obtained value of y is divided by a, the obtained value of x is divided by b, and the respective *remainders* are taken. Generally, in various operations after reduction of the chain, a and b are referred to as the takṣaṇas.

(8). In Āryabhaṭa I's equation $by = ax + c$, c is never less than 0. But Āryabhaṭa II obtains a solution for $by = ax - c$ ($c > 0$) when the number of quotients (including the first) is *even*, from a solution of $by = ax + c$ by subtracting the values of x and y from their respective takṣaṇas. But to find a solution for $by = ax - c$ ($c > 0$) when the number of quotients (including the first) is *odd*, Āryabhaṭa II simply finds a solution of $by = ax + c$.

No subtraction is needed in this case. Thus the method of solution in case of an odd chain and negative additive is analogous to that in case of an even chain and positive additive because no subtraction from the takṣaṇas is required (*MS* XVIII, 13). On the other hand, the method of solution in case of an even chain and negative additive is analogous to that in case of an odd chain and positive additive because in this case subtraction from the takṣaṇas is required (*MS* XVIII, 14):

एवमभीष्टविधिभवौ फलगुणकौ प्रस्फुटौ धनक्षेपे।
समवल्ल्यां विषमायामृणसंज्ञे क्षेपके स्याताम् ॥१३॥

समवल्ल्यामृणसंज्ञे धनसंज्ञे वा विषमवल्ल्याम्।
स्वविधौ फलगुणहीनौ सुदृढौ भाज्यच्छिदौ फलगुणौ स्तः ॥१४॥

N. *Śrīpati's Rules.*

Śrīpati treats pulverizer in the fourteenth chapter of his *Siddhāntaśekhara* (see e.g. Miśra, 1947, *Part II*, verses 22-31, pp. 118-127). Śrīpati's rules aim at the solutions of the problems of the first and second kinds. His rules pertaining to the problems of the first kind are similar to those of Brahmagupta. One can see this by comparing Śrīpati's *SSE* XIV, 28-29:

अल्पाग्रहृत्या बृहदग्रहारं
छित्त्वाऽवशेषं विभजेन्मिथोऽतः।
अग्रान्तरं तत्र युतिं प्रकल्प्य
प्राग्वद्गुणः स्यादधिकाग्रहारः ॥२८॥

तेनाहतः स्वाग्रयुतस्तदग्रं
छेदाहतिः सा द्वियुगं तथाऽग्रम्।
युगाद्व्यतीतं ग्रहयोः प्रदिष्टं
त्र्यादिग्रहाणामपि कुट्टकेन ॥२९॥

with Brahmagupta's *BSS* XVIII, 3-6 (3-5 have been stated above in the sub-section K.):

छेदवधस्य द्वियुगं छेदवधो युगगतं द्वयोरग्रम्।
कुट्टाकारेणैवं त्र्यादिग्रहयुगगतानयनम् ॥६॥

Brahmagupta, in turn, follows Āryabhaṭa I's rule.

Śrīpati's rule (see *SSE* XIV, 22-27) pertaining to the solution of

$$\frac{ax \pm c}{b} = y \ (c > 0)$$

is similar to that of Bhāskara I (i.e. the first quotient in the division of the dividend a by the divisor b is zero and is discarded; the final numbers obtained upon reduction of the chain are x and y, in order). However, Śrīpati (*SSE* XIV, 22d), like Āryabhaṭa II (*MS* XVIII, 3-4), states that the mutual division is to be carried until the (last) remainder is *one*, while Bhāskara I does not have this requirement. Furthermore, unlike Bhāskara I, Śrīpati allows the number of quotients in the mutual division to be even as well as odd. He says that when the number of quotients (discarding the first) is odd, the negative additive must be made positive and conversely, the positive additive must be made negative (see *SSE* XIV, 24a-b).

More precisely, Śrīpati's rule pertaining to the solution of a problem of the second kind is the following (*SSE* XIV, 22a-26b):

विभाज्यहारं च युतिं निजच्छिदा
समेन वाऽऽदावपवर्त्त्य सम्भवे।
विभाज्यहारौ विभजेत्परस्परं
तथा यथा शेषकमेव रूपकम् ॥२२॥

फलान्यधोऽधः क्रमशो निवेशयेन्
मतिं तथाऽधस्तदधश्च तत्फलम्।
इदं हतं केन युतं विवर्जितं
हरेण भक्तं सदहो निरग्रकम् ॥२३॥

समेषु लब्धेष्वसमेष्वृणं धनं
धनं त्वृणं क्षेपमुशन्ति तद्विदः।
मतिं विचिन्त्येति तदूर्ध्वगं तया
निहत्य लब्धं च तथा नियोजयेत् ॥२४॥

पुनः पुनः कर्म यथोत्क्रमादिदं
यदा तु राशिद्वयमेव जायते।
हरेण भक्तः प्रथमो गुणो भवेत्
फलं द्वितीयं तु विभाज्यराशिना ॥२५॥

विभाज्यहत्योरपवर्त्तनं यदा।
भवेद्युतौ नैव खिलं हि तत्तदा ॥२६a-b॥

We give the following translation of these verses (22a-26b):

If possible, first divide the dividend, divisor and additive by their common divisor. (Then) divide the dividend and divisor mutually so that the (last) remainder is unity. Place the quotients one below the other, in order (so that they form a chain); and underneath them the mati (t), and underneath that the (corresponding new) quotient (Q). (The mati is obtained as follows:) the (last remainder) multiplied by some number (mati) and added (by the positive additive) or subtracted (by the negative additive) becomes remainderless when (the resulting sum or difference is) divided by the divisor (corresponding to the last remainder, that is, by the last but one remainder. It is this division which yields the new quotient Q). (Such is the procedure) when the (number of) quotients (discarding the first) be even; (but) when the (number of) quotients (discarding the first) be odd, the negative additive (must be made) positive, but the positive additive (must be made) negative; the learned in this (subject) say so. Having chosen the mati in this way, and having multiplied by that (mati), the (number) above it, add the quotient (Q to the product obtained) in that manner. (Do) such an operation upwards again and again (that is, reduce the chain upwards). When just a pair of numbers is produced, the first (i.e. the upper number) divided by the divisor yields the multiplier (x), but the second (i.e. the lower) divided by the dividend yields the quotient (y). If (in a problem), a reducer (i.e. common divisor) of the (given) dividend and divisor is not a divisor of the additive, then indeed that (problem) is faulty.

Notice that Śrīpati's verses, (like those of Bhāskara I,) do not state that the first quotient from the mutual division of the dividend and the divisor is zero, and this quotient is to be discarded; though this is indicated by the fact that the pair of numbers in the reduced chain yields x and y in order.

Thus, Śrīpati's chains for

$$\frac{ax+c}{b} = y \ (c > 0),$$

when the number of quotients (discarding the first) is even = $2n$ or odd = $2n-1$, seem to be the following:

q_1		q_1

$$\begin{array}{lcl} q_1 & & q_1 \\ q_2 & & q_2 \\ \vdots & \text{or} & \vdots \\ q_{2n-1} & & q_{2n-1} \\ q_{2n} & & t \\ t & & Q = \dfrac{1 \cdot t - c}{r_{2n-1}} \\ Q = \dfrac{1 \cdot t + c}{r_{2n}} & & \end{array}$$

because in the mutual division the last remainders (r_{2n+1} and r_{2n}, respectively) are 1.

In order to obtain Śrīpati's chains for

$$\frac{ax - c}{b} = y \ (c > 0),$$

replacing $+c$ by $-c$ and $-c$ by $+c$ in the above chains, we get:

$$\begin{array}{lcl} q_1 & & q_1 \\ q_2 & & q_2 \\ \vdots & \text{or} & \vdots \\ q_{2n-1} & & q_{2n-1} \\ q_{2n} & & t \\ t & & Q = \dfrac{1 \cdot t + c}{r_{2n-1}} \\ Q = \dfrac{1 \cdot t - c}{r_{2n}} & & \end{array}$$

according as the number of quotients (discarding the first) is even = $2n$ or odd = $2n-1$.

Observe that Śrīpati's chains for

$$\frac{ax \pm c}{b} = y \ (c > 0),$$

when the number of quotients discarding the first is *even*, are identical with those of Bhāskara I except that (in case of Śrīpati) the last remainder r_{2n+1} is 1. Moreover, Śrīpati's chains for

$$\frac{ax + c}{b} = y \ (c > 0),$$

(when the number of quotients, discarding the first, is either *even* or *odd*), are identical with those of Āryabhaṭa I (Datta's first translation) except that (in case of Śrīpati) the last remainders (r_{2n+1} and r_{2n}, respectively) are 1. (See Āryabhaṭa I's unreduced columns under the Sub-cases (II.i) and (II.ii) given in the section 4.H. of this chapter.) Recall that Āryabhaṭa I does not allow a negative additive (see section 4.G., the Rationale of the Method of kuṭṭaka).

There are some similarities between the verses of Śrīpati and Bhāskara II, as will be seen later in our textual commentary (sub-section S. below).

O. *Bhāskara II's Treatment of the Kuṭṭaka—Comparison With Āryabhaṭa II's Treatment.*

As mentioned earlier, Bhāskara II solves mainly problems of the second kind using the method of kuṭṭaka in his treatises *Bījagaṇita* and *Līlāvatī*. Verses 46b-67d of the *Bījagaṇita* are devoted to kuṭṭaka (see *Text Alpha*). In verses 46b-51b, Bhāskara II describes his method of kuṭṭaka. The specifics of each verse will be treated in the textual commentary. In this sub-section, we propose to examine the treatment of Bhāskara II in relation to that of Āryabhaṭa II, whom he seems to have followed in many respects.

The significant points of similarity between the rules of Bhāskara II and Āryabhaṭa II include:

(1). Simplifications of the given equation, $by = ax + c$, by performing some preliminary operations. (Compare Bhāskara's *BG*, 46b-47b, p. 26 with Āryabhaṭa II's *MS* XVIII, 1-2; compare *BG*, 51c-52b, p. 27 with *MS* XVIII, 2a; and compare *BG*, 58c-59b, p. 29 with *MS* XVIII, 2b.)

(2). Discussion of particular cases (e.g. when b divides c etc.). (Compare Bhāskara's *BG*, 56a-b, p. 27 with Āryabhaṭa II's *MS* XVIII, 19.)

(3). Stopping the mutual division of a by b when the remainder is unity. (Compare Bhāskara's *BG*, 48c-d, p. 26 with Āryabhaṭa II's *MS* XVIII, 4a.)

(4). Not discarding the first quotient in the mutual division, and including it in the column of quotients. (Compare Bhāskara's *BG*, 49a-b, p. 26 with Āryabhaṭa II's *MS* XVIII, 66.)

(5). Choosing mati as 0 (see (10) below).

(6). Placing the additive under all the quotients. (Only Bhāskara puts mati under the additive). (Compare Bhāskara's *BG*, 49a-b, p. 26 with Āryabhaṭa II's *MS* XVIII, 4b.)

(7). Obtaining the least solution by dividing the upper and lower numbers in the reduced column by a and b respectively (taking equal quotients, see under difference between the treatments of Bhāskara II and Āryabhaṭa II); the remainders obtained from these divisions being the least values of y and x, in succession. (Āryabhaṭa II calls these upper and lower numbers in the reduced column as the "upper kuṭṭa" and the "lower kuṭṭa" respectively, and multiplies them by the additive, c. Then he divides the products by a and b respectively). (Compare Bhāskara's *BG*, 50a-b, p. 26 with Āryabhaṭa II's *MS* XVIII, 7a-8a.)

(8). Giving identical rules for finding a solution in case of an odd chain (i.e. odd number of quotients) and positive additive, as well as in case of an even chain and negative additive. (Compare Bhāskara's *BG*, 50c-51b, pp. 26-27 and *BG*, 53b, p. 27 with Āryabhaṭa II's *MS* XVIII, 13-14.)

(9). Giving similar rules for finding various (other) values of x and y (i.e. for the general solution). (Compare Bhāskara's *BG*, 57a-b, p. 27 with Āryabhaṭa II's *MS* XVIII, 20.)

(10). Giving a similar rule for solving a *saṃśliṣṭa* (conjunct) kuṭṭaka. (Compare Bhāskara's *BG*, 66a-d, p. 39 with Āryabhaṭa II's *MS* XVIII, 48b-49a.)

That mati is taken to be zero, is clear from our rationale. In sub-case (I.i), when the number of quotients is *even* (excluding the first quotient and including the last, in the case of Āryabhaṭa I (see section 4.H. of this chapter); or equivalently, including the first quotient and excluding the last which corresponds to remainder 0, in the cases of Āryabhaṭa II and

Bhāskara II), we have $y_{n+1} = c$ and, letting (mati) $x_n = 0$, we get $y_n = c$. Āryabhaṭa II and Bhāskara II ignore y_{n+1} and the last quotient q_{2n}. Similarly, in sub-case (I.ii), when the number of quotients is *odd* (reasoning as before), $x_n = -c$. Taking (mati) $y_n = 0$, we get $x_{n-1} = -c$. Āryabhaṭa II and Bhāskara II ignore x_n and the last quotient q_{2n-1}. Thus mati 0 is written under the additive by Bhāskara II (but ignored by Āryabhaṭa II). Of course, Bhāskara II does not write $-c$ in his chain, but writes instead c. Therefore, he obtains a solution to $by = ax - c$ (and not to $by = ax + c$), which he handles by subtracting from takṣaṇas. (See our commentary on verse 50c-51b). Recall that Āryabhaṭa II writes +1 (and not −1) in his chain when the number of quotients is odd, and subtracts from takṣaṇas, to get a solution for $by = ax + 1$ (see (7) under sub-section M. of this chapter).

The chains of quotients of Āryabhaṭa I for $by = ax + c$ ($c > 0$), of Āryabhaṭa II for $by = ax + 1$, and of Bhāskara II for $by = ax + c$ ($c > 0$) may be displayed as follows. The first quotient is excluded from the chain of Āryabhaṭa I, but included in the chains of Āryabhaṭa II and Bhāskara II.

First, we suppose the number of quotients to be even ($= 2n$). The quotient q_{2n} corresponding to remainder = 0 is to be included (explicitly according to Datta's first translation, though implicitly according to Datta's second translation) in Āryabhaṭa I's chain, but it is to be excluded from the chains of Āryabhaṭa II and Bhāskara II because these mathematicians do not carry on the division after the remainder 1 and the quotient q_{2n-1} have been obtained.

Āryabhaṭa I (Datta's first translation)	Āryabhaṭa I (Datta's second translation)	Āryabhaṭa II	Bhāskara II
		q	q
q_1	q_1	q_1	q_1
q_2	q_2	q_2	q_2
.	.	.	.
.	.	.	.
.	.	.	.
q_{2n-1}	q_{2n-1}	q_{2n-1}	q_{2n-1}
q_{2n}	$q_{2n}t+c$	1	c
t	t		0
$Q=\frac{r_{2n+1}t+c}{r_{2n}}$			

Note that in the cases of Āryabhaṭa II and Bhāskara II, since q_{2n-1} is the last quotient, q_{2n} does not exist (or it is ignored). Also then $r_{2n+1} = 0$, $r_{2n} = 1$ (and $q_{2n} = r_{2n-1}$, if calculated). Thus the chains of Āryabhaṭa II and Bhāskara II hold close similarities with the chain of Āryabhaṭa I (corresponding to Datta's *second* translation and not the first), when $t = 0$. Notice the omission by Āryabhaṭa II of the 0 under the 1.

Next we suppose the number of quotients to be odd $(= 2n-1)$. Then proceeding as in the previous case, the corresponding chains are:

Āryabhaṭa I (Datta's first translation)	Āryabhaṭa I (Datta's second translation)	Āryabhaṭa II	Bhāskara II
		q	q
q_1	q_1	q_1	q_1
q_2	q_2	q_2	q_2
.	.	.	.
.	.	.	.
.	.	.	.
q_{2n-2}	q_{2n-2}	q_{2n-2}	q_{2n-2}
q_{2n-1}	$q_{2n-1}t' - c$	1	c
t'	t'		0
$Q = \dfrac{r_{2n}t' - c}{r_{2n-1}}$			

As before, taking $t' = 0$ in Āryabhaṭa I's chain (Datta's second translation or Case I of the rationale), we *essentially* get the chains of Āryabhaṭa II and Bhāskara II but their additives are positive (i.e. 1 and c, respectively), whereas the additive of Āryabhaṭa I is negative (i.e. $-c$). That is why Āryabhaṭa II and Bhāskara II require subtraction from the takṣaṇas (see our textual commentary on Bhāskara II's verse 50c-51b).

The significant points of difference between the treatments of Āryabhaṭa II and Bhāskara II include:

(1). After reduction, the chain of quotients of Bhāskara gives a solution of $by = ax + c$ or $by = ax - c$, while that of Āryabhaṭa II gives a solution of $by = ax + 1$ or $by = ax - 1$.

(2). Āryabhaṭa II's chain of quotients has 1 in place of Bhāskara's c, and omits the mati 0.

(3). Bhāskara does not reject (Ganguli, 1931 – 32) the particular case when the additive is negative and b divides c. Thus, if $by = ax - c$ ($c > 0$) where $c = mb$, then

$$y = -m = -\frac{c}{b} \text{ and } x = 0.$$

Bhāskara takes $y = -m + at$ and $x = 0 + bt$; and chooses positive integral values of t such that $y > 0$ (*JIMS/NQ 19*, p. 160).

Note that this is a consequence of Bhāskara's verses 56a-b and 54b (see our commentary on verse 56a-b).

(4). To find the least values of y and x from the pair of numbers (in the reduced chain), Bhāskara takes *equal* quotients in the division of the upper and lower numbers by a and b, respectively. If the divisions do not yield equal quotients, Bhāskara chooses the equal quotient to be the *smaller* of the two quotients. Then the remainders are the least values of y and x, in order. But in this case, the method of Āryabhaṭa II is slightly different (see under verses 52c and 61b-62a, our textual commentary).

It is pertinent to add that here the word "remainders," after one has taken equal quotients on division by a and b, has a slightly generalized meaning. For example, 10 divided by 3 with quotient 2 leaves a "remainder" of 4. (Thus here the remainder is greater than the divisor.)

(5). Bhāskara II discusses the cases when the dividend or divisor is negative, but Āryabhaṭa II does not.

In light of the far greater number of points of similarity between the treatments of kuṭṭaka by Āryabhaṭa II and Bhāskara II, it is fair to say that the latter's treatment was anticipated by the former, who in turn built upon Āryabhaṭa I.

P. *Summary of the Major Innovations Pertaining to Kuṭṭaka.*

The sequence of innovations concerning kuṭṭaka can be described as follows:

The *Āryabhaṭīya* of Āryabhaṭa I is the earliest *known* extant work where the rule of kuṭṭaka is written (verses number 32-33, see e.g. Shukla & Sarma, 1976, p. 74).

Bhāskara I and some other commentators explain these verses. Bhāskara I also cites some problems, provides their solutions and makes his own innovations as well. The problems discussed by Bhāskara I are both non-astronomical and astronomical. Both Āryabhaṭa I and Bhāskara I ignore the first quotient obtained from the mutual division (of the divisors corresponding to the greater and smaller remainders, or dividend and divisor, according as the problem to be solved is of the first or second kind).

Brahmagupta basically follows Āryabhaṭa I. His treatment has 27 verses and is astronomical in content as is that of Bhāskara I for the most part (see e.g. Dvivedin, 1902, verses 3-29, pp. 294-308).

The problems formed by Mahāvīra are related mainly to distribution. Mahāvīra discards the first quotient as do his predecessors. He characterizes his rules and problems as 'vallikā', 'viṣama', 'sakala', 'suvarṇa' etc. kuṭṭīkāras. His principal rules are enunciated in verses $115\frac{1}{2}$ and $136\frac{1}{2}$ (see e.g. Raṅgācārya, 1912, pp. 80, 83). Mahāvīra's treatment is non-astronomical but covers a wide variety of daily-life problems.

Āryabhaṭa II gives a comprehensive treatment of kuṭṭaka in 66 verses (see e.g. Dvivedi, 1910, verses 1-66, pp. 224-245). In the first twenty verses, he gives rules and in the remaining verses, he discusses how to solve astronomical problems. Āryabhaṭa II carries the mutual division until the remainder is one, does not discard the first quotient and chooses mati to be zero.

Śrīpati treats kuṭṭaka in 10 verses (see Miśra, 1947, *Part II*, verses 22-31, pp. 118-127). He follows both Bhāskara I and Brahmagupta. But in his rule concerning the solution of a problem of the second kind, Śrīpati clearly states that the mutual division is to be carried until the (last) remainder is one, as does his predecessor Āryabhaṭa II.

Bhāskara II gives a rigorous treatment of kuṭṭaka in his *Līlāvatī* and *Bījagaṇita*. Like his predecessors Āryabhaṭa II and Śrīpati, Bhāskara II carries the mutual division until the remainder is one. Furthermore, following Āryabhaṭa II, Bhāskara II does not discard the first quotient and includes it in his chain. Also, Bhāskara II chooses mati to be zero as did Āryabhaṭa II (and perhaps Mahāvīra).

Thus Bhāskara II seems to follow Āryabhaṭa II, as far as his rules are concerned, whereas the latter is essentially following Āryabhaṭa I. But Bhāskara II displays his skill by improving, at places, upon the rules given by his predecessors. Though he borrows from Āryabhaṭa II, he also supplements from Śrīpati (as will be seen in the textual commentary). He considered the cases when the dividend or divisor is negative, which none of his predecessors seems to have done. In addition, to make his exposition lucid, easy and more interesting, Bhāskara II added non-astronomical examples (of the second kind) and their solutions. Sometimes, the solutions contain brief comments on the rules which are being applied.

On the other hand, Bhāskara II is followed by Nārāyaṇa. One can notice the similarities between Bhāskara's verses 46b-47b, 47c-48b, 63c-64b and Nārāyaṇa's corresponding verses 53, 54, 64 (see Shukla, 1970, *Part I*, pp. 29, 33). But Nārāyaṇa makes some innovations as well. For example, compare Nārāyaṇa's verse 62 (Shukla, 1970, *Part I*, p. 32) with Bhāskara's verse 54a (see our commentary on verse 54a and remark after our commentary on verse 60c-61a).

These are the principal innovations of the major contributors within the realm of kuṭṭaka. Of course, various commentators have provided valuable explanations to the often obscure verses of those contributors, in addition to providing unique examples of their own at some places.

Q. *Kuṭṭaka and Continued Fractions.*

A few modern authors such as Bag (1977, *IJHS 12*, pp. 1, 10-11) and Majumdar (1978, *IJHS 13*, p. 11; 1981a, *IJHS 16*, p. 116; 1983, *IJHS 18*, p. 204) are of the view that the general methods of solution of the indeterminate equations of the kind $by = ax \pm c$, given by Āryabhaṭa I, Bhāskara I and some other Indian mathematicians, involve a knowledge of continued fractions either explicitly or implicitly.

Bag (1977) defines the continued fraction as "a process of converting a fraction into a continued division," which appears to have arisen in the context of finding the approximate square-roots of non-square numbers which involves the use of "excess" or "defect" (*IJHS 12*, p. 1).

The method of continued fraction can be demonstrated using our rationale of the method of kuṭṭaka in which the equations obtained on division of a by b were the following:

$$a = bq + r_1$$
$$b = r_1 q_1 + r_2$$
$$r_1 = r_2 q_2 + r_3$$
$$r_2 = r_3 q_3 + r_4$$
$$\cdot \quad \cdot \quad \cdot$$
$$\cdot \quad \cdot \quad \cdot$$
$$\cdot \quad \cdot \quad \cdot$$
$$r_{m-2} = r_{m-1} q_{m-1} + r_m$$
$$r_{m-1} = r_m q_m + r_{m+1}.$$

Therefore,

$$\frac{a}{b} = q + \frac{r_1}{b} \qquad \text{(from the first equation)}$$

$$= q + \frac{1}{\left(\dfrac{b}{r_1}\right)} = q + \frac{1}{q_1 + \dfrac{r_2}{r_1}} \qquad \text{(from the second equation)}$$

$$= q + \frac{1}{q_1 + \dfrac{1}{\left(\dfrac{r_1}{r_2}\right)}}$$

$$= q + \frac{1}{q_1 + \dfrac{1}{\left(q_2 + \dfrac{r_3}{r_2}\right)}} \qquad \text{(from the third equation)}$$

If $r_{m+1} = 0$, then $\frac{r_{m-1}}{r_m} = q_m$.

Therefore, using the notation of continued fractions,

$$\frac{a}{b} = q + \frac{1}{q_1 +}\ \frac{1}{q_2 +}\ \frac{1}{q_3 +} \cdots \frac{1}{q_{m-1} +}\ \frac{1}{q_m}.$$

If

$$\frac{P_1}{Q_1},\ \frac{P_2}{Q_2},\ \frac{P_3}{Q_3},\ \ldots,\ \frac{P_{m+1}}{Q_{m+1}}$$

be the successive approximations of $\frac{a}{b}$, then

$$* \begin{cases} \frac{P_1}{Q_1} = \frac{q}{1} \\ \frac{P_2}{Q_2} = q + \frac{1}{q_1} = \frac{qq_1 + 1}{q_1} \\ \frac{P_3}{Q_3} = q + \frac{1}{q_1 +}\ \frac{1}{q_2} = \frac{q(q_1q_2 + 1) + q_2}{q_1q_2 + 1} \\ \frac{P_4}{Q_4} = q + \frac{1}{q_1 +}\ \frac{1}{q_2 +}\ \frac{1}{q_3} = \frac{q\left[q_1\left(q_2q_3 + 1\right) + q_3\right] + q_2q_3 + 1}{q_1\left(q_2q_3 + 1\right) + q_3} \\ \text{and so on.} \end{cases}$$

Ganguli (1931 – 32) explains that Āryabhaṭa II's solution of $by = ax \pm c$ is derived from the solution of $by = ax \pm 1$. Also the solution of $by = ax \pm 1$ depends on the chain

q
q_1
.
.
.
q_{m-2}
q_{m-1}
1

The process of deriving the roots x and y from this chain is the same as the process of simplifying the continued fraction

$$q+\frac{1}{q_1+}\ \frac{1}{q_2+}\ \frac{1}{q_3+}\cdots\frac{1}{q_{m-2}+}\ \frac{1}{q_{m-1}}\ \text{(see * above).}$$

When the quotient q_{m-1} gives the remainder 1 (i.e. $r_m = 1$), then the approximation

$$\frac{P_m}{Q_m} \text{ of } \frac{a}{b}$$

is such that $P_m = y$ and $Q_m = x$. Therefore $b \cdot P_m = a \cdot Q_m \pm 1$, where the sign is taken according as

$$\frac{P_m}{Q_m}$$

is of even or odd order (*JIMS/NQ 19*, p. 158).

The mathematical equivalence of this continued fraction method with kuṭṭaka is clear. It is not impossible to draw the conclusion that the Indian medieval mathematical authors had the knowledge of continued fractions (though no *explicit* mention of continued fractions seems to have been made in any of the original Sanskrit texts). It has recently been hypothesized by Hayashi, Kusuba and Yano (1990), on what seems to be persuasive grounds, that Mādhava (fl. 1400 A.D.) of Saṅgamagrāma used continued fractions in order to derive the corrections pertaining to his series for the circumference of a circle (*Centaurus 33*, pp. 149-150, 164-171).

R. *A Comparison Between Bhāskara's Treatments of Kuṭṭaka in his Līlāvatī and Bījagaṇita.*

The two treatments are almost the same except for a few notable differences. In the section on Kuṭṭaka in his *Bījagaṇita*, Bhāskara gives all the rules first and then the examples; but in the *Līlāvatī*, there exist examples in between the rules. Also, the cases of negative divisor or negative dividend have been treated in the algebraical treatise only. In general, the working of examples in the *Bījagaṇita* is a slightly condensed and modified version of that in the *Līlāvatī*. There exist emendations in the solutions at a few places, or replacements of a word or two of the *Līlāvatī* in the *Bījagaṇita*. The following examples may support the above conclusions:

(i). The verse 55 of the *Bījagaṇita* which is:

अथवा भागहारेण तष्टयोः क्षेपभाज्ययोः ।
गुणः प्राग्वत्ततो लब्धिर्भाज्याद्धतयुतोद्धृतात् ॥५५॥

is missing in the *Līlāvatī* (see Āpaṭe, 1937, *L II*, *ASS 107*). This verse contains a rule which states: When the dividend and the additive are divided by the divisor, how to find the quotient directly from the given equation when the multiplier is already known.

(ii). Similarly, the verse 60c-61a of the *Bījagaṇita* which is:

अष्टादश हताः केन दशाढ्या वा दशोनिताः ।
शुद्धभागं प्रयच्छन्ति क्षयगैकादशोद्धृताः ॥६०c-६१a॥

does not appear in the *Līlāvatī*. It contains a negative divisor.

(iii). Furthermore, Bhāskara's *L II*, 251, *ASS 107*, p. 261 is:

यद्गुणा गणक षष्टिरन्विता वर्जिता च दशभिः षडुत्तरैः ।
स्यात्त्रयोदशहृता निरग्रका तद्गुणं कथय मे पृथक् पृथक् ॥२५१॥

The corresponding verse in the *Bījagaṇita* is 59c-60b:

यद्गुणा क्षयगषष्टिरन्विता
वर्जिता च यदि वा त्रिभिस्ततः।
स्यात्त्रयोदशहृता निरग्रका
तं गुणं गणक मे पृथग्वद ||५९c-६0b||

Comparing the first half of these verses we see that the *Bījagaṇita* has क्षयग (negative) and त्रिभिः (by three) instead of गणक (oh calculator) and दशभिः षडुत्तरैः (by ten more than six i.e. by sixteen) of the *Līlāvatī*, so that the solutions also differ. This indicates that Bhāskara avoids a negative dividend in his *Līlāvatī*.

(iv). The *Bījagaṇita* text (p. 37) has a brief solution to verse 62b-63b:

येन पञ्च गुणिताः खसंयुताः
पञ्चषष्टिसहिताश्च तेऽथवा।
स्युस्त्रयोदशहृता निरग्रका-
स्तं गुणं गणक कीर्तयाशु मे ||६२b-६३b||

while the *Līlāvatī* (*L II*, 255, *ASS 107*, p. 266) has a detailed solution.

(v). In the *Bījagaṇita* (p. 36), Bhāskara solves the case when both the dividend and the additive have been divided by the divisor with reference to the verse 61b-62a:

येन संगुणिताः पञ्च त्रयोविंशतिसंयुताः।
वर्जिता वा त्रिभिर्भक्ता निरग्राः स्युः स को गुणः ||६१b-६२a||

This case is omitted in the *Līlāvatī* (see *L II*, 253, *ASS 107*, pp. 263-264). Note that this case is an application of the rule given by *BG*, 55, and this verse is also omitted in the *Līlāvatī*, as mentioned before.

(vi). Finally, in the solution pertaining to *BG*, 67a-d, p. 39:

कः पञ्चनिघ्नो विहृतस्त्रिषष्ट्या
    सप्तावशेषोऽथ स एव राशिः।
दशाहतः स्याद्विहृतस्त्रिषष्ट्या
    चतुर्दशाग्रो वद राशिमेनम् ॥६७a-d॥

Bhāskara adds (in the *Bījagaṇita*, p. 39) अयमेव राशिः and लब्धिः ३ । Also the *Bījagaṇita* (p. 39) has क्षेपः २१̇, while the *Līlāvatī* (*L II*, *ASS 107*, p. 274) has शुद्धिः २१ । Similar minor changes, omissions, augmentations or abridgements may be found in some other solutions as well.

S. *The Textual Commentary (Verses 46b-67d).*

The first verse is a tribute by Sūrya to the elephant-headed god Gaṇeśa.

(a). *The General Kuṭṭaka (Verses 46b-63b).*

Verse 46b-47b. Mathematical meaning: The equation is

$$by = ax + c, \qquad (I)$$

where a, b and c are integers and solutions in integers x and y will be found by means of the pulverizer. One should reduce a, b, c first by dividing each by the gcd (a, b, c). If there is a certain d such that $d | a$, $d | b$ but $d \nmid c$, then the equation has no solution.

Comments: Bhāskara states the preliminary operations i.e. the operations which need to be performed before carrying out the method of kuṭṭaka. The names are given as follows: b is the "divisor," a the "dividend" and c the "additive." The solutions, x and y, are given names as the "multiplier" and "result" (i.e. quotient) in verses 48c-50b.

The content and language of Bhāskara's lines 46b-c are similar to those of Śrīpati's *SSE* XIV, 22a-b:

विभाज्यहारं च युतिं निजच्छिदा।
समेन वाऽऽदावपवर्त्यं सम्भवे ॥२२a-b॥

Also, Āryabhaṭa II's *MS* XVIII, 1 has similar content:

भाज्यक्षेपच्छेदा यथोदिताः संस्थिताः क-विधिरेषः।
ते च करण्या भक्ता दृढाभिधाना अयं स-विधिः ॥१॥

Moreover, one can notice the similarity between the content of Bhāskara's 47a-b and that of Śrīpati's *SSE* XIV, 26a-b:

विभाज्यहृत्योरपवर्त्तनं यदा ।
भवेद्युतौ नैव खिलं हि तत्तदा ॥२६a-b॥

On the other hand, Nārāyaṇa's *BGV*, 53, p. 29, which is

भाज्यो हारः क्षेपः केनाप्यपवर्त्य कुट्टकस्यार्थम् ।
येन विभाज्यच्छेदौ (छिन्नौ) क्षेपो न तेन खिलम् ॥५३॥

has the same content and language as we find in Bhāskara's verse 46b-47b. For an application of the latter, see verse 57c-58b of the *Text Alpha*.

Verse 47c-48b. Mathematical meaning: Using "mutual division," that is, a process identical to Euclid's algorithm, find the d = gcd (a, b). Then $\frac{a}{d}$ and $\frac{b}{d}$ are the *reduced* dividend and divisor.

Comments: Note the similarity in the content and language of Bhāskara's 47c-d and Śrīpati's *SSE* XIV, 27a-b:

परस्परं भाजितयोस्तु शेषकं
तयोर्द्वयोरप्यपवर्त्तनं भवेत् ॥२७a-b॥

Also, the contents of Bhāskara's 48a-b and Āryabhaṭa II's *MS* XVIII, 1b (quoted before) are analogous.

Sūrya explains that the reducer of the given dividend and divisor is the (last non-zero) remainder of their mutual division, i.e. their greatest common divisor (gcd). Whenever such a reducer can be found, the given dividend, divisor and additive must be divided by that reducer. On division, those three become confirmed so that their further reduction is impossible.

Verses 48c-50b. Textual problems: Part of Sūrya's text pertaining to this verse was missing in the manuscripts of class A. This text has been supplied from its counterpart β because A seems to have omitted it due to homoeoteleuton.

Mathematical meaning: The entire process of finding one solution to equation (I), with $(a, b) = 1$, is given. In essence, one writes down all the equations of the Euclidean algorithm for a and b and then by reverse substitution arrives at an x and y which satisfy (I). The procedure is made more or less mechanical via manipulations of the "chain of results (i.e. quotients)."

Comments: Bhāskara's 48c-d and Śrīpati's similar verse which is *SSE* XIV, 22c-d, that is

विभाज्यहारौ विभजेत्परस्परं
तथा यथा शेषकमेव रूपकम् ॥२२c-d॥

are based on Āryabhaṭa II's *MS* XVIII, 4a:

भाज्यहारावन्योन्यं विभजेत् टा-शेषकं भवेद्यावत् ॥४a॥

We can see why, if possible, the given dividend, divisor and additive must be reduced as a preliminary operation to starting the method of kuṭṭaka. It is because, otherwise, in the mutual division of the dividend and divisor (which is carried out *for* accomplishing the kuṭṭaka), the remainder 1 (which is required by these mathematicians in the place of the dividend) will not be achieved. The examples of the formation of the chain of quotients and the manipulations applied to it will be given later.

The equivalent of Bhāskara's 49a-b is Āryabhaṭa II's *MS* XVIII, 66:

का-शेषे नो करणी फलान्यधोऽधः क्रमेण धार्याणि ।
करणीजं नो धार्यं वल्ली सा मध्यमा स्व-विधौ ॥६६॥

But here Bhāskara's exposition is more lucid than that of Āryabhaṭa II. Śrīpati's equivalent is *SSE* XIV, 23a-b (given below). Śrīpati's 23a and Bhāskara's 49a are very similar.

The equivalents of Bhāskara's two verses 48c-50b (which together describe his rationale of the method of kuṭṭaka) are Āryabhaṭa II's *MS* XVIII, 4a-8a:

भाज्यहरावन्योन्यं विभजेत् टा-शेषकं भवेद्यावत् ।
सा वल्ली तेन हतेऽन्त्येनोर्ध्वे कान्विते स्फुटा वल्ली ॥४॥

विषमसमत्वं ज्ञात्वाऽनष्टोपान्त्येन ताडिते स्वोर्ध्वे ।
स्वस्थानच्युतमन्त्यं योज्यमनेन प्रकारेण ॥५॥

राशी कुट्टाख्यौ स्तो वक्ष्येऽन्यौ तौ सदा विषमजाख्यौ ।
सकृदेवच्छेदहृते भाज्ये शेषं यदा टा स्यात् ॥६॥

लब्धं तदोर्ध्वकुट्टः शेषं चाधःस्थितो ज्ञेयः ।
कुट्टौ स्वक्षेपहतावूर्ध्वाधःस्थौ क्रमाद्व्यक्तौ ॥७॥

निजभाज्यच्छेदाभ्यां फलगुणकौ शेषकौ भवतः ॥८a॥

and Śrīpati's *SSE* XIV, 22c-25:

विभाज्यहारौ विभजेत्परस्परं
तथा यथा शेषकमेव रूपकम् ॥२२c-d॥

फलान्यधोऽधः क्रमशो निवेशयेन्
मतिं तथाऽधस्तदधश्च तत्फलम्।
इदं हतं केन युतं विवर्जितं
हरेण भक्तं सदहो निरग्रकम् ।।२३।।

समेषु लब्धेष्वसमेष्वृणं धनं
धनं त्वृणं क्षेपमुशन्ति तद्विदः।
मतिं विचिन्त्येति तदूर्ध्वगं तया
निहत्य लब्धं च तथा नियोजयेत् ।।२४।।

पुनः पुनः कर्म यथोत्क्रमादिदं
यदा तु राशिद्वयमेव जायते।
हरेण भक्तः प्रथमो गुणो भवेत्
फलं द्वितीयं तु विभाज्यराशिना ।।२५।।

The differences are that Āryabhaṭa II's chain of results has 1 for the additive and has no zero below it; whereas Śrīpati's chain has mati and a (new) quotient in place of Bhāskara's additive and 0 respectively.

In 50a-b, to distinguish the two numbers in the reduced chain, Bhāskara uses 'upper' and 'other' which is clearer than Śrīpati's 'first' and 'second' in his 25c-d. Śrīpati seems to discard the first quotient from the chain because his first (i.e. upper) number in the reduced chain gives the multiplier x, and his second number gives the quotient y.

At the end of his commentary to the present verses, Sūrya explains that the special symbolic word taṣṭa ("sliced") is used instead of a word meaning "divided," wherever one is interested in the remainder from division, and not in the result (phala i.e. quotient) from division; as in verse 50a-b, the "quotient" and "multiplier" are remainders which are obtained from division of the upper and lower numbers in the reduced chain by the reduced dividend and divisor respectively.

Verse 50c-51b. Mathematical meaning: The method of the previous verse gives a solution x, y to equation (I) when there is an even number of quotients in the chain. Otherwise $x' = b - x$ and $y' = a - y$ gives a solution.

Comments: Śrīpati's equivalent of this verse is *SSE* XIV, 24 (which has already been quoted), but his method is different from that of Bhāskara. Āryabhaṭa II's equivalents are *MS* XVIII, 13-14:

एवमभीष्टविधिभवौ फलगुणकौ प्रस्फुटौ धनक्षेपे।
समवल्ल्यां विषमायामृणसंज्ञे क्षेपके स्याताम् ॥१३॥

समवल्ल्यामृणसंज्ञे धनसंज्ञे वा विषमवल्ल्याम्।
स्वविधौ फलगुणहीनौ सुदृढौ भाज्यच्छिदौ फलगुणौ स्तः ॥१४॥

Bhāskara seems to follow Āryabhaṭa II's method.

What this verse means is that, when the number of quotients in Bhāskara's chain is even, then the method of kuṭṭaka gives a solution of $by = ax + c$, where c must be taken positive; but when the number of quotients is odd, this method gives a solution of $by = ax - c$, as is evident from our rationale. In the second case, the corresponding solution of $by = ax + c$ is $y'' = a - y'$ and $x'' = b - x'$, where x' and y' is a solution of $by = ax - c$. Bhāskara and Sūrya do not prove this simple statement. The proof is as follows (compare it with our commentary on verse 53b):

Suppose $y = y'$ and $x = x'$ is a solution of $by = ax - c$. Then $by' = ax' - c$. Let $y'' = a - y'$ and $x'' = b - x'$. We have

$$by'' = b(a-y') = ba - by' = ba - (ax' - c)$$
$$= a(b - x') + c$$
$$= ax'' + c.$$

Āryabhaṭa II's verse 13 also includes that if (i) the number of quotients (including the first) is odd and (ii) the additive is negative, then the method of solution is the same as that when the number of quotients is even and the additive is positive. That means in the case when the number of quotients including the first is odd, the column (chain) still has $+c$ but we get the solution for $by = ax - c$ without doing any subtraction from the takṣaṇas.

We give the following illustration to clarify these facts:

Let $63y = 10x + 9$.

Then the chain according to the rule of Bhāskara II gives

q	=	0	0	0	27
q_1	=	6	6	171	171
q_2	=	3	27	27	
c	=	9	9		
t	=	0			

Here the number of quotients (including the first) is *odd*. So $y = 27$ and $x = 171$ is a solution of $63y = 10x - 9$. To get the minimum positive solution we divide 27 and 171 by the takṣaṇas 10 and 63 respectively. When one obtains an equal quotient, 2, in the two divisions, the least solution of $63y = 10x - 9$ is $y = 7$, $x = 45$. According to the method of Bhāskara II, the corresponding solution of $63y = 10x + 9$ is given by $y = 10 - 7 = 3$, $x = 63 - 45 = 18$.

This subtraction from takṣaṇas is not required if we use the chain of Āryabhaṭa I (Datta's second translation with the number of quotients odd), with mati = 0. For then we get the chain for $63y = 10x + 9$ as follows:

$$
\begin{array}{rcrrr}
q_1 & = & 6 & 6 & -171 \\
q_2 & = & 3 & -27 & -27 \\
q_3 t' - c & = & -9 & -9 & \\
t' & = & 0 & &
\end{array}
$$

Then since $-171 = (-3)\,63 + 18$, the remainder $x = 18$.

Likewise $-27 = (-3)\,10 + 3$, gives $y = 3$. Note that the lower number in the reduced chain gives y because the first quotient in the mutual division is 0. Alternatively, putting $x = 18$ in $63y = 10x + 9$, we get $y = 3$.

Thus we have shown that when the number of quotients is odd and the additive c is positive, we don't have to subtract from the takṣaṇas if c is replaced by $-c$ in Bhāskara's chain. Incidentally, we have also proved Āryabhaṭa II's statement (see *MS* XVIII, 13) that, if the number of quotients is odd and no subtraction from takṣaṇas is made, we get a solution for a negative additive i.e. for $by = ax - c$, $c > 0$.

Note that in the present verse 50c-51b, Sūrya's demonstration refers to verses 48c-50b, and not to verse 50c-51b. The verse cited by Sūrya in connection with several multipliers and quotients is 57a-b. Also, Sūrya's statements, 'the inclusion of the quotient (multiplier) in the dividend (divisor) is seen,' mean that the quotient (respectively multiplier) depends on the dividend (respectively divisor); that is, the word 'inclusion' stands for dependence in these statements.

Next we comment on Sūrya's concise verses 1-9.

Verse 1. Comments: The first half is vague. It may just be Sūrya thinking of the equation $by = ax + c$ as $by = a\left(x + \frac{c}{a}\right)$. The second half means that the quotient and the multiplier are remainders upon division by the dividend and the divisor respectively.

Verse 2. Mathematical meaning: If $by = ax + 1$, then $b(yc) = a(xc) + c$.

Comments: If x is a multiplier for $by = ax + 1$, then $c \cdot x$ is a multiplier for $by' = ax' + c$.

Verse 3. Mathematical meaning: A particular solution x_0, y_0 to equation (I), if large, may be reduced by using the division algorithm $x_0 = bt + x, y_0 = at + y$.

Comments: Here the quotient t must be the same in both equations, though the verse does not mention this.

Verse 4. Mathematical meaning: The general solution to (I) is obtained from a specific solution x_0, y_0 by the formulas $x = bt + x_0, y = at + y_0$ (t arbitrary).

Verse 5. Comments: If x is known, but y is unknown, then y is found using (I). Similar is the procedure in the opposite case (see verses 6 and 7 below).

Verse 6. Mathematical meaning: $y = \frac{ax + c}{b}$.

Verse 7. Mathematical meaning: $x = \frac{by - c}{a}$.

Verse 8. Mathematical meaning: When $a < b$, we solve $b'y = a'x - c$, where $b' = a$ and $a' = b$, instead of $by = ax + c$, because we may discard the first quotient 0 in the mutual division. Clearly these two equations have the same solutions after interchange of x and y.

Comments: Sūrya does not state the obvious relation that, if in the original chain one retains the first quotient which is zero, then in the reduced chain the same y will correspond to both the first and the third quotients (see our illustration, $63y = 10x + 9$, under verse 50c-51b). Thus, if the first quotient 0 is removed from the original chain, then the upper of the two numbers in the reduced chain gives the "multiplier" x and the lower of the two numbers, which corresponds to the third quotient of the original chain, will give the "quotient" y. The change in sign of the additive results from the change in parity of the number of quotients in the chain when the first quotient 0 is removed.

Verse 9. Mathematical meaning: The solution to $by = ax - c$, does not require any subtraction of the obtained values of x and y from the takṣaṇas a and b, provided the number of quotients in the chain is odd.

Comments: This verse may be compared with Āryabhaṭa II's *MS* XVIII, 13. Following Bhāskara's 50c-51b and 53b, one has to subtract from takṣaṇas twice. But the rule of subtracting from takṣaṇas, if applied twice, will cancel each other as if no subtraction was done.

Verse 51c-52b. Textual problems: In the artha part, the β-recension has some text which seems to be a repetition. It is omitted by A and so also by us. We have put it in the Appendix #14.

Mathematical meaning: To solve the given equation $by = ax + c$ where $a = a'k$ and $c = c'k$, first solve $by' = a'x' + c'$. Then $y = y'k$, $x = x'$ is a solution of the given equation. Similarly, if $b = b'k$ and $c = c'k$, first solve $b'y' = ax' + c'$. Then $y = y'$ and $x = x'k$ is a solution of the given equation.

Comments: In the first case, $by = by'k = (a'x' + c')k = a'kx' + c'k = ax + c$. The second case is similar.

In the demonstration, Sūrya's statement 'the inclusion of the multiplier in the divisor is seen' means that x depends on b. Therefore if b is unaltered, so is x. Furthermore, the statement 'the multiplier becomes reduction-number-times less' means that multiplier $= \dfrac{1}{\text{reduction - number}} \times$ multiplier, if no reduction had been done; that is, $x' = \frac{1}{k}(x)$. For an example, see verse 58c-59b.

The present verse 51c-52b discusses operations which are similar to those contained in Āryabhaṭa II's *MS* XVIII, 2a:

भाज्यक्षेपौ ग-विधिः क्षेपच्छेदौ यदा तदा घ-विधिः ॥२a॥

Verse 52c. Mathematical meaning: If x and y are a solution to $by = ax + c$, then so are r and s where

$x = bt + r$ and $y = at + s$

provided t is the same in both equations.

Comments: In the above, *r* or *s* can be negative provided we allow negative solutions. (See our commentary on verse 61b-62a).

This verse may be compared with Āryabhaṭa II's *MS* XVIII, 15-16:

अन्यत्र प्रश्नोक्तावथ तत्सम्बधजे यदा लब्धी।
न समे गुण एव तदा ग्राह्यो हेयं फलं धनक्षेपे ॥१५॥

फलमृणसंज्ञे ग्राह्यं हेयो गुणको गुणात् फलोत्पत्तिम्।
वक्ष्ये फलतोऽपि तथा सर्वत्र समां गुणोत्पत्तिम् ॥१६॥

In these verses, Āryabhaṭa II says that when the results (quotients) on division (takṣaṇa) of the pair of numbers are not equal, then in case of the positive (respectively negative) additive, only the multiplier (respectively quotient) is to be retained. The editor and commentator Dvivedi (1910) explains that in the case of a positive additive, the result from the division of the lower number (by the divisor) is smaller because the divisor is smaller. But when the additive is negative, only the "quotient" (and not the "multiplier") is to be retained because then the dividend is smaller. Also Dvivedi claims that unequal results arise by division when the additive is larger than the product of the dividend and divisor (p. 228). The reader may refer to Bhāskara's verse 61b-62a, *Text Alpha* for an example.

Bhāskara's result in verse 52c is certainly a refinement over that of Āryabhaṭa II. Because by forcing equal results in the two divisions, such that that (common) result is the *smaller* of the two results which were formally obtained (in the two divisions), Bhāskara can find both the "multiplier" and "quotient" from the reduced chain of results. A perfect illustration is Bhāskara's solution to $3y = 5x + 23$ in verse 61b-62a:

येन संगुणिताः पञ्च त्रयोविंशतिसंयुताः ।
वर्जिता वा त्रिभिर्भक्ता निरग्राः स्युः स को गुणः ॥६१b-६२a॥

Sūrya explains that equal results in division of the two numbers in the reduced chain are to be taken because whatever multiple of the confirmed dividend is subtracted from the upper number, exactly the same multiple of the confirmed divisor is to be subtracted from the lower number.

Verse 53b. Textual problems: For the demonstration part of this verse 53b and for the subsequent text (which pertains to verses 53a, 54a-b and 55a-b), we have chosen text A and placed the corresponding text belonging to class β in the Appendix #15. Text β has the same verses though their demonstration is in a different, less logical order. Furthermore, the demonstration in text β has been to some extent borrowed from Sūrya's commentary on Bhāskara's *Līlāvatī*, the *Gaṇitāmṛtakūpikā* (see *Wai, PPM 9762*, f. 119r., 1-7), but does not seem to be borrowed by Sūrya himself. Sūrya wrote this commentary in 1541 A.D., i.e. three years after he wrote the *Sūryaprakāśa*.

Apparently, Sūrya did not want his readers to be passive. He usually leaves the easy explanations to his students. He writes "it is clear" when he thinks that some sūtra requires no deep explanation. Instead of augmenting a later recension of the *Sūryaprakāśa* with explanations from his *Gaṇitāmṛtakūpikā*, Sūrya presumably would have preferred that his readers make the effort to understand the text by themselves.

The chosen text A has a lacuna pertaining to verses 53a and 54a, as is evident from *Text Alpha*.

Mathematical meaning: If x' and y' are a solution to $by = ax + c$ ($c > 0$), then $x = b - x'$ and $y = a - y'$ are a solution to $by = ax - c$.

Comments: This verse may be compared with verse 50c-51b. An application of 53b can be found in the problem in verse 58c-59b, in which a solution of $100x + 90 = 63y$

is found to be $x = 18$, $y = 30$; and the corresponding solution of $100x - 90 = 63y$ is $x = 63 - 18 = 45$ and $y = 100 - 30 = 70$.

One may observe that using Bhāskara's verse 50c-51b and verse 53b for the solution of $63y = 10x - 9$ (see our commentary on verse 50c-51b), one has to subtract (the solution) $x = 45$, $y = 7$ from the respective takṣaṇas (63 and 10) twice because the number of quotients is odd and the additive is negative. But this process yields the same solution as the one obtained without subtraction. This confirms the statement of Āryabhaṭa II in his *MS* XVIII, 13 (already quoted, see our commentary to verse 50c-51b). A similar verse has been cited by Sūrya (see Sūrya's concise verse 9, before Bhāskara's verse 51c-52b).

Verse 54a. Textual problems: *Text Alpha* has a lacuna here (also, see textual problems under verse 53b). The explanations of verses 54a and 53a are intertwined.

Mathematical meaning (of verse 54a): If $by' = ax' + c$, then $x = b - x'$ and $y = -(a - y')$ is a solution to the equation $by = (-a)\,x + c$.

Comments: The verse 54a does not contain reference to the negative sign outside $(a - y')$.

The rule given by 54a is *essentially* the same as that in 53b, when only the additive is negative (i.e. subtractive). The commentator Kṛṣṇa (see *BP*, 31, pp. 106-107) informs us that in some manuscripts the reading 'bhājake' (भाजके) is found in place of 'bhājyaje' (भाज्यजे). The former is incorrect because the same procedure of subtraction is not needed when only the divisor is negative.

Thus, we can conclude that the correct rules seem to be the following (though 53b and 54a do not state all the specifics):

(i). When either of the additive and dividend is negative, the subtraction from the respective takṣaṇas is to be made as stated above. Moreover, when only the dividend is negative, the correct quotient has to be made negative, as is evident in view of the example in verse 59c-60b (see our commentary).

(ii). When both additive and dividend are negative, then $ax + c$ is negative (since, if one assumes x to be positive, then both ax and c are negative. Use *Text Alpha*, verse 3a-b). Then the subtraction from takṣaṇas is not required, only the *quotient* needs to be made negative (since when $ax + c = by$ is negative and b is positive, y must be negative: verse 5c-d).

(iii). The case when only the divisor is negative is the equivalent of the above case when both additive and dividend are negative (since, if one assumes x to be positive, then $by = -ax - c$ iff $-by = ax + c$). So only the *quotient* needs to be made negative (since $by = ax + c = (-b)(-y)$: verse 5c-d).

(iv). The case when both the additive and the divisor are negative is the equivalent of the case when only the dividend is negative (since, if one assumes x to be positive, then $-by = ax - c$ iff $by = -ax + c$). Likewise, the case when both the dividend and the divisor are negative is the equivalent of the case when only the additive is negative (when one assumes x to be positive). For an example, see verse 60c-61a.

(v). One can conclude that when all three—additive, dividend and divisor—are negative (if one assumes x to be positive), the method of solution is the same as that when they were all positive (since $-by = -ax - c$ iff $by = ax + c$).

In view of the relation between the rules given by 53b and 54a, it seems that these lines should be treated as forming a single verse.

Verses 53a and 54b. Textual Problems: See verse 53b. Also note that Vidyāsāgara (1878, p. 27) who is one of the editors of Bhāskara's *Bījagaṇita* writes another verse (i.e. 53b-54a) between these verses 53a and 54b, but Sūrya does not. Sūrya's arrangement keeps the continuity of the context. This indicates that Vidyāsāgara's 53a should come after his 54a, as does Sūrya's. Vidyāsāgara's arrangement is 53a, 53b, 54a, 54b, whereas Sūrya's arrangement is 53b, 54a, 53a, 54b. The verses in our *Text Alpha* follow Sūrya's order but Vidyāsāgara's numbering, because Sūrya does not give any numbering to Bhāskara's verses.

Mathematical meaning of verse 53a: Let $by = ax + c$, where $c = bt + c'$, $c > 0$. Then $b(y - t) = ax + c'$ i.e. $by' = ax' + c'$, where $x' = x$ and $y' = y - t$. A solution of $by' = ax' + c'$ can first be found using the previously described techniques.

Mathematical meaning of verse 54b: (i). Suppose x' and y' to be a solution of $by' = ax' + c'$ (as in 53a). Then a corresponding solution of $by = ax + c$ is $x = x'$ and $y = y' + t$.

(ii). Now let $by = ax - c$ where $c = bt + c'$, $c > 0$. Then $b(y + t) = ax - c'$ i.e. $by' = ax' - c'$, where $x' = x$ and $y' = y + t$. Solving this latter equation, suppose x' and y' to be a solution of $by' = ax' - c'$. Then a corresponding solution of $by = ax - c$ is $x = x'$ and $y = y' - t$.

Comments: Logically, it seems that 53a and 54b should be treated as forming a single verse.

An application of the above is the problem contained in verse 61b-62a, where one of the two indeterminate equations is $3y = 5x + 23$. Since $23 = 3 \cdot 7 + 2$, we can write $3(y - 7) = 5x + 2$ i.e. $3y' = 5x' + 2$ where $x' = x$ and $y' = y - 7$. A solution of $3y' = 5x' + 2$ is $x' = 2$, $y' = 4$. So a (corresponding) solution of $3y = 5x + 23$ is $x = x' = 2$ and $y = y' + 7 = 4 + 7 = 11$. Therefore, a solution of $-3y = 5x + 23$ is $y = -11$ and $x = 2$.

Again, when the additive 23 is negative, we have to solve $3y = 5x - 23$; i.e. $3(y + 7) = 5x - 2$ i.e. $3y' = 5x' - 2$ where $y + 7 = y'$ and $x = x'$. Now to solve $3y' = 5x' - 2$, we first solve $3y' = 5x' + 2$, a solution of which is $x' = 2$, $y' = 4$. So a solution of $3y' = 5x' - 2$ is $x' = 3 - 2 = 1$ and $y' = 5 - 4 = 1$. So a (corresponding) solution of $3y = 5x - 23$ is $x = x' = 1$ and $y = y' - 7 = 1 - 7 = -6$.

Note that Sūrya leaves the solution here; but Bhāskara says in his solution (*BG*, p. 36, 6-8) of his *BG*, 61b-62a, p. 34, the following:

नलब्ध्यर्थं द्विगुणे हरे क्षिप्ते जातौ तावेव लब्धिगुणौ ४ ७

That is, "for the sake of a positive quotient, when twice the (respective) divisors are added (to the quotient and multiplier), the quotient and multiplier are produced 4 and 7." Thus $x = 1 + 2(3) = 7$ and $y = -6 + 2(5) = 4$. Here Bhāskara is using the sūtra which is contained in his *BG*, 57a-b, p. 27.

Verse 55a-b. Textual problems: See verse 53b.

Mathematical meaning: Suppose $by = ax + c = (q_1 b + a_1)x + (q_2 b + c_1)$. Then $b(y - q_1 x - q_2) = a_1 x + c_1$ i.e. $by' = a_1 x' + c_1$, where $y' = y - q_1 x - q_2$ and $x' = x$. So a solution to $by = ax + c$ is found by first solving $by' = a_1 x' + c_1$, which yields an x', and then setting $x = x'$ and $y = \frac{ax + c}{b}$.

Comments: This verse has some similarity with verse 51c-d, since both discuss the multiplier. The differences are: (i) 55a-b discusses also the quotient; (ii) in 55a-b, the divisor of the additive c and dividend a is not their greatest common divisor. It is the given divisor *b*, which may not divide them exactly.

Sūrya does not discuss any example in connection with verse 55a-b, though he claims that one is forthcoming. However, we can apply it to verse 61b-62a as is shown by Bhāskara (see Vidyāsāgara, 1878, pp. 34-36):

येन संगुणिताः पञ्च त्रयोविंशतिसंयुताः।
वर्जिता वा त्रिभिर्भक्ता निरग्राः स्युः स को गुणः ॥६१b-६२a॥

Here we have to solve $3y = 5x + 23 = (1 \cdot 3 + 2)x + (7 \cdot 3 + 2)$; or equivalently, $3y' = 2x' + 2$. When we use the kuṭṭaka, the chain of results can be reduced as follows:

$$\begin{aligned} q &= 0 \quad 0 \quad 2 \\ q_1 &= 1 \quad 2 \quad 2 \\ c &= 2 \quad 2 \\ t &= 0 \end{aligned}$$

If we divide the upper 2 by the dividend 2, and the lower 2 by the divisor 3, and take the *equal* quotient 0 in division, the remainders are again 2, 2, so that $x' = 2$, $y' = 2$.

So $x = x' = 2$ and

$$y = \frac{5x+23}{3} = \frac{5(2)+23}{3} = 11.$$

Or (using $y = y' + q_1x + q_2$), $y = y' + 1 \cdot x + 7 = 2 + 1 \cdot 2 + 7 = 11$.

Verse 56a-b. Mathematical meaning: If $c = 0$, then $by = ax$ is solved by $x = 0$. If $c = bt$ then $by = ax + c$ $(c > 0)$ is solved by $x = 0$. In both cases, $y = \frac{c}{b}$.

Comments: The content of this verse is similar to that of Āryabhaṭa II's *MS* XVIII, 19:

स्वक्षेपे छेदहृते निरग्रके ना गुणः फलं लब्धिः ।
एवमृणक्षेपे नो ना-क्षेपे फलगुणौ नौ स्तः ॥१९॥

Bhāskara's verse means:

(i). When $c = 0$, then $by = ax$. So $x = 0$, $y = 0$ is a solution. This corresponds to Āryabhaṭa II's ना-क्षेपे फलगुणौ नौ स्तः ।

(ii). When $c = bt > 0$, then $by = ax + bt$, i.e. $b(y - t) = ax$, i.e. $by' = ax'$. So $x' = 0$, $y' = 0$ is a solution. Therefore, $x = x' = 0$, $y = y' + t = 0 + t = 0 + \frac{c}{b} = \frac{c}{b}$. This corresponds to Āryabhaṭa II's 19a.

(iii). Though Bhāskara does not state explicitly the case when $by = ax - c$, where $c > 0$ and $c = bt$, it is obvious in view of his 54b that when there is subtraction of the additive, then

$$x = 0,\ y = -t = -\frac{c}{b}$$

is a solution. Āryabhaṭa II, on the other hand, states एवमृणक्षेपे नो which means "such is not (the operation) when there is a negative additive." Thus Āryabhaṭa II seems to discard this case.

Note that this verse is equivalent to a special case of 53a and 54b (these lines should be treated as forming a single verse), in which

(i). When $c = 0$, then $c' = 0$ and $t = 0$. So, $by = ax$, whence $x = x' = 0$, $y = y' + t = 0 + 0 = 0$.

(ii). When $c > 0$, but $c' = 0$, then $t = \frac{c}{b}$. So, $b\ (y - t) = ax$, i.e. $by' = ax'$, whence $x = x' = 0$ and $y = y' + t = 0 + \frac{c}{b} = \frac{c}{b}$.

(iii). When there is subtraction of the additive, i.e. when $by = ax - c$, $c > 0$ and $c = bt$, then $t = \frac{c}{b}$. So, $by = ax - bt$ i.e. $b\ (y + t) = ax$, i.e. $by' = ax'$, whence $x = x' = 0$ and $y = y' - t = 0 - \frac{c}{b} = -\frac{c}{b}$.

In his demonstration of verse 56a-b, Sūrya says:

(i). When $c = 0$, the reduced chain will have only zeros. So it will yield (the remainders) $x = 0, y = 0$.

(ii). When $c = bt$, then $kc = b(kt)$ for any k. Since in this case, the reduced chain will have only *multiples* of c, therefore the lower of the two numbers (in this reduced chain) will also be a *multiple* of b. So, on division by b, it will yield the remainder zero which will give the multiplier $x = 0$. Therefore,

$$y = \frac{ax + c}{b} = \frac{0 + c}{b} = \frac{c}{b}.$$

Verse 57a-b. Mathematical meaning: If x', y' is a solution to $by = ax + c$, then many solutions can be obtained by

$$x = x' + bt,\ y = y' + at$$

where t is arbitrary.

Comments: The content of this verse is based on that of Āryabhaṭa II's *MS* XVIII, 20:

फलगुणकौ युक्तौ स्तः प्रश्नोक्ताभ्यामभीष्टगुणिताभ्याम् ।
भाज्यच्छिद्भ्यां बहुधा सुदृढाभ्यां चेष्टगुणिताभ्याम् ॥२०॥

Śrīpati too states one verse (*SSE* XIV, 27) which has similar content:

परस्परं भाजितयोस्तु शेषकं
तयोर्द्वयोरप्यपवर्त्तनं भवेत् ।
तदुद्धृतच्छेदविभाज्यकौ क्रमा-
दभीष्टनिघ्नौ तु गुणाप्तयोः क्षिपेत् ॥२७॥

For an application of the sūtra contained in verse 57a-b, see the next verse.

General Comment. Note that in verses 57c-58b to 62b-63b, the question asks only for the multiplier x. But both multiplier *and* quotient are found in Sūrya's as well as Bhāskara's solutions.

Verse 57c-58b. Mathematical meaning: Find an x such that $\frac{221x+65}{195}$ is an integer.

Setting out:	Dividend	221	Additive	65
	Divisor	195.		

For a solution, having reduced these by the gcd (221, 195, 65) = 13, the chain of results for $y = \frac{17x+5}{15}$ can be worked out as follows:

q	=	1	1	40
q_1	=	7	35	35
c	=	5	5	
t	=	0		

The takṣaṇas (divisors) of 40 and 35 are respectively 17 and 15. Obtaining the *equal* quotient 2 in both divisions, the remainders are respectively 6 and 5. They are the *basic* quotient y and multiplier x. Thus one solution is obtained.

Comments: Sūrya refers to verse 46b-47b and its commentary in the above solution. In order to find other (positive) solutions, Sūrya uses the sūtra of verse 57a-b, which gives $y = 17\ (1) + 6, x = 15\ (1) + 5$; $y = 17\ (2) + 6, x = 15\ (2) + 5$ and so on. In modern notation these solutions may be expressed as: $y = 17m + 6, x = 15m + 5$ where m is an arbitrary integer.

Verse 58c-59b. Mathematical meaning: Find an x such that

$$\frac{100x+90}{63}$$

is an integer and, likewise, another x such that

$$\frac{100x-90}{63}$$

is an integer.

Setting out: Dividend 100 Additive 90

Divisor 63.

The solution of $63y = 100x + 90$ can be found without any reduction, since gcd (100, 63, 90) = 1. The solutions of $63y = 100x + 90$ can also be found by using the reductions (or preliminary operations) suggested by Bhāskara in verse 51c-52b. Then the following reductions are possible:

(i). When one divides the dividend 100 and additive 90 by their gcd 10, the reduced equation is $63y_1 = 10x + 9$, where

$$y_1 = \frac{y}{10}.$$

By the kuṭṭaka, $x = 18, y_1 = 3$ is a solution. So a solution for $63y = 100x + 90$ is $x = 18$, $y = 3 \cdot 10 = 30$.

(ii). Dividing the divisor and additive by their gcd 9, we have $7y = 100x_1 + 10$, where

$$x_1 = \frac{x}{9}.$$

By the kuṭṭaka, $x_1 = 2, y = 30$. Therefore $x = 2 \cdot 9 = 18$ and $y = 30$ is a solution for $63y = 100x + 90$.

(iii). Dividing the dividend and additive by their gcd 10 and then the divisor and (reduced) additive by their gcd 9, we have

$$7\left(\frac{y}{10}\right) = 10\left(\frac{x}{9}\right) + 1.$$

Or $7y_1 = 10x_1 + 1$ where

$$y_1 = \frac{y}{10} \text{ and } x_1 = \frac{x}{9}.$$

Solving, $x_1 = 2, y_1 = 3$. So $x = 2 \cdot 9 = 18, y = 3 \cdot 10 = 30$.

Finally, since a solution of $63y = 100x + 90$ is $x = 18, y = 30$, therefore using verse 53b, a (corresponding) solution of $63y = 100x - 90$ is $x = 63 - 18 = 45$, $y = 100 - 30 = 70$.

Comments: Sūrya, following Bhāskara's solution (see *BG*, p. 32) of his *BG*, 58c-59b, p. 29, gives the other solutions of $63y = 100x - 90$ which may be written as $x = 45 + 1\,(63), y = 70 + 1\,(100)$; $x = 45 + 2\,(63), y = 70 + 2\,(100)$; and so on.

Furthermore, Sūrya does not discuss reduction (iii) but Bhāskara does (see *BG*, pp. 31-32). Bhāskara says

अथ वा भाज्यक्षेपौ हारक्षेपौ चापवर्त्य न्यासः

भा	१०	क्षे	१
हा	७		

and then gives the solution.

Verse 59c-60b. Mathematical meaning: Find those x such that

$$\frac{-60x \pm 3}{13}$$

are integers.

Setting out:	Dividend	–60	Additive	3
	Divisor	13.		

The various steps in the solutions of these two problems are as follows:

(i). A solution for $13y = 60x + 3$ is $x = 11, y = 51$.

(ii). A solution for $13y = (-60)x + 3$ is given by $x = 13 - 11 = 2$, $y = 60 - 51 = 9$ and y is made negative (see our commentary on verse 54a). So $x = 2, y = -9$.

(iii). To find a solution for $13y = -60x - 3$, we take the solution for $13y = 60x + 3$ and put a negative sign before the quotient (because when RHS is negative, LHS also has to be negative, so y should be negative). So a solution for $13y = -60x - 3$ is $x = 11$, $y = -51$. Clearly, this is also a solution for $-13y = 60x + 3$ (see our commentary on verse 54a).

Thus, alternatively, to find a solution of $13y = -60x - 3$, we may find a solution of $-13y = 60x + 3$, for which the procedure is: find a solution for $13y = 60x + 3$ and make the quotient negative. The reason is that, when the right hand side is positive, so is the left hand side, which is possible only when y is negative.

Comments: Sūrya does not discuss the alternative method in step (iii). He states that everything has been accomplished by verse 53b (that is, without the application of verse 54a).

Verse 60c-61a. Mathematical meaning: Find those x such that

$$\frac{18x \pm 10}{-11}$$

are integers.

Setting out:	Dividend	18	Additive	10
	Divisor	−11.		

The main points in the solution are:

(i). A solution for $11y = 18x + 10$ is $x = 8, y = 14$.

(ii). A solution for $11y = 18x - 10$ is $x = 11 - 8, y = 18 - 14$; i.e. $x = 3, y = 4$.

(iii). A solution for $-11y = 18x - 10$ is $x = 3, y = -4$, because the quotient y should be negative (as the RHS is positive).

(iv). A solution for $-11y = 18x + 10$ is $x = 8, y = -14$.

Comments: The points (i) – (iii) also form the essence of Sūrya's commentary. Sūrya stops here and does not discuss (iv). One can see that in (iii) alternatively, $-11y = 18x - 10$ if and only if $11y = -18x + 10$. The latter equation can be solved by following the method in verse 54a, when only the dividend is negative.

Remark. In view of the last two verses (i.e. 59c-60b and 60c-61a), it is clear that when the given dividend or divisor is negative, Sūrya and Bhāskara (perhaps because of applications to physical situations) allow the quotient to be negative but the multiplier is kept positive. In his commentary to *BG*, 60c-61a, p. 34 Bhāskara says (see Vidyāsāgara, 1878, p. 34, lines 13-14):

भाजके भाज्ये वा ऋणगते लब्धेः ऋणत्वं सर्वत्र ज्ञेयम्।

But Nārāyaṇa says that either the quotient or the multiplier should be made negative. More specifically, Nārāyaṇa states his rule in his *BGV*, 62, p. 32:

क्षयभाज्ये गुणलब्धी धनवत्साध्ये तु भाज्यतः क्षेपे ।
अल्पे तयोः क्षयं स्यादेकमनल्पे तु ते सकृद्धनगे ॥६२॥

He applies this rule in the solution of his *BGV*, 30, p. 32 (which is):

क्षयत्रिंशद्गुणो राशिस्त्रिभिर्युक्तोऽथवोनितः ।
सम्भक्तो [*sic*] निरग्रः स्यात्तं गुणं वद वेत्सि चेत् ॥३०॥

as follows, though there are several errors and omissions in the solution:

(i). A solution of $7y = 30x + 3$ is $x = 2, y = 9$. If one subtracts from their takṣaṇas, then $x = 7 - 2 = 5, y = 30 - 9 = 21$. (This is a solution for $7y = 30x - 3$).

(ii). A solution for $7y = -30x + 3$ is $x = -2, y = 9$ or $x = 5, y = -21$. (The first solution seems to have been supplied by the editor, Professor Shukla).

(iii). Likewise, a solution for $7y = -30x - 3$ is $x = 2, y = -9$ or $x = -5, y = 21$. (The manuscript has $x = 5, y = -21$ for the first solution. Professor Shukla has corrected it.)

The correct reading in the last verse above should be सप्तभक्तो ।

Verse 61b-62a. Mathematical meaning: Find those x such that

$$\frac{5x \pm 23}{3}$$

are integers.

Setting out:	Dividend	5	Additive	23
	Divisor	3.		

For a solution of $3y = 5x + 23$, Bhāskara's chain yields the pair of numbers 46, 23. So one solution is $y = 46, x = 23$. To get the minimum solution, 46 and 23 are to be divided by their respective takṣaṇas 5 and 3. Taking *equal* result (quotient) 7 in the two divisions, the remainders are 11 and 2. So the least solution is: the quotient $y = 11$, the multiplier $x = 2$.

The corresponding solution of $3y = 5x - 23$, is $y = 5 - 11 = -6$, $x = 3 - 2 = 1$. To get the *positive* value of the quotient y, we use verse 57a-b. Therefore $y = -6 + 2(5) = 4$ and $x = 1 + 2(3) = 7$.

For another method of solving the indeterminate equations $3y = 5x \pm 23$ (following Sūrya and Bhāskara), see our commentary under verses 53a and 54b.

Comments: Note that if we put the additive $= -c = -23$ in Bhāskara's chain, we get a solution for $3y = 5x - 23$ (without any subtraction from takṣaṇas) as follows:

$$\begin{array}{rcrrr} q & = & 1 & 1 & -46 \\ q_1 & = & 1 & -23 & -23 \\ -c & = & -23 & -23 & \\ t & = & 0 & & \end{array}$$

Now either $-46 = 5(-10) + 4$ and $-23 = 3(-10) + 7$ or $-46 = 5(-7) + (-11)$ and $-23 = 3(-7) + (-2)$ according as we keep both remainders (and hence both x and y) positive or negative. Clearly $y = 4$, $x = 7$ and $y = -11$, $x = -2$ are solutions of $3y = 5x - 23$. (Compare this commentary with our commentary on verse 50c-51b).

To demonstrate Āryabhaṭa I's method we may write $5x = 3y + 23$ i.e. $5y' = 3x' + 23$. Therefore, from the mutual division, $q = 0$, $q_1 = 1$, $q_2 = 1$, $q_3 = 2$. Now using the case when the number of quotients (excluding the first and including the last), is odd (i.e. using sub-case (I.ii) of our rationale, with $n = 2$, Datta's second translation), and choosing the mati $t' = 0$, we have

$$q_{2n-1}t' - c = q_3t' - c = 2(0) - 23 = -23.$$

Discarding $q = 0$, we have the following chain and its reduction:

$$\begin{array}{rcrrr} q_1 & = & 1 & 1 & -46 \\ q_2 & = & 1 & -23 & -23 \\ q_3t' - c & = & -23 & -23 & \\ t' & = & 0 & & \end{array}$$

as before. Here -46 will yield x' (which $= y$) and -23 will yield y' (which $= x$) because in the mutual division (of the dividend and the divisor), the first quotient is zero. The solutions will be the same as those obtained previously.

Verse 62b-63b. Mathematical meaning: Find x for which

$$\frac{5x}{13}$$

is an integer and also find x for which

$$\frac{5x+65}{13}$$

is an integer.

Setting out for the first example:

Dividend	5	Additive	0
Divisor	13.		

Using verse 56a-b, since the additive is zero, the multiplier $x = 0$ and the quotient

$$y = \frac{0}{13} = 0.$$

Setting out for the second example:

Dividend	5	Additive	65
Divisor	13.		

Again, using verse 56a-b, since the additive 65 is divisible by the divisor 13, the multiplier $x = 0$ and the quotient

$$y = \frac{65}{13} = 5.$$

Comments: One can verify that the same solutions can be found in both cases using the ordinary method of kuṭṭaka and without the application of the above short-cut which is mentioned in the verse 56a-b.

For the first example, Sūrya gives another solution as well: multiplier $x = 0 + 1 \cdot 13 = 13$, quotient $y = 0 + 1 \cdot 5 = 5$; but Bhāskara does not. On the other hand, Bhāskara gives another solution for the second example (which Sūrya does not): multiplier = 31 [*sic*], quotient = 10 (see *BG*, 62b-63b, p. 37). In fact, the multiplier should be $x = 0 + 1 \cdot 13 = 13$ since the quotient $y = 5 + 1 \cdot 5 = 10$. Therefore Vidyāsāgara's (1878) text has a misprint at this point.

(b). *The Constant Kuṭṭaka (Verses 63c-65d).*

This refers to the solution of the indeterminate equations of the form $by = ax \pm c$, by means of the solution of the indeterminate equations of the form $by = ax \pm 1$ where the additive is ± 1. Śrīpati discusses this constant kuṭṭaka in his Praśnādhyāya, as will be seen later.

Verse 63c-64b. Textual problems: The β-recension contains a one-sentence solution which corresponds to the positive additive 5. This portion of the text does not exist in the A-recension. It seems to have been borrowed from the *Gaṇitāmṛtakūpikā* (see *Wai, PPM 9762*, f. 120 v., 9) with minor changes. Since Sūrya presumably did not make this addition, we have discarded this text and included it in the Apparatus Criticus.

Furthermore, in the demonstration part the manuscripts of class β have some text which is different from that in the manuscripts of class A. In fact, β repeats with omissions and in a different order what A has. So we have chosen text A and placed the corresponding text pertaining to class β in the Appendix #16.

Mathematical meaning: To solve $by = ax \pm c$, first find a solution x', y' of $by = ax \pm 1$. Then a solution to the original equation is $x = cx', y = cy'$.

Setting out:	Dividend	17	Additive	1
	Divisor	15.		

So here the equation to be solved is $15y = 17x + 1$. The solution is $x = 7, y = 8$. Multiplying x and y by 5 and dividing the products by 15 and 17 respectively, the multiplier $x = 5$ and the quotient $y = 6$. This is a solution of $15y = 17x + 5$.

Again, a solution of $15y = 17x - 1$ is given by $x = 15 - 7 = 8$, $y = 17 - 8 = 9$. Multiplying x and y by 5 and dividing the products by 15 and 17 respectively as before, a solution of $15y = 17x - 5$ is $x = 10$, $y = 11$.

Comments: The rule contained in the present verse clearly relates to the *rule of three* or proportion. That is, if the dividend and divisor remain constant, the solutions to the indeterminate equations are proportional to the additives.

Sūrya does not discuss the solution of $15y = 17x + 5$, but Bhāskara does (see *BG*, 63c-64b, p. 37). Note that in the equations $15y = 17x + 1$ and $15y = 17x + 5$, the dividend and the divisor stay the same, only the additive changes. The use of this method mainly in astronomical problems may arise from the numerically very large additives normally found in such problems.

Though Bhāskara and Sūrya do not state these facts, we can see that a solution of $15y = 17x - 5$ can be obtained from that of $15y = 17x + 1$ by multiplying it (here $x = 7$, $y = 8$) by -5 and dividing the products by 15 and 17 respectively. Likewise, a solution of $15y = 17x + 5$ can be obtained from that of $15y = 17x - 1$, the multiplying factor being -5 also in this case. But Bhāskara and Sūrya seem to keep this factor (c) always positive.

Verses 64c-65d. Textual problems: Some part of the text pertaining to Sūrya's explanation (before his demonstration) differs in the two recensions A and β. The β-recension expands (in many sentences) on what A explains in (its last) two sentences. But β borrows this text from Sūrya's other commentary the *Gaṇitāmṛtakūpikā* (see *Wai, PPM 9762*, f. 121 r., 5-8). So this text had to be supplied from the A-recension and the borrowed text of β had to be placed in the Appendix #17 because of the reasons stated before (see our commentary to verse 53b).

Similarly, some portion of Sūrya's text belonging to the demonstration part has also been borrowed by the β-recension from Sūrya's above mentioned commentary on Bhāskara's *Līlāvatī* (See *Wai, PPM 9762*, f. 121 r., 9 to f. 121 v., 5). This β-text repeats with omissions what A-text says in detail. Consequently, the β-text is put in the Appendix #18, and the A-text has been chosen.

Mathematical meaning: In the computation of the position of a planet, $-s$ is the remainder of seconds (of arc) after a period of x_5 elapsed days. Therefore

$$\frac{60\,x_1 - s}{d} = y_1 = \text{total seconds per civil day;}$$

x_1 = number of minutes.

$$\frac{60\,x_2 - x_1}{d} = y_2 = \text{total minutes per civil day;}$$

x_2 = number of degrees.

$$\frac{30\,x_3 - x_2}{d} = y_3 = \text{total degrees per civil day;}$$

x_3 = number of zodiacal signs.

$$\frac{12\,x_4 - x_3}{d} = y_4 = \text{total zodiacal signs per civil day;}$$

x_4 = number of revolutions.

$$\frac{a\,x_5 - x_4}{d} = y_5 = \text{number of revolutions per civil day;}$$

x_5 = number of elapsed days;

a = number of revolutions in d civil days.

Comments: In the present verses, Bhāskara is proclaiming the method of computing the number of elapsed days of a planet from the remainder of seconds and so on. Similar verses have been stated by other mathematicians. These include the following:

Āryabhaṭa II's *MS* XVIII, 32b-35:

भगणाद्यग्राणि स्युः क्षेपा ऋणसंज्ञकाः क्वहाश्छेदः ॥३२b॥

भगणादीनां भाज्या भगणा यंसा गना तना तेना ।
विकलाशेषोत्पन्नं फलं विलिप्ता गुणः कलाशेषम् ॥३३॥

लिप्ताग्रोत्पन्नफलं लिप्ता गुणकोऽंशशेषं स्यात्।
लवशेषजफलमंशा गुणको राश्यग्रकं भवति ॥३४॥

राश्यग्रोत्पन्नफलं गृहाणि गुणको भवेद्गृहशेषम्।
मण्डलशेषप्रभवं फलं च चक्राण्यहर्गणो गुणकः ॥३५॥

Śrīpati's *SSE* XIV, 30-31 and *SSE* XX, 22:

चक्रर्क्षभागकलिकाविकलादिशेष-
मग्रं स्वहारविहृतं भगणादिभक्तम्।
न्यूनाग्रमत्र हि फलं भगणादिनाप्तं
लब्धं भवेद्दिनगणस्त्वपवर्तिते स्यात् ॥३०॥

युगाद्व्यतीतानयनं तथैव
त्र्यादिग्रहाणामपि कुट्टकेन।
--- --- ---
--- --- ॥३१॥

यो राशिशेषादथ भागशेषा-
ल्लिप्ताविलिप्तोद्भवशेषतो वा।
अहो गतं तत्परशेषतोऽपि
जानाति खेटं च स कुट्टकज्ञः ॥२२॥

Brahmagupta's *BSS* XVIII, 7:

भगणादिशेषमग्रं छेदहृतं सं च दिनजशेषहृतम्।
अनयोरग्रं भगणादिदिनजशेषोद्भूतं द्युगणः ॥७॥

Miśra (1947) comments that the former Indian mathematicians dealt with kuṭṭaka mainly for the sake of the above mentioned planetary computations (*Part II*, p. 129).

In the demonstration part, the sūtra cited by Sūrya from Bhāskara's *Siddhāntaśiromaṇi* is *GG I* Madhyamādhikāra, 4a (see Āpaṭe, 1939, *ASS 110*, p. 30). This sūtra involves the method of proportion.

The mathematical procedures connected with Bhāskara's verses 64c-65d, and Sūrya's demonstration on them, will be clear from the example which follows Sūrya's demonstration. This example we discuss next. Note that this example does not exist in Vidyāsāgara's (1878) edition of Bhāskara's *Bījagaṇita*, nor does it exist in Kṛṣṇa's commentary, the *Bījapallava*. This indicates that it is composed by Sūrya himself. Bhāskara claims to have given the relevant examples in his Tripraśnādhyāya. He says (see Bhāskara's commentary (*BG*, p. 39, 1) on his *BG*, 64c-65d, p. 38; or Bhāskara's commentary (*L II, ASS 107*, p. 271, 1) on his *L II*, 258, pp. 268-269): "अस्योदाहरणानि त्रिप्रश्नाध्याये।" However Śaṅkara (ca. 1500 – 1560 A.D.) and Nārāyaṇa (ca. 1540 – 1610 A.D.), the authors of the commentary entitled *Kriyākramakarī* (ca. 1556 A.D.) on Bhāskara's *Līlāvatī*, have 'Praśnādhyāya' instead of 'Tripraśnādhyāya' in their text pertaining to Bhāskara's gloss called *Vāsanā* on the present verses 64c-65d (see Sarma, 1975). Their text is the following: "अस्योदाहरणानि प्रश्नाध्याये द्रष्टव्यानि।" Furthermore, Śaṅkara and Nārāyaṇa comment that Praśnādhyāya is the eighth chapter of the *Mahābhāskarīya* (which was written by Bhāskara I) and they quote verse 13 of this chapter (along with its solution). This verse begins with: "नीता रवेर्बलवता" (*L, VIS 66*, p. 451). Sūrya's example is different from this example. Like Śaṅkara and Nārāyaṇa, Miśra (1947) mentions in his commentary on Śrīpati's *SSE* XX, 22 that, Bhāskara stated in his *Bījagaṇita* and *Līlāvatī* (see *Part II*, p. 331): "अस्योदाहरणानि प्रश्नाध्याये ।"

We now introduce the various *grammatical indicators* in connection with Sūrya's example.

Textual problems: The Apparatus Criticus shows that some parts of Sūrya's solution are missing from the β-recension. Perhaps they were missing from a later copy of the *Sūryaprakāśa* to which the author of β had access.

Mathematical meaning: Given that the imagined revolutions of a planet are 3, the civil days are 11 and the elapsed days are 3, to determine the remainder of the seconds. Conversely, to compute the position of the planet from the remainder of the seconds.

Setting out: Revolutions of the planet = 3; civil days = 11; elapsed days = 3.

For the first part of the solution, we are considering the proportion: If by 11 civil days 3 revolutions are obtained, then by 3 (elapsed) days what is obtained? So, employing repeatedly the sūtra *GG I* Madhyamādhikāra, 4a (which has been stated by Sūrya in his demonstration part, and involves the method of proportion), we will obtain (the mean longitude of the planet as) 0 revolutions, 9 signs, 24 degrees, 32 minutes, 43 seconds and 7 as remainder of seconds, in the following manner:

(i). $(3 \cdot 3) \div 11 = 0R9$, where the quotient is 0 revolutions and 9 is the remainder of revolutions.

(ii). $(9 \cdot 12) \div 11 = 9R9$, where the quotient is 9 signs and 9 is the remainder of signs.

(iii). $(9 \cdot 30) \div 11 = 24R6$, where the quotient is 24 degrees and 6 is the remainder of degrees.

(iv). $(6 \cdot 60) \div 11 = 32R8$, where the quotient is 32 minutes and 8 is the remainder of minutes.

(v). $(8 \cdot 60) \div 11 = 43R7$, where the quotient is 43 seconds and 7 is the remainder of seconds.

Conversely, for the second part of the solution, following verses 64c-65d, we should assume the remainder of seconds (which is 7) as subtractive, 60 as dividend and 11 as the divisor. Going backwards thus and applying the method of (constant) kuṭṭaka repeatedly, we can find from the remainder of seconds, the number of elapsed days and revolutions of the planet, which determines the position of the planet. In particular, employing kuṭṭaka repeatedly, we will be solving the following indeterminate equations, and in each case obtaining *one* solution as shown:

$60x - 7$ (sec) $= 11y$; $x = 8$ (surplus of) minutes, $y = 43$ seconds.

$60x - 8$ (min) $= 11y$; $x = 6$ (surplus of) degrees, $y = 32$ minutes.

$30x - 6$ (deg) $= 11y$; $x = 9$ (surplus of) signs, $y = 24$ degrees.

$12x - 9$ (signs) $= 11y$; $x = 9$ (surplus of) revolutions, $y = 9$ signs.

$3x - 9$ (revolutions) $= 11y$; $x = 3$ (elapsed) days, $y = 0$ revolutions.

Comments: Sūrya is referring to this example in his *Gaṇitāmṛtakūpikā* when he says: तद्बीजभाष्ये सोदाहरणत्वेन व्याख्यातमतोऽत्र संक्षिप्योक्तम् (see *Wai, PPM 9762*, f. 121 v., 9), which means: "This is explained with an example in the commentary on the *Bīja*. For this reason, here it is described briefly". Furthermore, the verse

लब्धयो विषमा यत्र क्षेपः शुद्धिर्भवेद्यदि ।
यौ तत्र लब्धिगुणकौ तावेव हि परिस्फुटौ ॥

is the last of the nine concise verses stated by Sūrya (before Bhāskara's verse 51c-52b). Sūrya has quoted this verse from his *Gaṇita*, for he mentions this in his *Gaṇitāmṛtakūpikā* (see *Wai, PPM 9762*, f. 118 v., 5-6): तदुक्तमस्माभिः स्वगणिते ।

(c). *The Conjunct Kuṭṭaka (Verses 66a-67d).*

This refers to the method for solving the equations which arise in connection with a problem of the first kind i.e. a problem which involves remainders on division of a given number. These equations give rise to simultaneous indeterminate equations of the first degree, in which the divisor stays the same.

Verse 66a-d. Textual problems: The Apparatus Criticus shows that after Sūrya's explanation of the meaning of the present verse, text β has a disorder because it contains an explanation of the next verse (67a-d) before its lemma. The first sentence (with slight change) and the last two sentences of this text β are in the *Gaṇitāmṛtakūpikā* (*Wai, PPM 9762*, f. 122 r., 4-6). So we have discarded this text β and placed it in the Appendix #19. Text A has been chosen. But text A omits the introduction, lemma and explanation to the next verse (67a-d). Consequently, *Text Alpha* has a *lacuna* at this point.

Mathematical meaning: Given the two equations

$$\frac{a_1 x}{b} = y_1 + \frac{c_1}{b}, \quad \text{i.e.,}\, by_1 = a_1 x - c_1,$$

$$\frac{a_2 x}{b} = y_2 + \frac{c_2}{b}, \quad \text{i.e.,}\, by_2 = a_2 x - c_2,$$

where x is the unknown quantity or dividend, solve the equation $by = (a_1 + a_2)\,x - (c_1 + c_2)$ by using the pulverizer.

Comments: The implication is that in this verse, though a_1 and a_2 are multipliers, the sum of the "multipliers" (i.e. $a_1 + a_2$) is to be called the dividend, because, in order to solve this problem by the method of kuṭṭaka the notation needs to be changed as usually the dividend is known (and not the multiplier). The sum of the remainders is to be assumed to be the negative additive. The quotient obtained will be equal to the sum of the quotients y_1 and y_2 (which may be found by the use of the kuṭṭaka).

Such a kuṭṭaka is called *saṃśliṣṭa* (conjunct, expanded), because it refers to that multiplier or quantity which is obtained using addition of the given multipliers and remainders separately.

In line 66a, the word 'two multipliers' (गुणकौ) suggests an *ellipsis*, for the number of multipliers can be more than two. Sūrya (or Bhāskara) does not state anything to this effect.

Āryabhaṭa II's equivalent of Bhāskara's present verse 66 is *MS* XVIII, 48b-49a:

गुणकैक्यं संश्लिष्टे भाज्यः शेषैक्यकं भवेत् क्षेपः।
तुल्यच्छेदे कर्म मन्दार्थं कथ्यते विततः ॥४८b-४९a॥

Verse 67a-d. Textual problems: See under verse 66a-d.

Mathematical meaning: Solve for x the simultaneous equations

$$\frac{5x}{63} = y_1 + \frac{7}{63} \quad \text{and} \quad \frac{10x}{63} = y_2 + \frac{14}{63}$$

i.e. $63y_1 = 5x - 7$ and $63y_2 = 10x - 14$.

Setting out:

Multiplier	5	Remainder	7	|	Multiplier	10	Remainder	14
Divisor	63.			|	Divisor	63.		

Solving by kuṭṭaka the equation $(5 + 10)x - (7 + 14) = 63y$, i.e., $21y = 5x - 7$, we get $x = 14$ and $y = 3$.

Comments: One can find using kuṭṭaka that $y_1 = 1$ and $y_2 = 2$ so that $y = y_1 + y_2$. Also, $x = 14$ satisfies the given equations as well.

Ayyangar (1929 – 30) has remarked that the least value of x satisfies the given equations only when $a_1 + a_2$ is prime to b (though in Bhāskara's above problem it satisfies them by chance).. Ayyangar has given the following example: If the given equations be $5x - 7 = 63y_1$ and $30x - 42 = 63y_2$, then (adding) $35x - 49 = 63y$ or

$5x - 7 = 9y$. This last equation gives least $x = 5$. But this x does not satisfy the given equations. This is so because $a_1 + a_2 = 5 + 30 = 35$ is not prime to $b = 63$. So we have to try the next higher value, $x = 5 + 9 = 14$, which satisfies the given equations (*JIMS/NQ 18*, p. 7).

One can check that a solution of $5x - 7 = 63y_1$ is $x = 14$, $y_1 = 1$. Also a solution of $30x - 42 = 63y_2$ is $x = 14$, $y_2 = 6$. In addition, the least solution of $35x - 49 = 63y$ is $x = 5$, $y = 2$. Obviously, $y = 2 \neq 1 + 6 = y_1 + y_2$. The next higher solution of $35x - 49 = 63y$ is $x = 14$, $y = 7$. Clearly, $y = 7 = 1 + 6 = y_1 + y_2$. Also $x = 14$ satisfies the two given equations.

Ayyangar (1929 – 30) believes that Bhāskara overlooked this possibility and evidently so did Sūrya because in Bhāskara's example, 5 + 10 = 15 is not prime to 63 and yet the smallest x by chance works. Gaṇeśa, in his commentary *Buddhivilāsinī* (1545 A.D.) on Bhāskara's *Līlāvatī*, noticed this possibility and warns the reader (*JIMS/NQ 18*, p. 7).

A modern author Ganguly (1926) claims that Bhāskara considered the general case of a conjunct pulverizer, when the divisors also vary. In other words, Bhāskara considered the general case of simultaneous indeterminate equations of the first degree which might be stated as $a_1x \pm c_1 = b_1y_1, a_2x \pm c_2 = b_2y_2$, and so on. He says that he has found four palm-leaf manuscript copies of Bhāskara's *Līlāvatī* which contain the same rule and the same illustrative example pertaining to the general case:

संश्लिष्टबहुसामान्यकुट्टकसूत्रम् -

हारे विभिन्ने गुणके च भिन्ने

स्यादाद्यराशेर्गुणकस्तु साध्यः ।

द्वितीयभाज्यघ्नतदाद्यजो गुणः

क्षेपो भवेत् क्षेपयुतो द्वितीये ॥

द्वितीयभाज्यघ्नतदाद्यहारो
भाज्यो भवेत्तत्र हरो हरः स्यात्।
एवं प्रकल्प्यापि च कुट्टकेऽथ
जातो गुणश्चाद्यहरेण निघ्नः ॥
गुणो भवेदाद्यगुणेन युक्तो
हरघ्नहारोऽत्र हरः प्रदिष्टः।
अथ तृतीयेऽपि तथैव कुर्या-
देवं बहूनामपि साधयेत्तु ॥

अत्रोदाहरणमाह।

कः सप्तनिघ्नो विहृतो द्विषष्ट्या
त्रिकावशेषोऽथ स एव राशिः।
षडाहतः सैकशतेन भक्तः
पञ्चाग्रकश्चाथ स एव राशिः ॥
अष्टाहतः सप्तशशाङ्कभक्तो
नवाग्रको मे वद राशिसंख्याम्।
धनाग्रकेणापि तदेव राशेः
किं स्याद्धनं कुट्टविधानमाशुः ॥

Two of these manuscripts are from Puri, in Oriya characters, with the commentary of an Oriya mathematician Śrīdhara Mahāpātra, written in 1717 A.D. The other two are in Andhra characters and are preserved in the Oriental Libraries of Madras and Mysore. One of Ganguly's arguments in support of his claim is that if some example was missing in some copies of the *Līlāvatī* which Mahāpātra consulted, then Mahāpātra added a note: "इदमुदाहरणं क्वचित् क्वचित् पुस्तके न दृश्यते।" But no such note corresponds to the case under consideration; so this suggests that the case occurred in the copy of Gaṇeśa's commentary which Mahāpātra had consulted and criticized. Moreover, when

Mahāpātra provided any rules or examples which were not attributed by him to Bhāskara, he made clear indications to that effect, such as: "अथ ग्रन्थकृताऽनुक्तमपि केनचित् कृतमुदाहरणान्तरं दर्शयामः।" "मम तातपादा अपि।" "केशवोऽप्याह।" (*BCMS 17*, pp. 90-91, 97-98)

However, Ayyangar (1929 – 30) thinks that the rule and example could belong to some commentator and not to Bhāskara because (i) they have not been mentioned by any earlier commentator of the *Līlāvatī* and (ii) they do not appear in any manuscript of the *Bījagaṇita* though the same treatment of kuṭṭaka occurs in these two works (*JIMS/NQ 18*, pp. 3-5). On the other hand, Datta and Singh (1962) accept Ganguly's conclusion because the four manuscript copies do not seem to have been drawn from the same source (*Part II*, p. 140).

In the matter under discussion, Ganguly's arguments seem to carry more weight than those of Ayyangar. The original manuscripts of Bhāskara's text and commentaries thereon are not available. It is likely that the rule disappeared from the manuscripts because it had no application to astronomy.

Verse without number. Comments: This last verse has been composed by Sūryadāsa. It is known as the verse of upasaṃhāra (उपसंहार) and marks the end of the third chapter of the *Text Alpha*, that is, the section of the *Sūryaprakāśa* which deals with kuṭṭaka.

Colophon. Textual problems: This part of the *Sūryaprakāśa* has been supplied from the manuscripts of class A because it is missing from the manuscripts of class β.

Comments: This last sentence is the colophon which identifies the author and the work.

5. *Concluding Remarks.*

As was mentioned in the abstract to this thesis, our present study ends with the chapter on kuṭṭaka. The remaining adhyāyas included by Bhāskara in his *Bījagaṇita* discuss the following topics: indeterminate equations of the second degree (वर्गप्रकृतिः); the cyclic method (चक्रवालम्); linear equations (एकवर्ण समीकरणम्); quadratic equations (मध्यमाहरणम्); linear equations and quadratic equations having more than one unknown (अनेकवर्ण समीकरणम् and मध्यमाहरणभेदाः); operations with products of several unknowns (भावितम्); a section about the author Bhāskara and his work (ग्रन्थसमाप्तिः). The chapter of the *Sūryaprakāśa* following that of kuṭṭaka deals with some indeterminate equations of the second degree, in particular, equations of the form $Nx^2 \pm c = y^2$, known as vargaprakṛtiḥ (वर्गप्रकृतिः) or kṛti-prakṛtiḥ (कृतिप्रकृतिः) (literally, square nature). In the succeeding chapter, which is on चक्रवालम्, Bhāskara discusses the cyclic method for solving the indeterminate equation of the type $Nx^2 + 1 = y^2$. This method is so-called because of its iterative character, that is, the same sets of operations are applied again and again. Bhāskara says that this name is due to previous writers: "चक्रवालमिदं जगुः", meaning thereby the mathematicians Jayadeva and others.

It was stated in Chapter I.2.G., that the equation $Nx^2 + 1 = y^2$, which has been (incorrectly) called Pell's equation by many mathematicians over the years, has been called Jayadeva-Bhāskara equation by Selenius (1975, *HM 2*, p. 168), a modern historian of Mathematics. Srinivasiengar (1967) had suggested the name Brahmagupta-Bhāskara equation (p. 110). The translator Strachey (1816) had asserted that the Indian indeterminate analysis is based on the principle of continued fractions. Moreover, the knowledge involved in the Indian indeterminate analysis "was in Europe first communicated to the world by Bachet and Fermat in the seventeenth century, and by Euler and De La Lagrange in the eighteenth" (*AR 12*, pp. 160-161). Likewise, the translator Colebrooke (1817) had remarked that Euler and Lagrange had rediscovered the Indian methods (Dissertation, pp. xviii-xix). Selenius (1975) was so impressed by the achievements of the Indian mathematicians in this area that he remarked: "No European performances in the whole field of algebra at a time much later than Bhāskara's, nay nearly up to our times, equalled the marvellous complexity and ingenuity of Cakravāla" (*HM 2*, p. 180).

Therefore we wish to conclude by mentioning a few papers below which deal with वर्गप्रकृतिः and the famous method of चक्रवालम्, which the readers might find useful for studying beyond the topic of kuṭṭaka:

1. Datta, B. (1928). "The Hindu Solution of the General Pellian Equation." *Bulletin of the Calcutta Mathematical Society. Vol. 19*, No. 2, 87-94.
2. Ganguli, S. K. (1928). "The Source of the Indian Solution of the So-called Pellian Equation." *Bulletin of the Calcutta Mathematical Society. Vol. 19*, No. 4, 151-176.
3. Majumdar, P. K. (1981b). "Bījagaṇitam of Bhāskara II and the Continued Fraction." *Journal of the Asiatic Society of Bengal. Vol. 23*, Nos. 1-2, 124-136, 1981.
4. Potts, D. H. (1946). "Solution of a Diophantine System Proposed by Bhāskara." *Bulletin of the Calcutta Mathematical Society. Vol. 38*, 21-24.
5. Selenius, C. (1975). "Rationale of the Cakravāla Process of Jayadeva and Bhāskara II." *Historia Mathematica. Vol. 2*, 167-184.
6. Shukla, K. S. (1950). "On Śrīdhara's Rational Solution of $Nx^2+1=y^2$." *Gaṇita. Vol. 1*, No. 2, 1-12.
7. Shukla, K. S. (1954). "Ācārya Jayadeva, The Mathematician." *Gaṇita. Vol. 5*, No. 1, 1-20.
8. Van der Waerden, B. L. (1976). "Pell's Equation in Greek and Hindu Mathematics." *Russian Mathematical Surveys. Vol. 31*, No. 5, 210-225.

GLOSSARY

OF

TECHNICAL TERMS

अंकः	Number.
अंतःपातित्वं	Inclusion.
अंतःपाती	That which is within.
अंतरं	Difference.
अंतरकरणी	Surd of the difference.
अंतर्गत	Within.
अंतरज	Arising from the difference.
अग्रं	Surplus.
अधिक	Greater.
अधिमासः	Intercalary month.
अनंत	Infinite.
अनंतत्वं	Infinity.
अन्त्यं	Final.
अन्वित	Increased by.
अपवर्त्तः	Division, reduction.
अपवर्त्तनं	Reducer.
अपवर्त्तांकः	Reduction-number.
अपवर्त्तित	Reduced.
अपवर्त्त्यं	To be reduced (by a common measure), when one has reduced.
अभिहतिः	Multiplication, product.
अभीप्सित	Arbitrary.
अभीष्ट	Arbitrary, chosen.
अभीष्टा करणी	Chosen surd.
अभ्यासः	Multiplication.
अल्पा करणी	Small(er) surd.
अवमं	Omitted tithi.
अव्यक्त	Unmanifest, unknown.

अव्यक्तगणितं	Mathematics of the unknown (algebra), computation of the unmanifest.
अवशेषः	Remainder.
असमजातिक	Having different classes.
अहर्गणः	The number of elapsed days.
आगत	Obtained.
आगमनं	Arrival, acquisition.
आढ्य	United with, increased by.
आनयनं	Computation, derivation.
आनेतुं	To compute.
आनेयं	To be derived.
आप्तिः	Quotient.
आसन्नमूलं	Approximate square-root, near(er) square-root.
इच्छारूपत्व	Arbitrariness.
इष्ट	Arbitrary, chosen.
उचित	Correct.
उत्तरोत्तरं	Further and further.
उत्तरोत्तरः	Each succeeding one.
उत्पत्स्यमाना करणी	The karaṇī which is going to be produced.
उत्पद्	To produce.
उत्पन्न	Produced.
उत्पादकः	Generator.
उदयांतरः	Difference in risings.
उदाहरणं	Example.
उदाहृत	Exemplified.
उद्देशक्रमः	Order of instructions.
उद्धृत	Divided.
उपनिबद्ध	Composed.
उपन्यस्	To lay down.

उपपत्तिः .... Demonstration.

उपपन्न............................. Demonstrated.

उपान्तिम .... Penultimate.

उभयत्र............................. In both places.

उर्वरित .... Remaining.

ऊर्ध्व................................ The upper, the one higher.

ऊनित .... Diminished by.

ऋण .... Negative.

ऋणगत............................ That which has become negative.

ऋणत्वं............................ Negativity.

ऋणभाजक....................... Having a negative divisor.

एकसंख्यांक................... Numeral one.

ऐक्यं.............................. Sum.

करणी .... Surd.

कर्म................................ Operation.

कल्प .... Kalpa.

कविः.............................. Wise man, poet.

कुट्टः.............................. Pulverizer.

कुट्टकः........................... Pulverizer.

कुदिनं............................ Civil day.

कृत .... Devised.

कृतिः.............................. Square.

कृत्वा............................. When one has operated (in realization of the fact).

क्रमः.............................. Procedure.

क्रमेण............................ In order.

क्रमात्............................ In order.

क्रान्तिः.......................... Declination.

क्रिया .... Working.

क्वहः.............................. Civil day.

क्षयः.............................. Negative quantity.

क्षेपः.............................. Additive.

क्षेपकः.............................. Additive.

खं.............................. Zero.

खच्युत.............................. Subtracted from zero.

खण्डः, खण्डं.............................. Part.

खयुक्त.............................. Added to zero.

खहरः.............................. That which has zero as its divisor.

गणकः.............................. Calculator, mathematician.

गणितं.............................. Computation, calculation, mathematics.

गुणः.............................. Multiplier.

गुणकः.............................. Multiplier.

गुणकारः.............................. Multiplier.

गुणनं.............................. Multiplication.

गुण्य.............................. Multiplicand.

ग्रहगणितं.............................. Computation of (the longitudes of) planets.

ग्राह्य.............................. To be obtained.

घनः.............................. Cube.

घातः.............................. Multiplication, product.

घ्न.............................. Multiplied.

चतुष्टयं.............................. A quartet.

चरः.............................. Half-equation of daylight.

च्युत.............................. Subtracted.

छेदः.............................. Divisor.

ज्योतिःशास्त्रं.............................. Science of mathematics, astronomy and astrology.

तक्षणं.............................. Division.

तक्षणः.............................. Divisor.

तष्ट	Divided.
त्यज्	To subtract, abandon.
त्रयं	A triad.
दंडान्वयः	Logical order.
दिनं	Day.
दिवसः	Day.
दुष्ट	Faulty.
दृढ	Confirmed.
धन	Positive.
निघ्न	Multiplied.
नियमः	Certainty, necessity, rule.
नियोजनीय	Should be applied.
नित्य	Invariable.
निरग्र	Without a remainder.
निरग्रक	Without a remainder, without a surplus.
निरवशेष	Remainderless.
निष्पन्न	Obtained.
निरूप्	To investigate.
न्यायः	Reasoning.
न्यासः	Setting out, lay out.
न्यूनीकृ	To make smaller.
पदं	Square-root.
परस्परभजनं	Mutual division.
पूर्वपूर्व	Each preceding one.
पूर्ववत्	As before.
पृथक्	Separately.
पृथक्स्थितिः	Putting separately.
प्रकल्प्य	When one has assumed.
प्रकारः	Procedure.

प्रकृते	In the matter under discussion, in the case under discussion.
प्रच्युत............................	Subtracted.
प्रमाणं............................	Criterion.
प्रमित	Measured by.
प्रयोजना	Purpose.
प्रस्तुत	Described.
प्राक्तन	Previous, former.
प्राग्वत्...........................	As previously.
फलं	Result, quotient.
फलवल्ली	Chain of results.
बह्वंकः............................	Larger number.
बह्वी करणी...................	Great(er) surd.
बीजं...............................	Source, algebra.
भगणः............................	Revolution.
भजनं..............................	Division.
भागः..............................	Division, degree.
भागहारः.........................	Division, divisor.
भाजकः..........................	Divisor.
भाज्य..............................	Dividend.
भिन्नांकः..........................	Fractional number.
मध्यग्रह..........................	Mean (longitude of a) planet.
महती.............................	Great(er surd).
मान................................	Measure.
मिथःसमजातीयत्वं...........	Mutual homogeneity.
मूलं................................	Origin, root, square-root.
मूलकरणी	Root-surd.
मूलगतकरणी.................	Surd in a square-root.
मूलव्यवस्था....................	Condition of being the square-root.

यावत्तावत् .... An unknown, as much as, so much.
युक् .... Combined with.
युज् .... To combine with.
युक्त .... Joined with, increased by.
युत .... Combined with, joined, increased by.
युतिः .... Sum, additive.
योगः .... Addition, sum.
योगकरणी .... Surd of addition.
योगगतकरणी .... Surd in a sum.
योगज .... Arising from addition.
योगसूत्रं .... Addition-sūtra.
योज्य .... To be added.
रविः .... The Sun.
रहित .... Diminished by.
राशिः .... Heap, quantity, sign (of the zodiac).
राशियुग्मं .... Pair of quantities.
रूपं .... Form, (any) number, the number one.
लघु .... Small.
लघ्वी .... Small, small(er surd).
लघ्वीकरणं .... Diminishment.
लब्धं .... Result.
लब्धिः .... Quotient, result.
लवः .... Degree.
लाभः .... Result.
वधः .... Multiplication.
वर्गः .... Square, squaring.
वर्गमूलं .... Square-root.
वर्गराशिः .... Square-quantity.
वर्जित .... Diminished.

वर्णः................................ Colour.
विधिः.............................. Method, rule.
विपर्यास.......................... Opposite.
विभाज्य.......................... Dividend.
वियत्.............................. Zero.
वियोगः........................... Subtraction.
वियोगज.......................... Arising from subtraction.
विवर्जित......................... Decreased by, diminished.
विशुद्धिः.......................... Subtractive.
विश्लेषसूत्रं...................... Separation-sūtra.
विषम.............................. Odd (number).
विषमजातीयत्वं............. Non-homogeneity.
वृहत्.............................. Large.
वैपरीत्यं.......................... Reversal.
वैलोम्यं.......................... Inversion.
व्यक्त.............................. Manifest, known.
व्यक्तगणितं.................... Mathematics of the manifest (arithmetic).
व्यतिरेकः....................... Exclusion.
व्यत्ययं.......................... Reversal.
व्यवकलनं..................... Subtraction.
व्याकुल......................... Confused.
व्युत्पत्तिः........................ Definition.
शरः................................ Latitude.
शुद्ध................................ Subtracted.
शुद्धभाग.......................... Without a fractional part.
शुद्धिः.............................. Subtraction, the state of being reduced (without a remainder), subtractive.
शुध्................................ To subtract.
शून्यं............................... Zero.

शेष:, शेषं .... Remainder.

शेषविधि: .... Method of the remainder.

षड्विध .... Six-fold (operation).

संकलनं .... Addition.

संकलितमित .... Measured by a sum (or sums).

संक्रमणसूत्रं .... Concurrence-sūtra.

संख्या .... Enumeration, calculation, counting, number.

संख्याक .... Number.

संख्यानं .... Calculation.

संयुत .... Increased.

संशोध्यमान .... Going to be subtracted.

संश्लिष्ट .... Conjunct.

संस्कार: .... Correction.

सम .... Even (number).

समजातिक .... Having the same character or class.

समजातीयत्वं .... Homogeneity.

समपवर्त्तित .... Reduced together.

समानजाति: .... Having the same character.

समानीत .... Derived.

सवासन .... With demonstrations.

सहित .... Combined with.

सांख्यं .... Sāṅkhya (philosophy).

सांख्य: .... Sāṅkhya philosopher, wise man, Sāṅkhya (philosophy).

साध् .... To compute.

साधनं .... Computation.

साधित .... Established, accomplished.

साध्य .... To be obtained.

सिद्ध .... Established, accomplished, determined.

सूत्रं..................................	Sūtra, (short) rule, aphorism.
सूत्रक्रमः..........................	Procedure of the sūtra, rule of the sūtra, method of the sūtra.
सूत्रप्राप्तिः.........................	What is obtained from the sūtra.
स्थाप्य.............................	To be placed.
स्पष्ट.................................	Correct, clear.
स्पष्टीकरणं.....................	Computing the true (longitude).
स्फुट..............................	Accurate.
स्व..................................	Positive (quantity).
स्वरूपं.............................	Form.
हरः.................................	Divisor.
हारः................................	Divisor.
हृत.................................	Divided.
ह्रियमाण.........................	Being carried out.

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